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A Gossiping Approach to Sampling Clock Synchronization in Wireless Acoustic Sensor Networks

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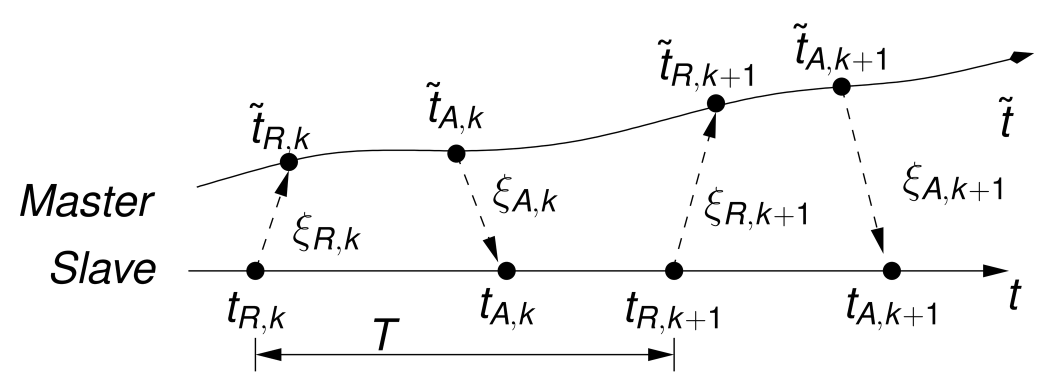
Introduction

- Wireless acoustic sensor network (WASN)
  - Task: Acoustic sensor network for cooperative signal processing
  - Problem: Diverging clocks cause sampling rate mismatch
- Proposed approach: Gossiping of Kalman Filter state estimates for WASN synchronization

Time Stamp Exchange

- Two-way message exchange every T seconds [Chaudhari 2012]



- Temporal relationship between message exchanges

$$\tilde{t}_{R,k} = (t_{R,k} + \xi_{R,k}) \cdot (1+\epsilon) + \varphi$$
$$\tilde{t}_{A,k} = (t_{A,k} - \xi_{A,k}) \cdot (1+\epsilon) + \varphi$$

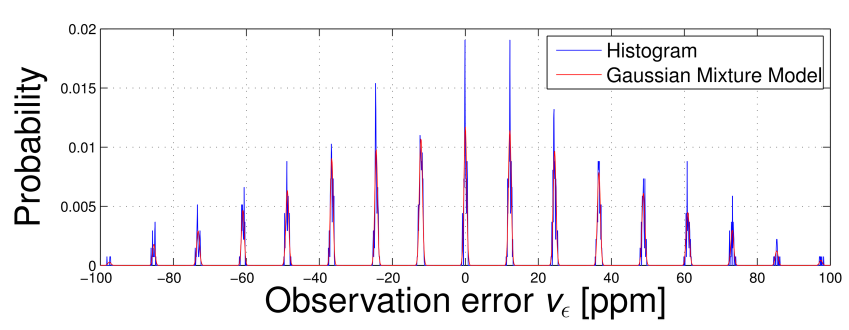
- Clock synchronization between master and slave node measured in terms of frequency deviation ϵ and phase offset φ with $f_M = (1+\epsilon)\cdot f_S$
- Estimate of deviation ϵ between oscillator frequencies

$$\tilde{\epsilon} = \frac{\Delta\tilde{t}^{+} - \Delta\tilde{t}^{-}}{\Delta t^{+} - \Delta t^{-}} - 1 \approx \epsilon + \frac{(\xi_{R,k+1} - \xi_{R,k}) + (\xi_{A,k} - \xi_{A,k+1})}{(t_{R,k+1} - t_{R,k}) + (t_{A,k+1} - t_{A,k})}$$

with $\Delta\tilde{t}^{+} = (\tilde{t}_{R,k+1} - \tilde{t}_{A,k})$ and $\Delta\tilde{t}^{-} = (\tilde{t}_{R,k} - \tilde{t}_{A,k+1})$

Observation Error Distribution

- Model unknown transmission times ξ over ZigBee network as sum of three contributions: $\xi = T_c + l \cdot T_d + T_{r,\epsilon}$
  - T_c: Constant minimum delay
  - $l \cdot T_d$: Multiples of fixed delay
  - T_r: Exponentially distributed random component



$$\tilde{\epsilon} \approx \epsilon + \underbrace{h \cdot (T_d/(2T))}_{\text{large-scale error}: h\cdot\mu_\epsilon} + \underbrace{(T_r'/(2T))}_{\text{small-scale error}} =: \epsilon + v_\epsilon$$

