

# DoA-Based Microphone Array Position Self-Calibration Using Circular Statistics

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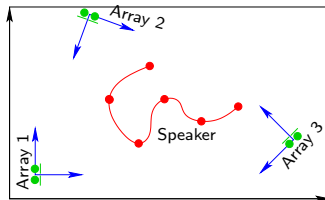
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## Problem statement

- Application areas of spatially distributed microphone arrays:
  - ▶ Speaker localization & tracking
  - ▶ Signal enhancement (e.g. beamforming)
- ⇒ Geometry calibration required:
  - ▶ Position & orientation of microphone arrays

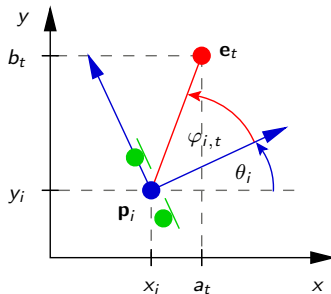


## Task

- Automatic calibration procedure
- Input: Reverberant speech signal
- Direction-of-arrival measurements per array (avoid inter array clock sync.)

## System overview

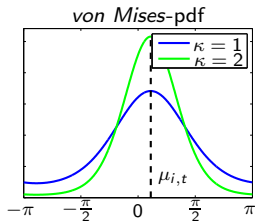
- Unknown parameters:
  - ▶ Speaker position  $\mathbf{e}_t = [a_t, b_t]$
  - ▶ Array position  $\mathbf{p}_i = [x_i, y_i]$
  - ▶ Array orientation  $\theta_i$
- Measured data:
  - ▶ Direction of arrival  $\varphi_{i,t}$



## Observation model of a microphone pair

- Measurement:  $\varphi_{i,t} := \mu_{i,t} + \text{Noise}$
- Observation modeled by *von Mises*-pdf:

$$p(\varphi_{i,t}; \mu_{i,t}, \kappa_{i,t}) = \frac{\exp(\kappa_{i,t} \cos(\varphi_{i,t} - \mu_{i,t}))}{2\pi I_0(\kappa_{i,t})}$$



# Maximum-Likelihood Geometry Calibration

## Loglikelihood function

- Moving speaker  $\Rightarrow$  Multiple measurements to a speaker position unavailable
- Independent observations  $\varphi_{i,t}$  from  $I$  microphone pairs and  $T$  speaker positions:

$$\ell(\boldsymbol{\mu}) := \sum_{t=1}^T \sum_{i=1}^I \kappa_{i,t} \cos(\varphi_{i,t} - \mu_{i,t})$$

- #Unknowns =  $2 \cdot I \cdot T$
  - #Observation =  $I \cdot T$
- $$\Rightarrow \# \text{Unknowns} > \# \text{Observation}$$

## How to overcome these drawbacks?

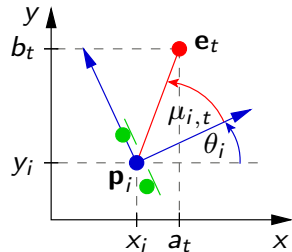
- Incorporate geometric relation:

$$\mu_{i,t} := \mu(\mathbf{p}_i, \theta_i, \mathbf{e}_t) = \text{atan} \left( \frac{b_t - y_i}{a_t - x_i} \right) - \theta_i$$

- Approximate  $\kappa_{i,t}$ :

$$\kappa_{i,t} \propto d_{i,t} = |\mathbf{e}_t - \mathbf{p}_i|$$

$$\Rightarrow \# \text{Unknown} = 3 \cdot I + 2 \cdot T$$



## Optimization problem

- Resulting equation:

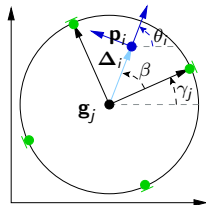
$$\langle \mathbf{p}_{2:l}^*, \theta_{2:l}^*, \mathbf{e}_{1:T}^* \rangle = \underset{\mathbf{p}_{2:l}, \theta_{2:l}, \mathbf{e}_{1:T}}{\operatorname{argmin}} \left\{ \sum_{t=1}^T \sum_{i=1}^I d_{i,t} [1 - \cos(\varphi_{i,t} - \mu(\mathbf{p}_i, \theta_i, \mathbf{e}_t))] \right\}$$

