Sampling Rate Synchronisation in Acoustic Sensor Networks with a Pre-Trained Clock Skew Error Model

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12.09.2013

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Introduction

Acoustic sensor network

- Consists of sensor nodes connected by wire or wirelessly
- Nodes record (multi-)channel audio signals and process them
- Applications: ad-hoc teleconferencing, monitoring, surveillance

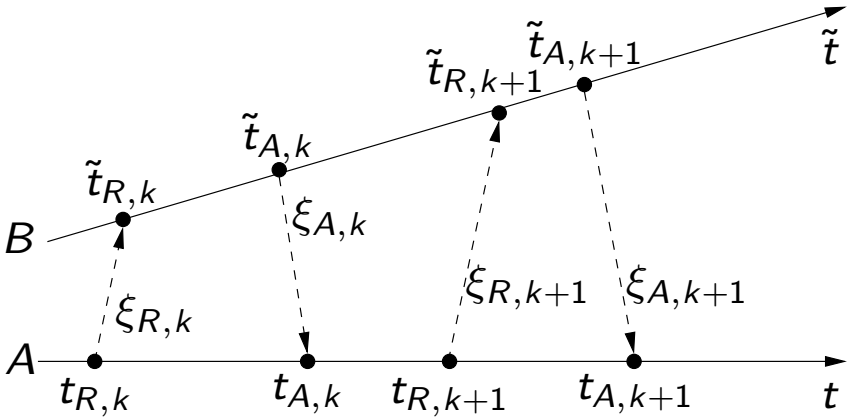
Problem statement

- Sampling clock oscillators at nodes differ typically by ± 50 ppm
  - $\Rightarrow$ Time base of nodes diverges rapidly
  - $\Rightarrow$ Precludes application of certain algorithms (e.g. TDOA estimation across sensor nodes)

Our approach

- Estimate clock frequency and phase offset via time stamp exchange protocol
- Improve estimate by Kalman Filter with dedicated error model
- Adjust sampling frequency through direct digital synthesis (DDS) on hardware platform

Clock Frequency Offset Estimation



Time stamp exchange

- Times at nodes A and B

$$\tilde{t}_{R,k} = (t_{R,k} + \xi_{R,k}) \cdot \omega + \varphi$$
$$\tilde{t}_{A,k} = (t_{A,k} - \xi_{A,k}) \cdot \omega + \varphi$$

- ω: clock freq. offset ($\omega = 1$: perfect sync.)

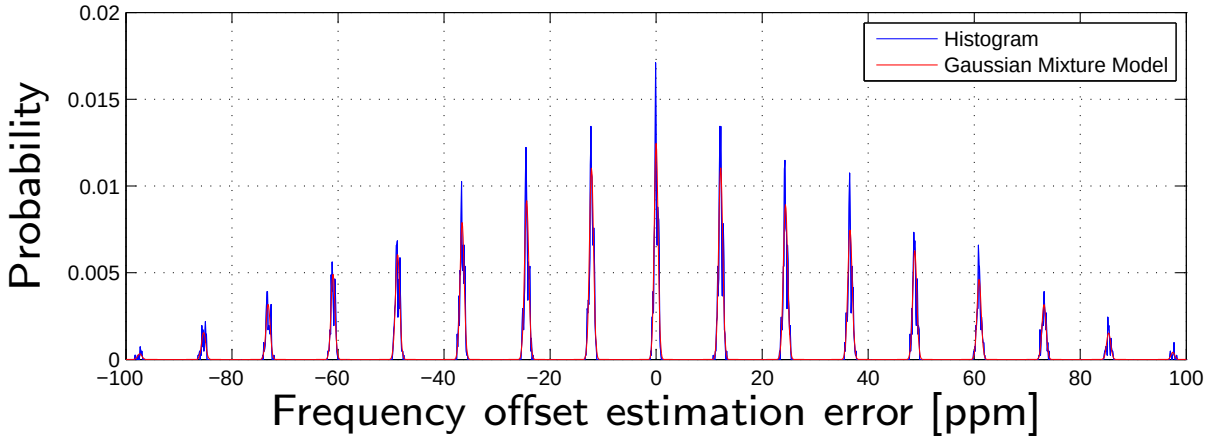
Clock Frequency Offset Estimation [Chaudhari 2012]

