Investigations into Bluetooth Low Energy Localization Precision Limits

Joerg Schmalenstroer and Reinhold Haeb-Umbach
University of Paderborn, Germany
http://nt.uni-paderborn.de

Bluetooth Low Energy (BLE) localization system for location-based services

- Task: Precise localization & tracking (2D) of persons
- Approach: Use RSSI measurements of BLE beacons (positions known) to estimate the current position of the smartphone
- Application fields: Industry 4.0, indoor navigation, shopping,...

Contributions of this publication

- Study on influence of directional radio patterns of Bluetooth Low Energy beacons on localization precision
- Investigations into optimal beacon network planning to minimize localization error

BLE RSSI Model

- BLE beacons emit periodically advertisement packages with predefined signal strengths
- Log-normal fading model: \( P = P_0 - 10 \eta \ln (d/d_0) + n = \mathcal{P} + n \)
  - \( P_0 \): RSSI level at reference distance \( d_0 \)
  - \( \eta \): Room specific constant
  - \( \mathcal{P} \): Mean RSSI value at distance \( d \)
  - \( n \): Measurement noise (\( n \sim \mathcal{N}(0, \sigma_n^2) \))

![Measured RSSI values in dB at 1 m distance in antenna lab for different azimuth and elevation angles.]

Figure 1: Measured RSSI values in dB at 1 m distance in antenna lab for different azimuth and elevation angles.

- Antenna lab measurements show significant dependency on azimuth and elevation angles
- Angular dependent RSSI model (2D)
  - Assume beacons and sensors to be on a common plane
  - Describe \( P_0(\alpha) \) by Fourier polynomial
    \[
    P_0(\alpha) = \frac{a_0}{2} + \sum_{k=1}^{M} a_k \cos(k\alpha) + b_k \sin(k\alpha)
    \]
    with \( a_k = \frac{2}{\pi} \int P_0(\alpha) \cos(k\alpha) \) and \( b_k = \frac{2}{\pi} \int P_0(\alpha) \sin(k\alpha) \)

![Fourier polynomial approximation]

Figure 2: Measurements and Fourier polynomial approximation

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Cramér-Rao lower bound

- Probability density function of RSSI values at user position \( U_j \)
  \[
  p_{\mathcal{P}}(P_{ij}|U_j) = \frac{1}{\sqrt{2\pi\sigma_n}} e^{-\frac{1}{2\sigma_n^2}(P_{ij} - \mathcal{P})^2}
  \]
- Log likelihood: \( I_j = -\sum_{i=1}^{N} \ln \left( \sqrt{2\pi\sigma_n} \right) - \frac{1}{\sigma_n^2} \sum_{i=1}^{N} (P_{ij} - \mathcal{P})^2 \)
- CRLB of \( k \)-th dim at \( U_j \): \( \text{var}(\hat{U}_{jk}) \geq (J^\dagger(U_j))_{kk} \) with
  \[
  J(U_j) = -E \left[ \begin{pmatrix} \frac{\partial^2}{\partial u_i^2} I_j & \frac{\partial^2}{\partial u_i \partial u_j} I_j & \frac{\partial^2}{\partial u_j \partial u_i} I_j & \frac{\partial^2}{\partial u_j^2} I_j \end{pmatrix} \right].
  \]
- Lower bound for RMSE of \( \hat{U}_j \): \( \text{RMSE}(\hat{U}_j) \geq \sqrt{\text{tr}(J^\dagger(U_j))} \)

Simulation results

- Experiment: Random orientations for predefined geometry
  - Localization precision depends on sensor direction selection
  - Grid based localization approach
    1. Accumulate log-likelihoods of RSSI observations at grid points
    2. Select grid point with max. log-likelihood sum as position hypothesis

![Comparison between root CRLB and RMSE of grid approach estimator.]

Figure 3: Comparison between root CRLB and RMSE of grid approach estimator.

Sensor placement optimization

- Task: Find beacon placement with minimum expected RMSE
  - Use evolutionary optimization to handle computational complexity

![Genetic optimization procedure and histogram of average root CRLB values of 1000 randomly generated geometries, and the result of the evolutionary algorithm.]

Beacon network setup

- \( N \) Bluetooth low energy beacons distributed in room
  - Beacon positions: \( S_i = [s_{ix}, s_{iy}]^T \)
  - User position: \( U_j = [u_{ix}, u_{iy}]^T \)
  - Distance to \( i \)-th beacon: \( d_j = \sqrt{(s_{ix} - u_{ix})^2 + (s_{iy} - u_{iy})^2} \)
  - Angle to \( i \)-th beacon: \( \alpha_j = \arctan(\frac{(s_{iy} - u_{iy})}{(s_{ix} - u_{ix})}) \)

Conclusions

- Fourier polynomial based radiation pattern description
- Analytic solution for CRLB of position estimator
- Cramér-Rao lower bound used for network geometry planning
- Evolutionary optimization procedure enables network planning for large-scale setups