Joint Parameter Estimation and Tracking in a Multi-Stage Kalman Filter for Vehicle Positioning

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***Abstract*—In this paper we present a novel vehicle tracking method which is based on multi-stage Kalman filtering of GPS and IMU sensor data. After individual Kalman filtering of GPS and IMU measurements the estimates of the orientation of the vehicle are combined in an optimal manner to improve the robustness towards drift errors. The tracking algorithm incorporates the estimation of time-variant covariance parameters by using an iterative block Expectation-Maximization algorithm to account for time-variant driving conditions and measurement quality. The proposed system is compared to an interacting multiple model approach (IMM) and achieves improved localization accuracy at lower computational complexity. Furthermore we show how the joint parameter estimation and localizaiton can be conducted with streaming input data to be able to track vehicles in a real driving environment.**

I. INTRODUCTION

In recent years there has been a lot of interest in the field of car-to-car communication. Driver assistance systems can help avoid accidents by making use of information about the traffic and road. An important issue is the accurate localization of the own vehicle to be able to exchange position information with neighboring cars and to assess the relevance of received messages.

Location information can be obtained from a Global Positioning System (GPS) device and an internal Inertial Measurement Unit (IMU) consisting of e.g. a gyroscope and an accelerometer [1]. These sensor data are fused employing a model of vehicle movements to obtain an improved location estimate. To account for different driving conditions a piecewise linear kinematic model has been proposed which consists of switching between the 'constant velocity' (CV), 'constant acceleration' (CA), and sometimes also the coordinated turn model (CT) [2]. However, inference in such models, e.g. by the Interacting Multiple Model (IMM) algorithm, is known to be computationally demanding [7], certainly a drawback in light of the desired real-time processing on resource-limited devices.

As an alternative to this we explore here a time-variant single model approach, where system and observation covariance matrices are estimated alongside the tracking of the vehicle by employing the Expectation-Maximization (EM) algorithm. By updating these covariance parameters to account for different driving conditions we avoid the need for multiple dynamical models. In the Expectation step we employ a multi-stage (MS) filter, which consists of separate state estimators for the GPS and IMU data and an optimal estimator combination: Angle estimates by an external GPS device and the internal gyroscope are fused optimally taking into account their respective error covariances. By this we avoid the complexity of a single high-dimensional filter. In the Maximization step state and observation covariances are reestimated. We also show how the EM-algorithm is extended to achieve (quasi) online-ability such that vehicle tracking with a time-variant kinematic model is possible in a real driving environment. The experiments conducted show that the MS approach achieves higher positioning accuracy at greatly reduced computational complexity compared to the IMM and that the EM-based covariance estimate further improves positioning accuracy slightly.

This paper is organized as follows. In the next section, we give an overview of the equations governing the IMU unit. Then we describe our multi-stage filtering approach and derive the EM-algorithm for process noise and observation noise covariance estimation. In section IV we show how the calculations in the proposed joint parameter estimation and tracking system can be organized to achieve online vehicle tracking with time-variant model parameters, while section V presents simulation results. The paper finishes with conclusions drawn in section VI.

II. INERTIAL MEASUREMENT UNIT

The quality of position estimates via GPS can be improved by employing measurements from an inertial measurement unit (IMU). In this paper we assume that the IMU consists of a gyroscope and an accelerometer.

A gyroscope measures the coriolis acceleration caused by angular rotation. It produces voltage outputs proportional to the angular velocity of the vehicle around the principal axis of the device. The second IMU device is the accelerometer, which measures the linear acceleration of the vehicle in the body frame along three orthogonal axes.

Usually the inertial measurement unit (IMU) provides measurements of an accelerometer and a gyroscope at a much higher rate than the GPS device. We assumed a sampling period of $\Delta T_2 = 1/100\ sec.$, where the measurements of the IMU are the accelerations $\mathbf{a}^b$ and the angular velocities $\boldsymbol{\omega}^b$ with respect to the body frame. The orientation of the device is represented by a vector of the three Euler angles $\tilde{\boldsymbol{\mu}} = [\gamma, \theta, \varphi]^T$, where φ is yaw and θ denotes pitch and γ the roll angle. Disregarding the effect of earth rotation and gravity for simplicity we obtain [1]

$$\dot{\mathbf{p}} = \mathbf{v}; \quad \dot{\mathbf{v}} = \mathbf{a} = \mathbf{C}\mathbf{C}_b^n \mathbf{a}^b; \quad \dot{\tilde{\boldsymbol{\mu}}} = \boldsymbol{\Pi}\boldsymbol{\omega}^b, \tag{1}$$