- Gaussian Mixture Model: $p(v_\epsilon) = \sum_{h=-M}^{+M} \gamma_h \cdot \mathcal{N}(v_\epsilon; h\cdot\mu_\epsilon, \sigma_\epsilon^2)$

Kalman filter

- State vector: $\boldsymbol{x}(n) = [\varphi(n), \epsilon(n), \Delta\epsilon(n)]^T$
- Observations: $\boldsymbol{z}(n) = [\tilde{\varphi}(n) - \hat{h}\cdot\mu_\varphi, \tilde{\epsilon}(n) - \hat{h}\cdot\mu_\epsilon]^T$
- System equation: $\boldsymbol{x}(n+1) = \boldsymbol{F}\cdot\boldsymbol{x}(n) + \boldsymbol{v}_S(n)$
- Measurement equation: $\boldsymbol{z}(n) = \boldsymbol{H}^T \cdot \boldsymbol{x}(n) + \boldsymbol{v}(n)$

$$\boldsymbol{F} = \begin{pmatrix} 1 & T\cdot f_0 & 0 \\ 0 & 1 & T \\ 0 & 0 & 1 \end{pmatrix}; \boldsymbol{H} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}; E[\boldsymbol{v}_S\boldsymbol{v}_S^T] = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \sigma^2_{\Delta\epsilon} \end{pmatrix}; E[\boldsymbol{v}\boldsymbol{v}^T] = \begin{pmatrix} \sigma_\varphi^2 & 0 \\ 0 & \sigma_\epsilon^2 \end{pmatrix}$$

Reduction of Observation Error

- Idea: Identification of large scale error via Kalman Filter
  - Assumption: Kalman filter prediction $\hat{\epsilon}(n|n-1)$ is close to true value $\epsilon(n)$

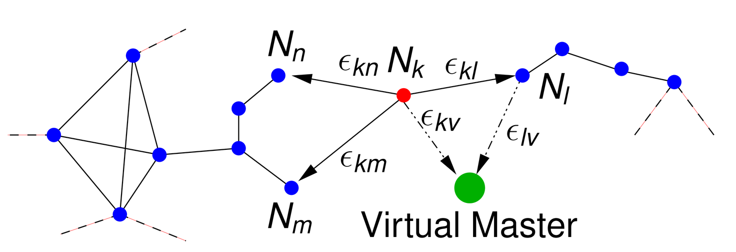
$$|\hat{\epsilon}(n|n-1) - \epsilon(n)| \ll \frac{1}{2}\mu_\epsilon,$$

  - Large-scale observation error can be uniquely determined

$$\hat{h} = \underset{h}{\texttt{argmin}}\, |\hat{\epsilon}(n|n-1) - (\tilde{\epsilon}(n) - h\cdot\mu_\epsilon)|$$

Gossiping Algorithm

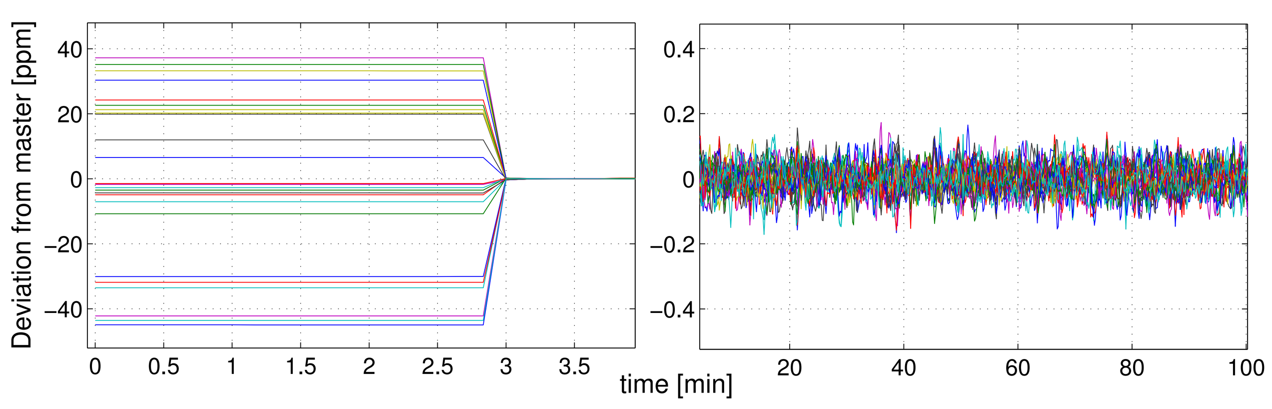
- Local message exchange results in global convergence to virtual master ($\hat{=}$ avg. sampl. frequency)



