UNIVERSITÄT PADERBORN
Die Universität der Informationsgesellschaft

Computer Science, Electrical Engineering and Mathematics
NT Communications Engineering
Prof. Dr.-Ing. Reinhold Häb-Umbach

Online Observation Error Model Estimation for Acoustic Sensor Network Synchronization

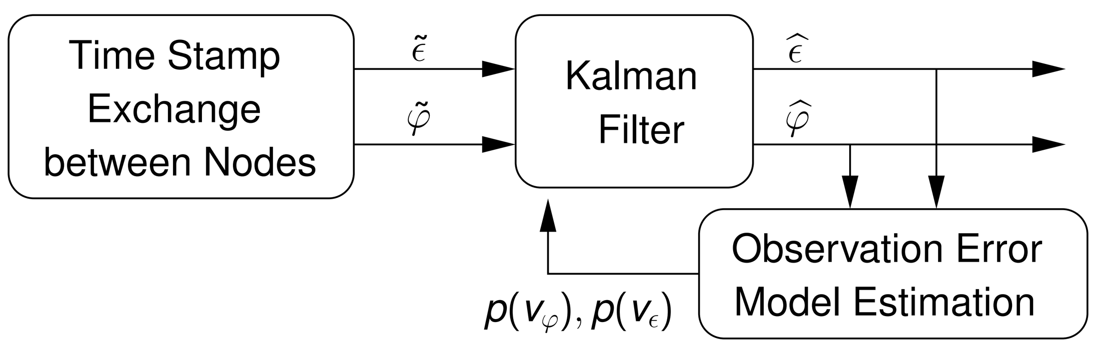
Joerg Schmalenstroeer, Weile Zhao, Reinhold Haeb-Umbach

University of Paderborn, Germany
{schmalen,haeb}@nt.uni-paderborn.de
http://nt.uni-paderborn.de

Introduction

- Wireless acoustic sensor network (WASN)
  - Task: Sensor network for cooperative signal processing
  - Problem: Diverging clocks cause sampling rate mismatch
- Our Approach
  - Clock frequency deviation and phase offset "measurement" via time stamp exchange between nodes
  - Postfiltering of measurements by Kalman filter
- Here: Online estimation of parameters of Kalman filter measurement equation (observation error model)

Time Stamp Exchange



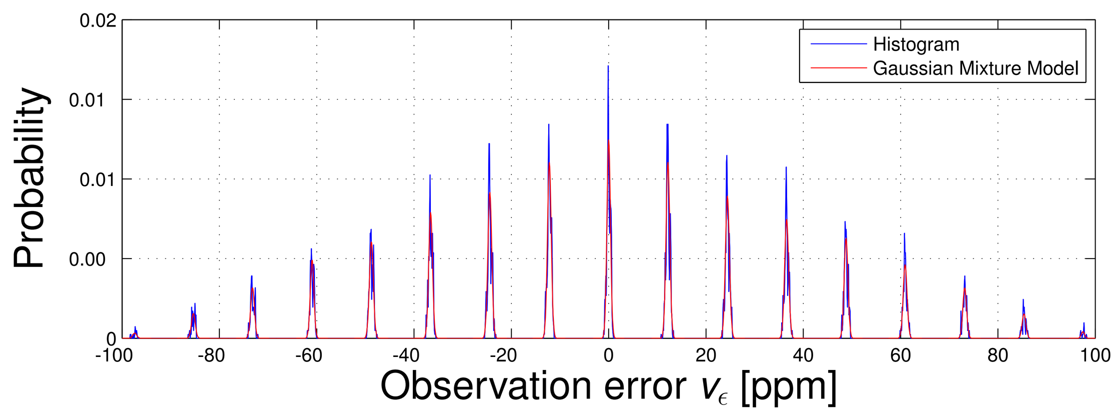
- Sensor node frequencies given by $f_M = (1+\epsilon) f_S$
- Two-way message exchange algorithm [Chaudari 2012] to estimate frequency deviation $\epsilon(k)$ and phase offset $\varphi(k)$

$$\tilde{\epsilon}(k) \approx \epsilon(k) + \underbrace{\frac{(\xi_{R,k} - \xi_{R,k-1}) + (\xi_{A,k-1} - \xi_{A,k})}{(t_{R,k} - t_{R,k-1}) + (t_{A,k} - t_{A,k-1})}}_{v_\epsilon(k)} = \epsilon(k) + v_\epsilon(k)$$

$$\tilde{\varphi}(k) \approx \varphi(k) + \underbrace{\frac{1}{2}(\xi_{R,k} - \xi_{A,k})}_{v_\varphi(k)} = \varphi(k) + v_\varphi(k)$$