where $\mathbf{C}_b^n$ transforms data from navigation to body frame with

$$\mathbf{C}_b^n = \begin{bmatrix} \cos\theta\cos\varphi & -\cos\gamma\sin\varphi & \sin\gamma\sin\varphi \\ & +\sin\gamma\sin\theta\cos\varphi & +\cos\gamma\sin\theta\cos\varphi \\ \cos\theta\sin\varphi & \cos\gamma\cos\varphi & -\sin\gamma\cos\varphi \\ & +\sin\gamma\sin\theta\sin\varphi & +\cos\gamma\sin\theta\sin\varphi \\ -\sin\theta & \sin\gamma\cos\theta & \cos\gamma\cos\theta \end{bmatrix} \tag{2}$$

and $\boldsymbol{\Pi}$ is the transformation matrix for the angular velocities with

$$\boldsymbol{\Pi} = \begin{bmatrix} 1 & \sin\gamma\tan\theta & \cos\gamma\tan\theta \\ 0 & \cos\gamma & -\sin\gamma \\ 0 & \sin\gamma/\cos\theta & \cos\gamma/\cos\theta \end{bmatrix}. \tag{3}$$

The vector $\mathbf{p}$ contains the current vehicle position $[x, y, z]$ and matrix $\mathbf{C}$ transforms the data from the NED to the ENU frame. The errors (drift and biases) of accelerometer $\boldsymbol{\epsilon}_a^b = [\epsilon_{a_x}^b, \epsilon_{a_y}^b, \epsilon_{a_z}^b]^T$ and $\dot{\boldsymbol{\epsilon}}_a^b$ and of gyroscope $\boldsymbol{\epsilon}_\omega^b = [\epsilon_{\omega_x}^b, \epsilon_{\omega_y}^b, \epsilon_{\omega_z}^b]^T$ and $\dot{\boldsymbol{\epsilon}}_\omega^b$ are modeled as deterministic processes with $\epsilon(t) = C_{\epsilon,2} + C_{\epsilon,1}(1 - \exp(-t/T_\epsilon))$, where C_ϵ and T_ϵ are assumed to be time-variant parameters which are known [5]. The initial conditions of the parametrized model are $\epsilon(0) = C_{\epsilon,2}$ and $\dot{\epsilon}(0) = C_{\epsilon,1}/T_\epsilon$. Further the IMU sensors are influenced by quantization and scaling errors. We assume a quantization error of 0.1 deg./sec. and a scaling error factor of 4 %.

III. EM-Based Covariance Estimation

Let $\mathbf{s}_{1:N} = \mathbf{s}_1, \ldots, \mathbf{s}_N$ and $\mathbf{z}_{1:N} = \mathbf{z}_1, \ldots, \mathbf{z}_N$ denote the state and observation vector sequence. The estimation of some parameter θ with the EM algorithm is based on the iterative optimization of the objective function

$$Q(\theta, \hat{\theta}^{(l)}) = E\left[\log\, p(\mathbf{s}_{1:N}, \mathbf{z}_{1:N}; \theta) | \hat{\theta}^{(l)}, \mathbf{z}_{1:N}\right], \tag{4}$$

where l is the iteration index. The expectation can be written

$$Q(\theta, \hat{\theta}^{(l)}) = E\left[\log\, p(\mathbf{z}_{1:N} | \mathbf{s}_{1:N}; \theta) | \hat{\theta}^{(l)}, \mathbf{z}_{1:N}\right] + E\left[\log\, p(\mathbf{s}_{1:N}; \theta) | \hat{\theta}^{(l)}, \mathbf{z}_{1:N}\right]. \tag{5}$$

For the estimation of the observation noise covariance $\mathbf{R}$ only the first term on the right hand side and for the estimation of the process noise covariance $\mathbf{Q}$ only the second term on the right hand side is relevant, as the other term is constant, respectively:

$$\hat{\mathbf{R}}^{(l+1)} = \text{argmax}_{\mathbf{R}} E\left[\log\, p(\mathbf{z}_{1:N} | \mathbf{s}_{1:N}; \theta) | \hat{\theta}^{(l)}, \mathbf{z}_{1:N}\right] \tag{6}$$

$$\hat{\mathbf{Q}}^{(l+1)} = \text{argmax}_{\mathbf{Q}} E\left[\log\, p(\mathbf{s}_{1:N}; \theta) | \hat{\theta}^{(l)}, \mathbf{z}_{1:N}\right] \tag{7}$$

In the following we show how the Expectation step, eq. (5), and the Maximization step, eqs. (6) and (7) can be computed.

