

Multiobjective Optimal Control Methods for the Development of an Intelligent Cruise Control

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1 Introduction

Electrically powered vehicles (EV) have gained more and more attention during the last years due to an increasing awareness of the impact of CO₂ emissions on climate change and the limitation of fossil fuels. New research challenges arise due to limited battery storage densities, high battery costs and a considerably reduced range in comparison to conventionally powered vehicles. Therefore, range increasing driving strategies play an important role in electromobility (cf. e.g. [8]).

Different control and optimization strategies have been suggested for vehicle applications in the past, see [16] for an overview, [7, 9] for model predictive control for trucks, [5] for an application of dynamic programming, [13] for indirect or [3] for direct optimal control methods.

In this paper, an "intelligent cruise control" is developed by taking into account topographic data of a given travel route. We formulate the challenge of range extension as a multiobjective optimal control problem, transforming it into a multiobjective optimization problem by using a direct approach. We then use numerical methods to compute the so-called Pareto set of optimal compromises between the concurrent objectives "maximize battery charge" and "maximize driven distance". Pareto optimal accelerator pedal position profiles of the EV are computed by using two different multiobjective optimization methods.

The paper is organized as follows: In Section 2, the mathematical problem formulation and solution methods for multiobjective optimal control problems are given. The EV model and computational results are presented in Section 3 followed by a conclusion in Section 4.

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2 Multiobjective Optimal Control

Searching for a control strategy of an EV which maximizes the driving distance is an example of an *optimal control problem*. In general, the technical system is represented by a model, typically of the form $\dot{x}(t) = f(x(t), u(t))$. Further, the objectives subject to optimization have to be modeled. By convention, we always consider minimization problems. Typically, objective functionals are of the form

$$J(x, u) = \int_0^T C(x, u)dt + \Psi(x(T)) \quad (1)$$

with running costs $C(\cdot, \cdot)$ depending on the system's states x and controls u and a final cost $\Psi(\cdot)$ depending on the final state $x(T)$. Finally, there might be different kinds of constraints, e.g. boundary conditions $g(x(0), x(T)) = 0$ and box constraints $b_l \leq \begin{pmatrix} x(t) \\ u(t) \end{pmatrix} \leq b_u$ for all $t \in [0, T]$.

In many applications, there arise several objective functionals that have to be minimized simultaneously. This leads to vector-valued objective functionals, denoted by $\mathbf{J}(x, u)$ with $\mathbf{J} = (J_1, \dots, J_k)$, $k \geq 1$ and, for all $i \in \{1, \dots, k\}$, J_i as in (1). Altogether, we obtain a *multiobjective optimal control problem* (MOCP)

$$\min_u \mathbf{J}(x, u) \quad \text{w.r.t. } \dot{x} = f(x, u), \quad g(x(0), x(T)) = 0, \quad b_l \leq \begin{pmatrix} x(t) \\ u(t) \end{pmatrix} \leq b_u \quad \forall t \in [0, T].$$

The minimization of the vector valued functional $\mathbf{J}(x, u)$ is understood w. r. t. the partial order \leq_p on \mathbb{R}^k , defined as follows: Let $v, w \in \mathbb{R}^k$, then the vector v is *less than* w ($v \leq_p w$), if $v_i < w_i$ for all $i \in \{1, \dots, k\}$. The relation \leq_p is defined analogously. By this relation, we can introduce the concept of dominance and Pareto optimality (cf. [6], for instance).

Definition 1 (Dominated and Pareto Optimal Solutions). Let (x, u) and (x^*, u^*) be admissible points, i.e. they satisfy the restrictions of the MOCP.

- a) The point (x, u) is *dominated* by the point (x^*, u^*) w.r.t. $\mathbf{J}(x, u)$, if $\mathbf{J}(x^*, u^*) \leq_p \mathbf{J}(x, u)$ and $\mathbf{J}(x, u) \neq \mathbf{J}(x^*, u^*)$, otherwise (x, u) is *non-dominated* by (x^*, u^*) .
- b) The point (x^*, u^*) is called *Pareto optimal* if there exists no admissible (x, u) which dominates (x^*, u^*) .
- c) The set of all Pareto optimal points (x^*, u^*) is called the *Pareto set* and its image under \mathbf{J} the *Pareto front*.

