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Investigations into Bluetooth Low Energy Localization Precision Limits

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Introduction

- Bluetooth Low Energy (BLE) localization system for location-based services
  - Task: Precise localization & tracking (2D) of persons
  - Approach: Use RSSI measurements of BLE beacons (positions known) to estimate the current position of the smartphone
  - Application fields: Industry 4.0, indoor navigation, shopping, ...
- Contributions of this publication
  - Study on influence of directional radio patterns of Bluetooth Low Energy beacons on localization precision
  - Investigations into optimal beacon network planing to minimize localization error

BLE RSSI Model

- BLE beacons emit periodically advertisement packages with predefined signal strengths
- Log-normal fading model: $P = P_0 - 10\,\eta\,\ln(d/d_0) + n = \overline{P} + n$
  - P_0: RSSI level at reference distance d_0
  - η: Room specific constant
  - $\overline{P}$: Mean RSSI value at distance d
  - n: Measurement noise ($n \sim \mathcal{N}\left(0, \sigma_n^2\right)$)

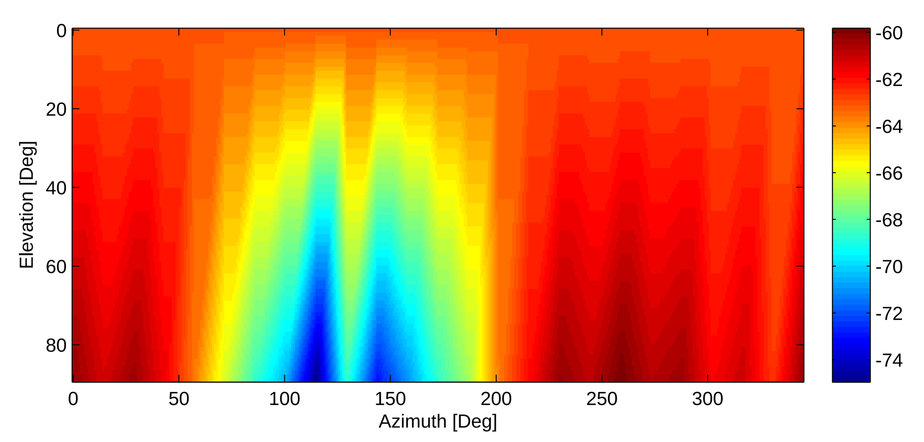
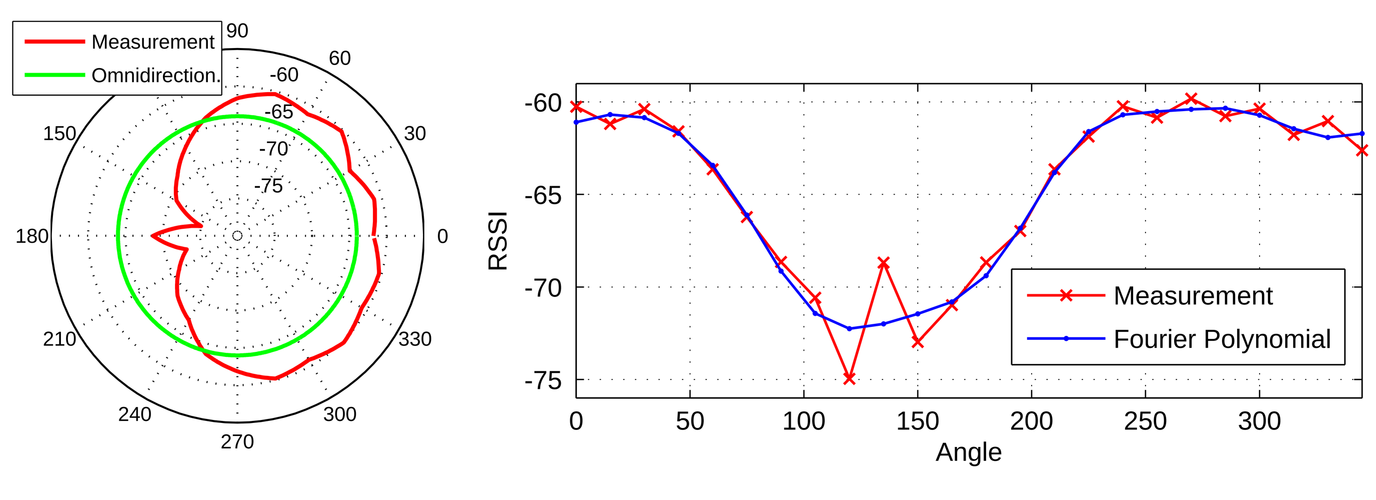


Figure 1 : Measured RSSI values in dB at 1 m distance in antenna lab for different azimuth and elevation angles.