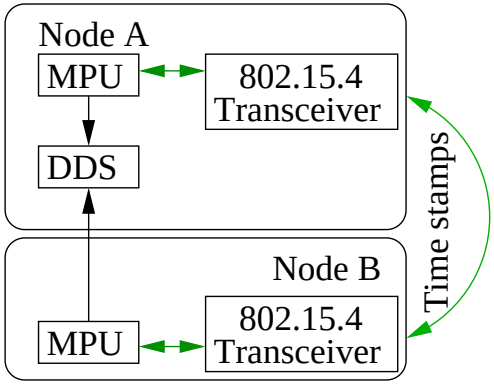
$$\frac{\Delta\tilde{t}^{+}}{\Delta t^{+}} = \frac{\tilde{t}_{R,k+1} - \tilde{t}_{A,k}}{t_{R,k+1} - t_{A,k}} = \left(1 + \frac{\xi_{R,k+1} + \xi_{A,k}}{t_{R,k+1} - t_{A,k}}\right)\omega \quad \text{and} \quad \frac{\Delta\tilde{t}^{-}}{\Delta t^{-}} = \left(1 - \frac{\xi_{R,k} + \xi_{A,k+1}}{|t_{R,k} - t_{A,k+1}|}\right)\omega$$

$$\widehat{\omega} = \frac{\Delta\tilde{t}^{+} - \Delta\tilde{t}^{-}}{\Delta t^{+} - \Delta t^{-}} = \left(1 + \frac{(\xi_{R,k+1} - \xi_{R,k}) + (\xi_{A,k} - \xi_{A,k+1})}{(t_{R,k+1} - t_{R,k}) + (t_{A,k+1} - t_{A,k})}\right)\omega$$

- Influence of transmission times vanishes with increasing temporal distance between k-th and $(k+1)$-st time stamp exchange

Transmission Error Model





Experiment: Two nodes connected to a single crystal oscillator

- Measured observation error distribution $p(\hat{\omega}|\omega = 0)$ (6 hours of data)
- Two types of errors
  - Largecscale: Packet losses, protocol dependent wait states, medium access control (caused by network)
  - Small-scale: Estimation error of time stamp exchange protocol, MPU hardware interrupts, I/O latencies
- Gaussian mixture model (GMM) to approximate measured histogram

$$p(v_o) = \sum_{k=1}^{K} p(k)p(v_o|k) = \sum_{k=1}^{K} \gamma_k \mathcal{N}(v_o; \mu_{k,o}, \sigma^2_{k,o})$$

- GMM trained offline

Kalman Filter (1/2)

Kalman filter

- Simple kinematic model to model oscillator frequency drifts ($\mathbf{x} = [\omega, \Delta\omega]^T$)

$$\mathbf{x}(n+1) = \underbrace{\begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}}_{\mathbf{F}} \mathbf{x}(n) + \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{\mathbf{G}} \cdot \underbrace{\begin{bmatrix} 0 \\ v_s(n) \end{bmatrix}}_{\mathbf{v}_s}$$

- Measurement equation: $z(n) := \widehat{\omega}(n) = \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{\mathbf{H}^T} \mathbf{x}(n) + v_o(n)$

- Minimum mean square error (MMSE) estimate of the system state

$$\widehat{\mathbf{x}}(n|n) = \widehat{\mathbf{x}}(n|n-1) + \mathbf{K}(n) \cdot \Big(z(n) - \mathbf{H}^T \widehat{\mathbf{x}}(n|n-1) - E[v_o(n)] \Big)$$