For classical, i.e. single-objective optimal control problems, direct methods have shown to be well suitable in many applications (cf. [1], for instance). Such methods transform the control problem into a high dimensional optimization problem by a time discretization. For solving MOCPs, multiobjective optimization techniques have to be applied to the discretized problem, cf. e.g. [10, 12, 15]. A number of methods exist for the computation of single Pareto points (cf. [6] for an overview). To approximate the whole Pareto front, methods such as evolutionary algorithms

(cf. [2]), set-oriented techniques, or path following methods (cf. [4, 14, 15] and the short overview given below) can be applied.

To transform the MOCP into an optimization problem with multiple objectives, we introduce a discrete time grid $\Delta t = \{t_0 = 0, t_1, \dots, t_N = T\}$. The control u is approximated by a discrete control $u_d = \{u_k\}_{k=0}^{N-1}$ with u_k being an approximation of u on the interval $[t_k, t_{k+1}]$ for $k = 0, \dots, N-1$. A discrete state trajectory $x_d = \{x_k\}_{k=0}^N$ with $x_k \approx x(t_k)$ can be obtained by a numerical integration scheme, $x_{k+1} = \Phi_{t_k}^{t_{k+1}}(x_k, u_k)$ with $x_0 = x(0)$ and for $k = 0, \dots, N-1$. Together with an approximation of all objective functionals on the discrete time grid, we obtain a *multiobjective optimization problem*

$$\min_{u_d} J_d(x_d, u_d) = \sum_{k=0}^{N-1} C_d(x_k, u_k) + \Psi_d(x_N), \quad (2)$$

$$\text{w.r.t. } x_{k+1} = \Phi_{t_k}^{t_{k+1}}(x_k, u_k), \forall k < N, g_d(x_0, x_N) = 0, b_l \leq \begin{pmatrix} x_k \\ u_k \end{pmatrix} \leq b_u \forall k \leq N. \quad (3)$$

Set-Oriented Subdivision

The aim of the subdivision method is to approximate the Pareto set by a successive refinement and selection of boxes, cf. [4, 15]. The procedure starts with a box that covers the admissible set of optimization parameters. Then, subdivision and selection steps are applied alternatingly. In a subdivision step, all active boxes are subdivided into smaller boxes. For the selection, a number of test points are chosen in all boxes and the objective functions are evaluated. Then, all boxes not containing any non-dominated test points are deleted and one proceeds with the next subdivision step (cf. Fig. 1, left). This a gradient-free sampling technique which, amongst other set-oriented algorithms, is implemented in the software package GAIO [4, 15].

Scalarization by Reference Point Techniques

A discretization with a fine time grid Δt leads to a high number of optimization parameters $u_d = \{u_0, \dots, u_{N-1}\}$ in the transformed MOCP (2), (3). In this case, scalarization techniques have shown to be well suitable, cf. e.g. [10, 14]. More concretely, we apply a reference point method which defines auxiliary scalar optimization problems. To this aim, nonadmissible target points P in image space are defined, and the distance to the image of the admissible set is minimized,

$$\min_{u_d} \|J_d(x_d, u_d) - P\| \quad \text{w.r.t. constraints as in (3).}$$

As a result, single Pareto points on the boundary of the admissible set can be found. The target points are defined iteratively by a continuation method in image space as depicted in Fig. 1 to the right. These points are not necessarily Pareto optimal. How-

ever, dominated points can be easily eliminated by a subsequent non-dominance test. The auxiliary optimization problems can be efficiently solved by *sequential quadratic programming* (SQP) methods (cf. e.g. [1, Sect. 5.4] and the references therein).

While the set-oriented subdivision method works globally but it is restricted to moderate dimensions of the parameter space, SQP methods are suitable for high numbers of optimization parameters.