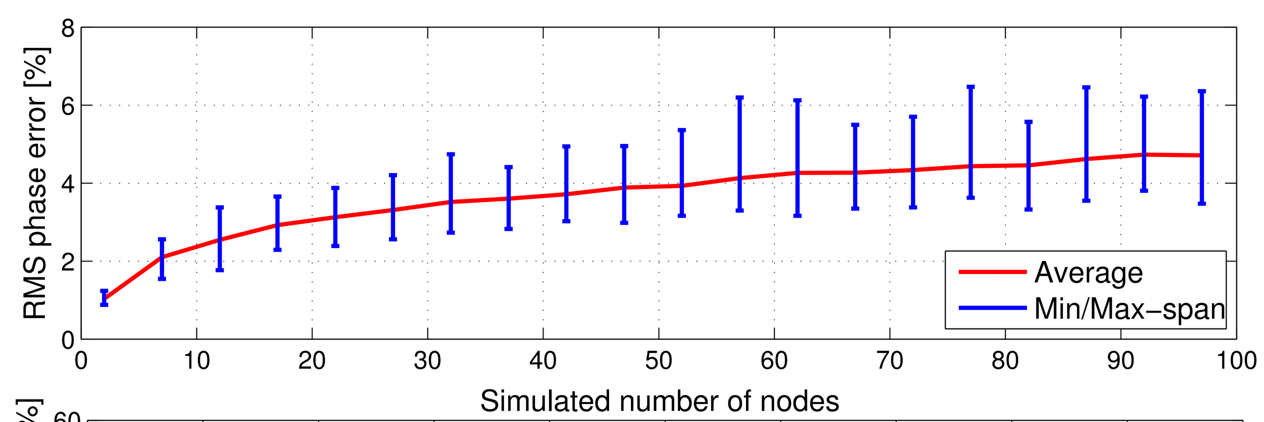
- Gossip message exchange between N_k and N_l
  1. Node N_k sends information $[\epsilon_{kl}, \epsilon_{kv}]$ to N_l
  2. Node N_l calculates
     - Average deviation $\bar{\epsilon}_{lk} = \left(\frac{\epsilon_{lk}}{2} + \frac{-\epsilon_{kl}}{2}\right)$
     - Deviation after adjustment $\bar{\epsilon}_{lk}^{(A)} = (\bar{\epsilon}_{lk} - \epsilon_{lv} + \epsilon_{kv})$
  3. Node updates deviation towards virtual master $\epsilon_{lv} \leftarrow \epsilon_{lv} + \frac{\bar{\epsilon}_{lk}^{(A)}}{2}$
  4. Node N_l sends information $\left[\bar{\epsilon}_{lk}^{(A)}, \epsilon_{lv}\right]$ to N_k for updating ϵ_{kv}

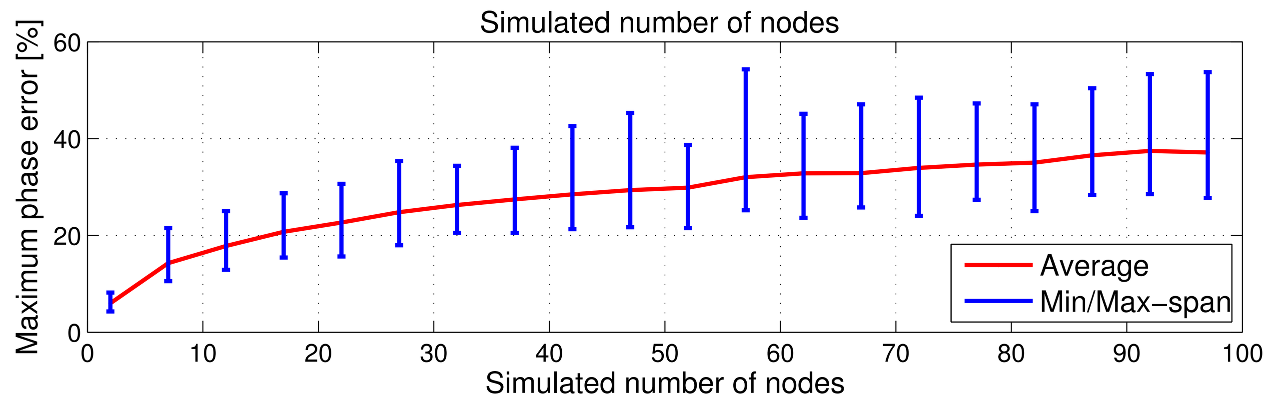
Simulation Results

- Network synchronization example: 25 nodes
  - Deviation from virtual master remains below 0.2 ppm



- Simulation of random networks
  - RMS phase error below 7% of a sample interval
  - Maximum phase error on average below half a sample interval





Conclusions

- Synchronization of acoustic sensor networks via gossiping
- Kalman filter for improved estimates of ϵ and φ
- Small clock phase errors even in large networks