Observation error model

- PDF of error term v_ϵ can be approximated by GMM



$$p(v_\epsilon) = \sum_{h=-M}^{M} \pi_{\epsilon,k} \mathcal{N}\left(v_\epsilon; h \cdot \mu_\epsilon, \sigma_\epsilon^2\right)$$

- Large-scale error ($h \cdot \mu_\epsilon$): MAC, timeouts
- Small-scale error (σ_ϵ^2): random variations
- Idea: Identification of large scale error via Kalman Filter
  - Assumption: Kalman filter prediction $\hat{\epsilon}(k|k-1)$ is close to true value $\epsilon(k)$

$$|\hat{\epsilon}(k|k-1) - \epsilon(k)| \ll \frac{1}{2}\mu_\epsilon,$$

  - Large-scale observation error can be uniquely determined

EM Parameter Estimation

- To be estimated: $\Theta_\epsilon = \{\mu_\epsilon; \sigma_\epsilon^2; \boldsymbol{\pi}_\epsilon\}$, Θ_φ
- Hidden variables
  - $\epsilon(k)$: True frequency deviation
  - $c_\epsilon(k) \in [-h, h]$: Mixture label underlying k-th observation $\tilde{\epsilon}(k)$

Expectation Step

- Compute posteriors of hidden variables
  - Kalman filter: $\hat{\epsilon}(k|k-1)$
  - Posterior probability of mixture component

$$\gamma_h(k) := P(c_\epsilon(k){=}h | \tilde{\epsilon}(k), \hat{\epsilon}(k|k{-}1); \Theta_\epsilon^{(\kappa)}) = \frac{\mathcal{N}(\tilde{\epsilon}(k); \hat{\epsilon}(k|k{-}1) + h\mu_\epsilon^{(\kappa)}, (\sigma_\epsilon^2)^{(\kappa)}) \cdot \pi_{\epsilon,h}^{(\kappa)}}{\sum_{m=-M}^{M} \mathcal{N}(\tilde{\epsilon}(k); \hat{\epsilon}(k|k{-}1) + m\mu_\epsilon^{(\kappa)}, (\sigma_\epsilon^2)^{(\kappa)}) \cdot \pi_{\epsilon,m}^{(\kappa)}}$$

  - Uses state prediction of Kalman filter: $\hat{\epsilon}(k|k-1)$
- Class label of most likely hypothesis: $\hat{c}_\epsilon(k) = \underset{h}{\mathrm{argmax}}\{\gamma_h(k)\}$
- Kalman filter is fed with observations: $z(k) = \tilde{\epsilon}(k) - \hat{c}_\epsilon(k) \cdot \mu_\epsilon$

Maximization step

- Auxiliary function using approximations

$$Q(\Theta_\epsilon; \Theta_\epsilon^{(\kappa)}) \propto \sum_{k=1}^{N} \underbrace{\sum_{h=-M}^{M} \ln\left\{\pi_{\epsilon,h} \mathcal{N}(\tilde{\epsilon}(k); \hat{\epsilon}(k|k) + \hat{c}_\epsilon(k)\mu_\epsilon, \sigma_\epsilon^2(k))\right\} \gamma_h(k)}_{Q_k(\Theta_\epsilon; \Theta_\epsilon^{(\kappa)})},$$