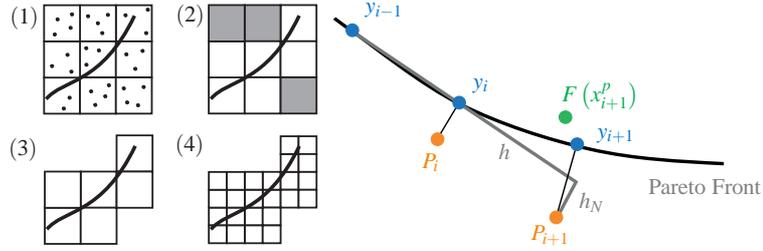


Fig. 1 *Left*: Subdivision method. Alternatingly, dominated boxes are removed and non-dominated boxes are subdivided. *Right*: Image space continuation method. If two points are known (say y_{i-1} and y_i), a target T_{i+1} is calculated. Then, the scalar minimization problem with initial guess x_{i+1}^p yields y_{i+1} .

3 Application to the Electric Vehicle

A Matlab/SIMULINK model is used to describe the EV dynamics. It is based on a drivetrain efficiency map and considers vehicle properties such as rolling friction and air drag, as well as environmental conditions like slope and ambient temperature. The model holds several state variables such as position, velocity and state of charge. These variables depend on the input variables accelerator pedal position profile u and the inclination profile α . For a more detailed model description, we refer to [3, 11].

Since we aim to compute the Pareto set for the objectives "final state of charge $SOC(T)$ " and "driven distance $s(T)$ " with a fixed final time T , we set the vector of objective functionals (cf. Eq. (1)) to $\mathbf{J}(x, u) = \Psi(x(T)) = (SOC(T), s(T))$. As an example scenario we choose a track with a periodic inclination profile superimposed by a linear increase:

$$\alpha(s) = 4^\circ \sin\left(360^\circ \frac{s}{2000 \text{ m}}\right) + 1^\circ \frac{s}{2000 \text{ m}}.$$

This defines the height profile h . In this way, we ensure that most of the Pareto points (except for the solutions with very short driven distances) are computed for tracks with both uphill and downhill sections.

To compute the Pareto set we apply the algorithms presented in Section 2. The two solutions $u(t) = 0$ and $u(t) = 100$, respectively, correspond to the two endpoints of the Pareto front, where one objective becomes minimal while the other becomes maximal. To improve numerical accuracy, these values have been used to normalize both objectives to the interval $[0, 1]$ with the optimum being 0. For the results shown in the following, the normalization has been reversed. In this case, a maximization of both objectives is desired.

We start the **subdivision algorithm** with a box of dimension n (number of parameters) with the center at 50 and a radius of 50 so that it covers the whole pedal position profile $u_i \in [0, 100]$, $i = 1, \dots, n$. We then apply $4n$ subdivision steps. Figure 2 shows the resulting Pareto front for different pre-image dimensions on the left and one EV simulation with a Pareto optimal pedal position profile and the resulting velocity profile on the right. As has been observed before (cf. [3]), a high engine torque on positive slopes but lower torque on negative slopes is beneficial to the energy consumption.

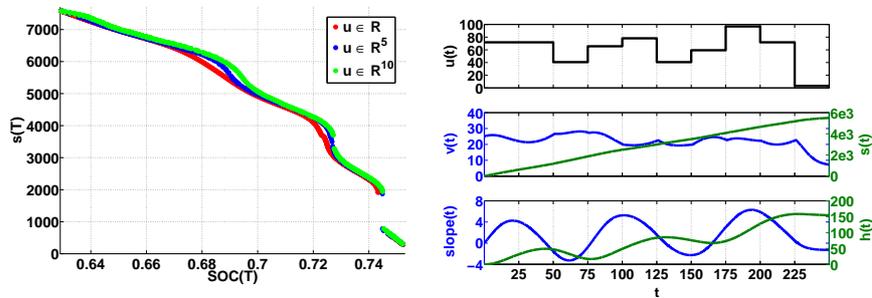


Fig. 2 *Left*: Pareto front computed by the subdivision algorithm for different pre-image dimensions (boxes represented by their center points). *Right*: EV simulation with a Pareto optimal pedal position profile ($u \in \mathbb{R}^{10}$, $SOC(T) = 0.6914$, $s(T) = 5800$ m).