- Predicted MMSE estimate of frequency offset

$$\widehat{\omega}^{(KF)}(n) = \mathbf{H}^T \widehat{\mathbf{x}}(n|n-1)$$

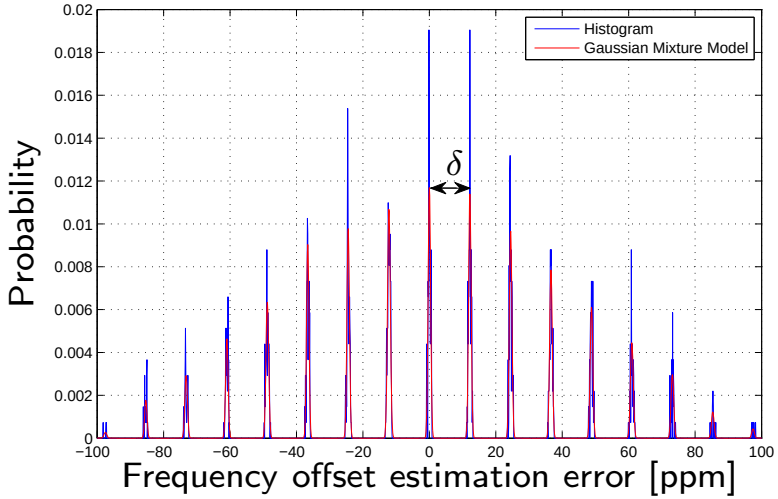
Kalman filter (2/2)

Assumption

- Kalman filter prediction $\widehat{\omega}^{(KF)}(n)$ is close to true value ω

$$|\widehat{\omega}^{(KF)}(n) - \omega| \ll \delta,$$

- $\delta = \min_{k,l} \left|\mu_{k,o} - \mu_{l,o}\right|$



Removal of large-scale observation errors

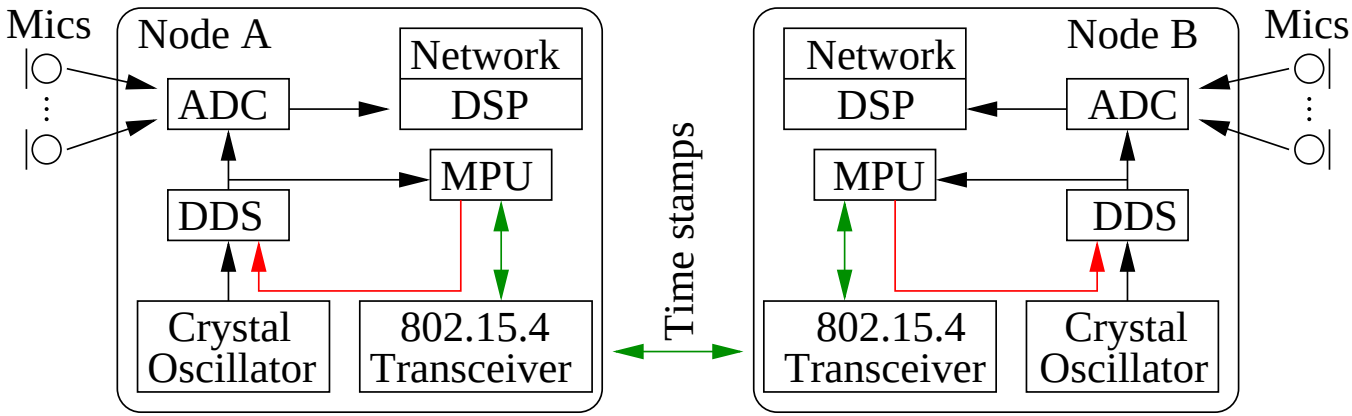
- Contributions of large scale effects to the observation error can be identified

$$\hat{k} = \underset{k}{\text{argmin}} \left|\widehat{\omega}^{(KF)}(n) - \mu_{k,o}\right| = \underset{k}{\text{argmax}}\ p(v_o|k)$$

- Large-scale observation error can be removed:

$$\widehat{\mathbf{x}}(n|n) = \widehat{\mathbf{x}}(n|n-1) + \mathbf{K}(n)\left((z(n) - \mu_{\hat{k},o}) - \mathbf{H}^T\widehat{\mathbf{x}}(n|n-1)]\right)$$