- Antenna lab measurements show significant dependency on azimuth and elevation angles
- Angular dependent RSSI model (2D)
  - Assume beacons and sensors to be on a common plane
  - Describe $P_0(\alpha)$ by Fourier polynomial

$$P_0(\alpha) = \frac{a_0 + a_m}{2} + \sum_{k=1}^{m} a_k \cos(k\alpha) + b_k \sin(k\alpha)$$

with $a_k = \frac{2}{M}\sum_{i=1}^{M} P(\alpha_i)\cos(k\alpha_i)$ and $b_k = \frac{2}{M}\sum_{i=1}^{M} P(\alpha_i)\sin(k\alpha_i)$



(a) Measurements (b) Fourier polynomial approximation

Figure 2 : Comparison between omnidirectional radiation characteristic ((a) green line), and laboratory measurements ((a) and (b), red curve); approximation of measured radiation pattern by Fourier polynomial of order $m = 6$ ((b) blue curve).

Beacon network setup

- N Bluetooth low energy beacons distributed in room
  - Beacon positions: $\boldsymbol{S}_i = [s_{x,i}, s_{y,i}]^{\mathsf{T}}$
- User position: $\boldsymbol{U}_j = [u_{x,j}, u_{y,j}]^{\mathsf{T}}$
  - Distance to i-th beacon: $d_{ij} = \sqrt{(s_{x,i} - u_{x,j})^2 + (s_{y,i} - u_{y,j})^2}$
  - Angle to i-th beacon: $\alpha_{ij} = \mathrm{atan2}\left((s_{y,i} - u_{y,j}), (s_{x,i} - u_{x,j})\right)$

Cramér-Rao lower bound

- Probability density function of RSSI values at user position $\boldsymbol{U}_j$

$$p_{P|\boldsymbol{U}}(P_{i,j}|\boldsymbol{U}_j) = \frac{1}{\sqrt{2\pi}\sigma_n} e^{-\frac{1}{2\sigma_n^2}\left(P_{i,j} - \overline{P}_{i,j}\right)^2}$$

$$\overline{P}_{i,j} = P_0(\alpha_{i,j}) - 5\eta\ln\left((s_{x,i} - u_{x,j})^2 + (s_{y,i} - u_{y,j})^2\right)$$

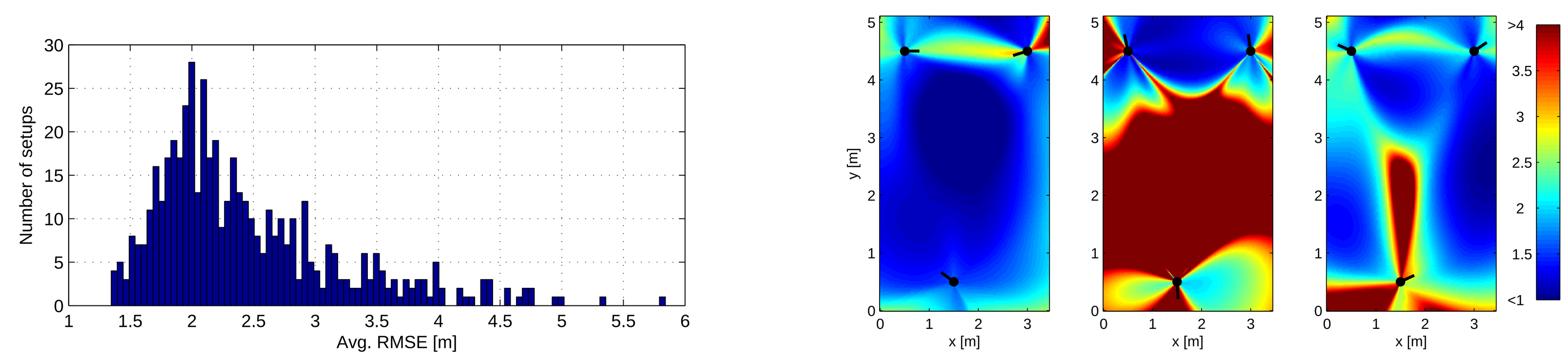
- Log likelihood: $l_{\boldsymbol{U}_j} = -\sum_{i=1}^{N} \ln\left(\sqrt{2\pi}\sigma_n\right) - \frac{1}{2\sigma_n{}^2}\sum_{i=1}^{N}\left(P_{i,j} - \overline{P}_{i,j}\right)^2$
- CRLB of k-th dim at $\hat{\boldsymbol{U}}_j$: $var(\hat{\boldsymbol{U}}_{j,k}) \geq (J^{-1}(\boldsymbol{U}_j))_{kk}$ with

$$J(\boldsymbol{U}_j) = -\mathrm{E}\left\{\begin{bmatrix} \frac{\partial^2}{\partial u_{x,j}\partial u_{x,j}} l_{\boldsymbol{U}_j} & \frac{\partial^2}{\partial u_{x,j}\partial u_{y,j}} l_{\boldsymbol{U}_j} \\ \frac{\partial^2}{\partial u_{y,j}\partial u_{x,j}} l_{\boldsymbol{U}_j} & \frac{\partial^2}{\partial u_{y,j}\partial u_{y,j}} l_{\boldsymbol{U}_j} \end{bmatrix}\right\}.$$