It is obvious that solutions with a higher pre-image space dimension always have to be at least as good as the lower dimensional solutions (cf. Fig. 2). Additionally, the difference between the solutions is largest in the middle section. This is due to a higher variability in this part while near the ends of the front, the pedal position has to be close to the maximal or minimal value at all times.

When looking at the Pareto points around $SOC(T) \approx 0.745$ (as well as $SOC \approx 0.725$ for $u \in \mathbb{R}^{10}$), one observes a gap which is caused by the EV's recuperation technique. The last point at the low distance part of the Pareto front corresponds to a stop at the top of a hill. Increasing the pedal position profile only slightly results in a final position with a negative inclination $\alpha(s(T))$. Since the EV can roll down the slope and recharge its battery via recuperation, a slight reduction of the objective $SOC(T)$ leads to a huge increase of the second objective $s(T)$ which then results in a gap in the front. The varying inclination of the Pareto front is a result of the track slope alternating between positive and negative values.

It should be mentioned that due to the relatively long EV simulation time (≈ 1 s), the number of testpoints for each box was set to a comparably low value of 30 which may cause boxes to be either eliminated or identified as non-dominated by mistake. The first case leads to spurious gaps in the Pareto front (cf. e.g. the Pareto front of $u \in \mathbb{R}^{10}$ in Fig. 2 for $SOC(T) \approx 0.68$) while the second case leads to boxes apart from the Pareto front.

A comparison of the results of the subdivision and the **image space continuation algorithm** (cf. Fig. 3, left) shows good agreement for the case $u \in \mathbb{R}^{10}$, indicating that the image space continuation method also yields good results despite its local nature. Having shown the continuation algorithm's applicability, Pareto sets of higher dimension are computed (cf. Fig. 3, left).

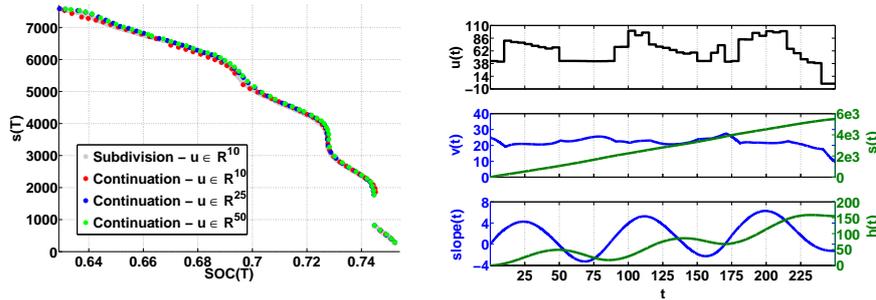


Fig. 3 *Left*: Pareto front computed by the image space continuation algorithm for different dimensions of pre-image space. *Right*: EV simulation with a Pareto optimal pedal position profile ($u \in \mathbb{R}^{50}$, $SOC(T) = 0.6934$, $s(T) = 5750$ m)

To improve the simulation time and convergence rate, each Pareto point from a lower pre-image space dimension serves as the initial guess for the next higher dimensional solution. As can be seen in Fig. 3, the resulting improvements become smaller quickly. The choice of the pre-image space dimension should be considered carefully since computation time increases significantly with the number of optimization variables. This effect is even strengthened by an observed decreasing convergence rate for high-dimensional cases, presumably caused by inaccuracies in the numerical differentiation of the EV model required for the SQP method.

4 Conclusion

In this paper, we apply two MOCP algorithms for the development of an intelligent cruise control. Pedal position profiles can be chosen as optimal compromises between energy consumption and travel distance.

For future work, it will be interesting to compute the Pareto set with a constant travel distance instead of a constant driving time. Moreover, Model Predictive Control methods can be applied to realize real time optimization.

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