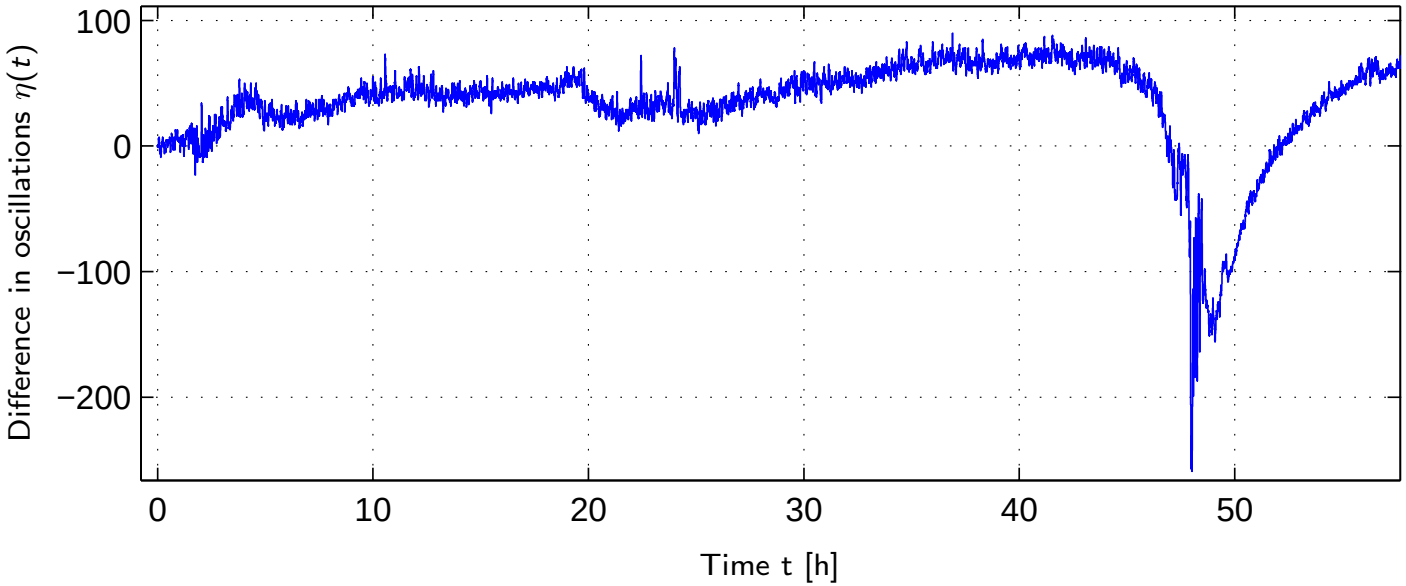
Hardware platform



Hardware platform

- Network-connected multi-channel acoustic sensor nodes (own development)
- ADC with an oversampling factor of 512 to generate a 16 kHz sampling rate
- Direct Digital Synthesis (DDS): Generates arbitrary frequencies with sub-hertz resolution: 0.0279 Hz$\hat{=}$0.003 41 ppm@16 kHz
- Time stamps: MPU counts oscillations
- Time stamp exchange via wireless link (IEEE 802.15.4 MAC & physical layer)
- Stacked on top: BeagleBoard XM (DSP & Ethernet connection)