- Lower bound for RMSE of $\hat{\boldsymbol{U}}_j$: $\mathrm{RMSE}(\hat{\boldsymbol{U}}_j) \geq \sqrt{\mathrm{tr}\left\{J^{-1}(\boldsymbol{U}_j)\right\}}$

Simulation results

- Experiment: Random orientations for predefined geometry
  - Localization precision depends on sensor direction selection



(a) Histogram of average root CRLB (b) Best, worst and an average result

- Grid based localization approach
  1. Accumulate log-likelihoods of RSSI observations at grid points
  2. Select grid point with max. log-likelihood sum as position hypothesis

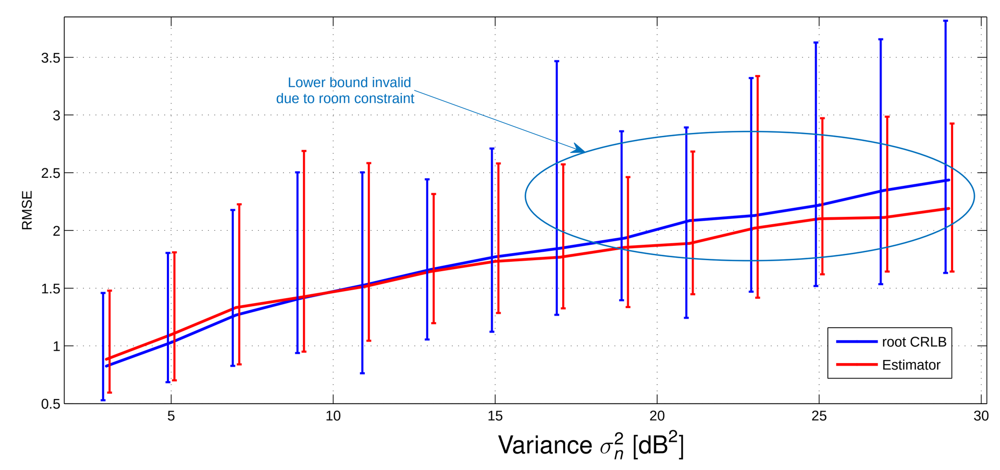
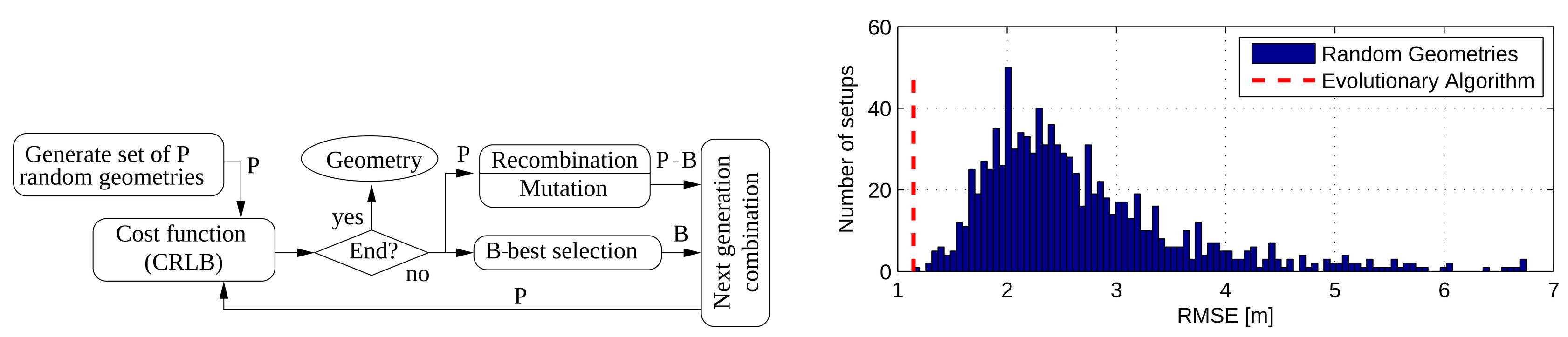


Figure 3 : Comparison between root CRLB and RMSE of grid approach estimator.

Sensor placement optimization

- Task: Find beacon placement with minimum expected RMSE
  - Use evolutionary optimization to handle computational complexity



(a) Genetic optimization procedure (b) Histogram of average root CRLB values of 1000 randomly generated geometries, and the result of the evolutionary algorithm

Conclusions

- Fourier polynomial based radiation pattern description
  - Analytic solution for CRLB of position estimator
- Cramér-Rao lower bound used for network geometry planing
  - Evolutionary optimization procedure enables network planing for large-scale setups