- Parameter update of weights, mean and variance

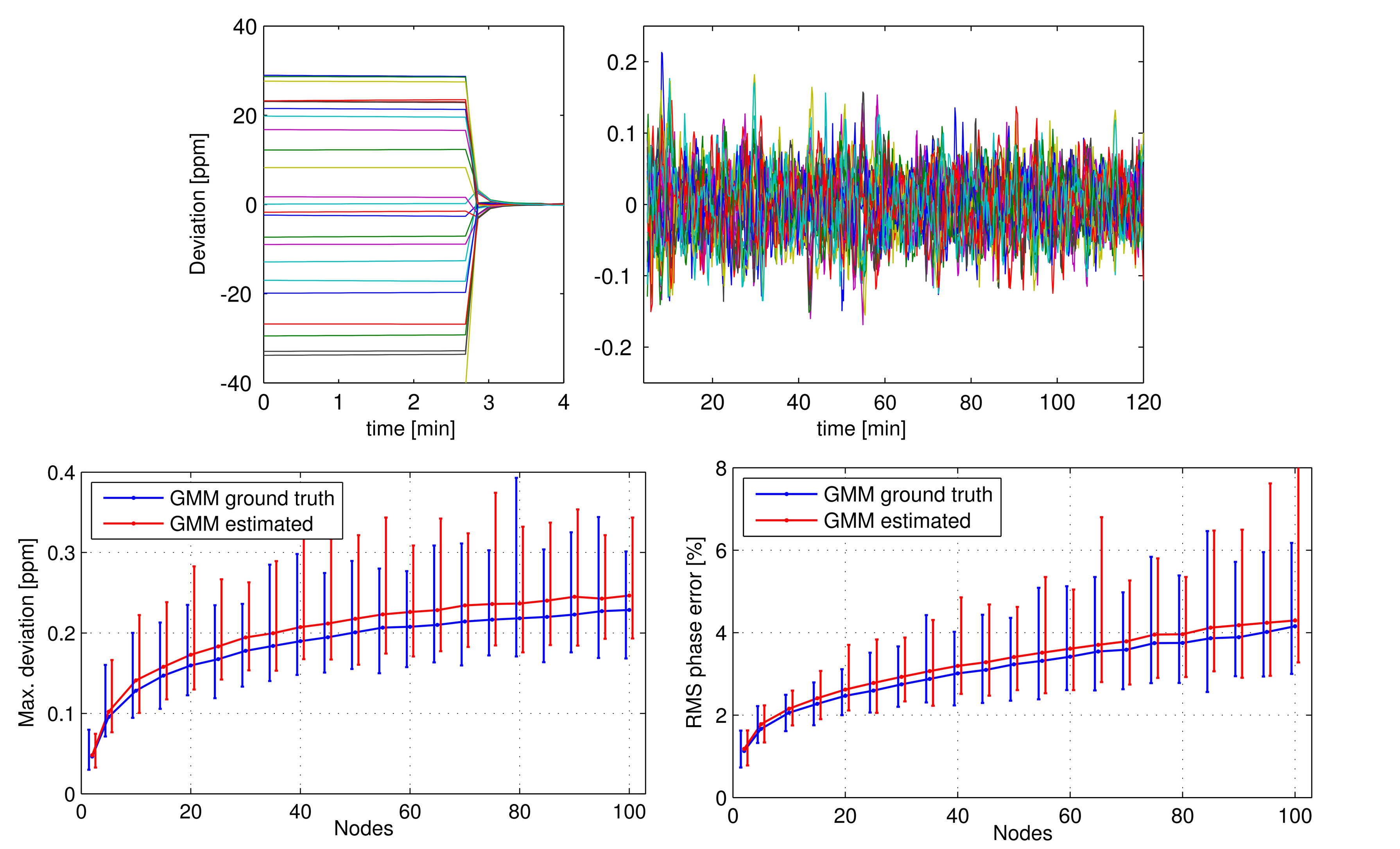
$$\pi_{\epsilon,h}^{(\kappa+1)} = \frac{1}{N}\sum_{k=1}^{N} \gamma_h(k) \qquad \mu_\epsilon^{(\kappa+1)} = \frac{\sum_{k=1}^{N}\sum_{h=-M}^{+M} \gamma_h(k) \cdot h \cdot (\tilde{\epsilon}(k) - \hat{\epsilon}(k|k))}{\sum_{k=1}^{N}\sum_{h=-M}^{+M} \gamma_h(k) \cdot h^2}$$

$$(\sigma_\epsilon^2)^{(\kappa+1)} = \left(\sum_{k=1}^{N}\sum_{h=-M}^{+M} \gamma_h(k) \cdot (\tilde{\epsilon}(k) - \hat{\epsilon}(k|k) - h\mu_{\epsilon,\kappa})^2\right) \Big/ \left(\sum_{k=1}^{N}\sum_{h=-M}^{+M} \gamma_h(k)\right)$$

- Replace iterative block EM by recursive online EM

$$\Theta_\epsilon(k) = \Theta_\epsilon(k-1) + \tau \cdot \nabla_{\Theta_\epsilon}\left\{Q_k(\Theta_\epsilon; \Theta_\epsilon(k-1))\right\}\Big|_{\Theta_\epsilon = \Theta_\epsilon(k-1)}$$

Simulation results



Conclusions

- Derived online EM algorithm for estimating observation error model parameters
- Integration of Kalman filter predictions into the expectation step of the EM algorithm