Experiment on Wired Connection

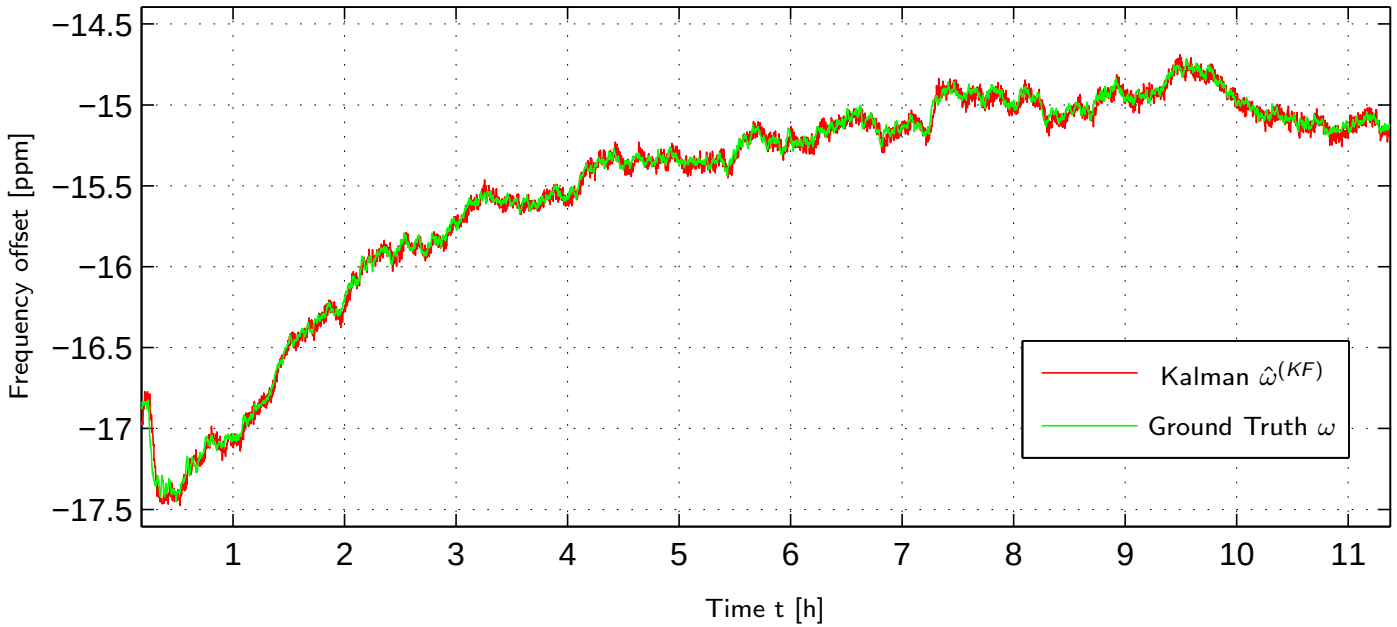


Setup: Wired connection, DDS of slave node adjusted

- USART connection between two sensor nodes for time stamp exchange
  - Absence of large-scale errors: GMM turns into single Gaussian distribution
- Difference in oscillations: $\eta(t) = \int_0^t (f_M(\tau) - f_S^{(A)}(\tau))d\tau$
  - $f_M(t)$: Frequency of master node
  - $f_S^{(A)}(t)$: Adjusted frequency of slave node
- Maximum difference was kept below 250 oscillations

  $\Rightarrow$ Maximum sampling error below a half sample (oversampling factor of 512!)