- Iterative optimization using Newton root finding algorithm
- Recovers array positions, orientations and speaker positions
- Result has arbitrary scaling

# Absolute calibration using circular arrays

## How to recover the scaling?

- **Literature:** Knowledge of distance between two sensor nodes
- **Proposed:** Obtain scaling information from radius of circular microphone arrays



## Calibration using circular arrays

- Express position of microphone pair relative to center of circular array:
 
$$\mathbf{p}_i = \mathbf{g}_j + \Delta_i \text{ and } \theta_i = \gamma_j + \beta_i$$
- Determine position and orientation of circular arrays instead of microphone pairs

$$\langle \mathbf{g}_{2:J}^*, \gamma_{2:J}^*, \mathbf{e}_{1:T}^* \rangle = \underset{\mathbf{g}_{2:J}, \gamma_{2:J}, \mathbf{e}_{1:T}}{\operatorname{argmin}} \left\{ \sum_{t=1}^T \sum_{j=1}^J \sum_{i=1}^I d_{i,t} [1 - \cos(\varphi_{i,t} - \mu(\mathbf{g}_j, \gamma_j, \Delta_i, \mathbf{e}_t))] \right\}$$

- Apply same optimization as before

# Simulation Framework

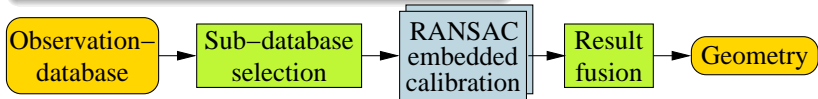
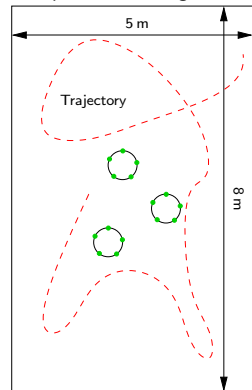
## Scenario

- 3 circular arrays with 5 microphones
- 5 array arrangements, approx. distance of 1 m
- 6 min long speaker trajectory
- Reverberation times form 0 ms to 500 ms
- DoA estimates obtained by adaptive beam-forming, on blocklength of 128 Samples @ 16 kHz

## Random Sample Consensus Framework

- Outlier rejection
- Handle large data sets

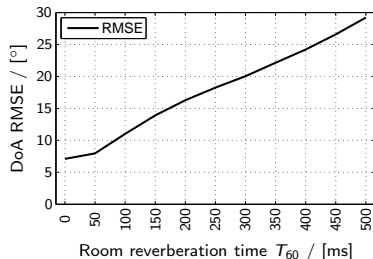
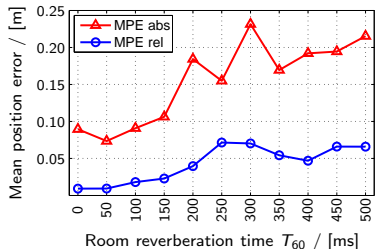
sample sensor configuration





## Error measure

- **Mean position error:** Average distance between real and estimated sensor position
- Apply rigid body transformation to match real and estimated positions
- Distinguish between 2 error types:
  - ▶ **Geometric + Scaling errors (MPE abs):** Rotation + Translation
  - ▶ **Geometric errors (MPE rel):** Rotation + Translation + Scaling



## Summary

- Geometry calibration framework base on reverberated speech input
  - Circular arrays enable absolute geometry calibration
  - ML-Estimator in case of *von Mises* distributed observations
  - RANSAC-Framework: Outlier rejection and handling of large data sets
  - Calibration Error:
    - ▶ Relative positioning error  $< 10$  cm
    - ▶ Absolute positioning error  $< 25$  cm
- } Even in case of high reverberation



**Thank you for your attention!**

**Questions ?**

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