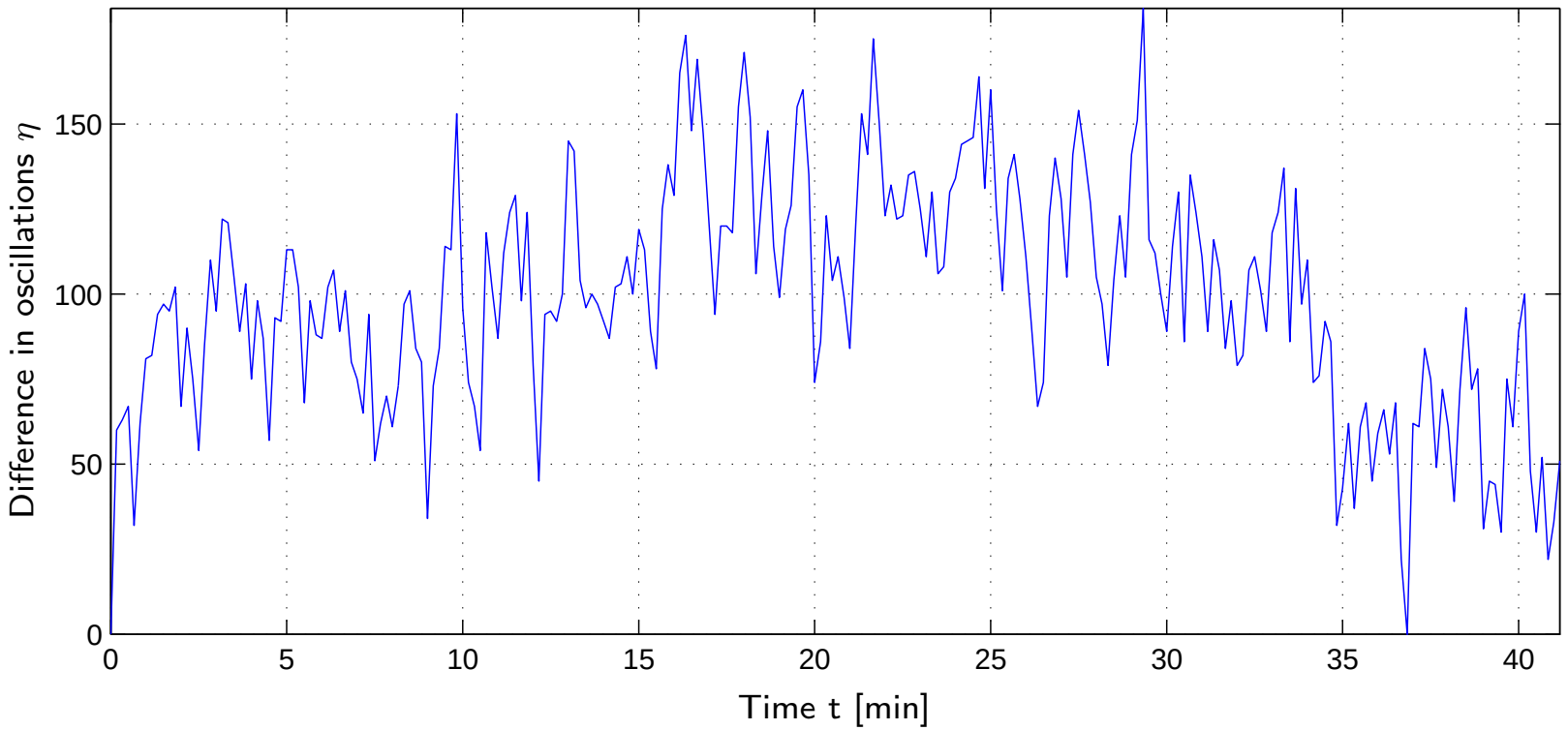
Experiment (1/2) on Wireless Connection



Setup: wireless connection, DDS not adjusted

- Wireless 802.15.4 connection (ZigBee)
- Comparison between
  - Ground truth $\omega(n)$ (measured by extra hardware device)
  - Kalman filter estimate $\widehat{\omega}^{(KF)}(n)$
- Mean square error was measured to be 0.0019 ppm

Experiment (2/2) on Wireless Connection



Setup: wireless connection, DDS adjusted

- Wireless 802.15.4 connection (ZigBee)
- Difference between data streams remains below $180/512 = 0.35$ samples
- Network load caused by exchange of 64-Bit time stamps each 10 s:
  $4 \cdot 64\,\text{Bit}/10\,\text{s} = 25.6\,\text{Bit/s}$

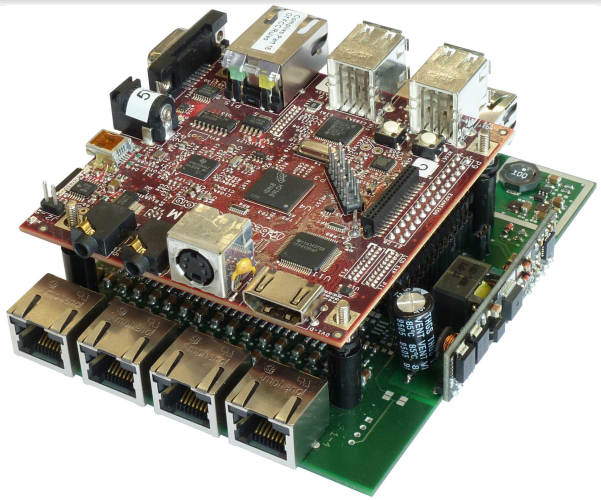
Conclusions and Outlook

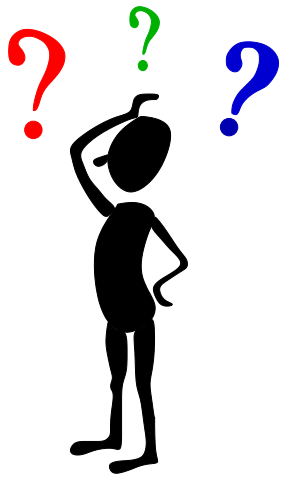
Conclusions

- Clock frequency synchronisation of distributed sensor nodes by time stamp exchange protocol
- Improved clock frequency offset estimates by post filter
  - Kalman filter which exploits the characteristics of the estimation error
- Implementation on microprocessor units
- Communication via an IEEE 802.15.4 wireless network
- Low network load by time stamp exchange
- Long term experiments: Difference between two data streams is kept below a maximum of half a sample

Outlook

- Improved feedback control
- Online error model estimation





Thank you for your attention!

Questions ?

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