A. E-Step: Filtering with a Multi-Stage Kalman Filter

Fig. 1 gives an overview of the multi-stage Kalman filter, where the estimates are labelled by ($\hat{}$). The GPS position measurements $\mathbf{z}^{GPS}$ are assumed to be obtained every $\Delta T_1 = 1$ *sec.* with respect to the local ENU coordinate system:

$$\mathbf{z}_{k_1}^{GPS} = \mathbf{H}\mathbf{s}_{k_1}^{GPS} + \mathbf{w}_{k_1}^{GPS}, \quad \mathbf{w}_{k_1}^{GPS} \sim \mathcal{N}(\mathbf{0}, \mathbf{R}_{k_1}^{GPS}). \tag{8}$$

Here k_1 is the time index which counts multiples of ΔT_1. $\mathbf{s}^{GPS} = [x, \dot{x}, \ddot{x}, y, \dot{y}, \ddot{y}, z, \dot{z}, \ddot{z}]^T$ is the state vector of Kalman Filter KF1 and $\mathbf{w}_{k_1}^{GPS}$ denotes white Gaussian observation noise with (time-variant) covariance $\mathbf{R}_{k_1}^{GPS}$. The composition of the matrix $\mathbf{H}$ is obvious from the definition of state and observation vectors. The state equation of KF1 is given by

$$\mathbf{s}_{k_1+1}^{GPS} = \mathbf{F}^{GPS}\mathbf{s}_{k_1}^{GPS} + \mathbf{v}_{k_1}^{GPS}, \quad \mathbf{v}_{k_1}^{GPS} \sim \mathcal{N}(\mathbf{0}, \mathbf{Q}_{k_1}^{GPS}), \tag{9}$$

where the block-diagonal matrix $\mathbf{F}^{GPS} = blkdiag(\mathbf{F}^{CA}, \mathbf{F}^{CA}, \mathbf{F}^{CA})$ depends on the constant acceleration (CA) state transition matrix $\mathbf{F}^{CA} = \begin{bmatrix} 1 & \Delta T_1 & (\Delta T_1)^2/2 \\ 0 & 1 & \Delta T_1 \\ 0 & 0 & 1 \end{bmatrix}$ and the covariance matrix $\mathbf{Q}_{k_1}^{GPS}$ of the white Gaussian process noise.

Corresponding to eq. (1) we use an Unscented Kalman Filter (UKF1) for the gyroscope measurements. The state vector is $\mathbf{s}^{Gyro} = [\tilde{\boldsymbol{\mu}}^T, \dot{\tilde{\boldsymbol{\mu}}}^T, (\boldsymbol{\epsilon}_\omega^b)^T, (\dot{\boldsymbol{\epsilon}}_\omega^b)^T]^T$ and the measurements are the angular velocities.

A disadvantage of inertial navigation is the drift which results in error accumulation. So, after transforming the filtered GPS data to spherical coordinates via an unscented transform (UT) (state vector $\mathbf{s}^{GPS} \rightarrow \boldsymbol{\mu}^{GPS} = [\theta^{GPS}, \varphi^{GPS}]^T$ and error covariance $\mathbf{P}^{GPS} \rightarrow \mathbf{P}_\mu^{GPS}$), we combine pitch and yaw of $\boldsymbol{\mu}^{GPS}$ and the vector $\boldsymbol{\mu}^{Gyro}$ of UKF1 in an optimal manner to an estimate $\boldsymbol{\mu}^{EC} = [\theta^{EC}, \varphi^{EC}]^T$ (EC: estimator combination), using their respective estimation error covariances [3], [6]:

$$\begin{aligned} (\mathbf{P}_{k_1}^{EC})^{-1}\boldsymbol{\mu}_{k_1}^{EC} &= (\mathbf{P}_{\mu,k_1}^{Gyro})^{-1}\boldsymbol{\mu}_{k_1}^{Gyro} + (\mathbf{P}_{\mu,k_1}^{GPS})^{-1}\boldsymbol{\mu}_{k_1}^{GPS}, \\ (\mathbf{P}_{k_1}^{EC})^{-1} &= (\mathbf{P}_{\mu,k_1}^{Gyro})^{-1} + (\mathbf{P}_{\mu,k_1}^{GPS})^{-1}. \end{aligned} \tag{10}$$

To improve robustness the result is fed back to the prediction step of UKF1, where the Euler angles and the corresponding error covariances are replaced correspondingly. With the estimator combination of (10) we make the simplifying assumption that the errors of KF1 and UKF1 are statistically independent, which is actually not the case, due to the feedback of the EC result to UKF1.

Finally, another UKF (UKF2) is used to bring all, i.e. the output of the combiner, the accelerometer and the GPS measurements, together. The control input of UKF2 is $\mathbf{u} = [\gamma^{Gyro}, (\boldsymbol{\mu}^{EC})^T]^T$. The state vector is $\mathbf{s}^{Acc} = [x, \dot{x}, \ddot{x}, y, \dot{y}, \ddot{y}, z, \dot{z}, \ddot{z}, (\boldsymbol{\epsilon}_a^b)^T, (\dot{\boldsymbol{\epsilon}}_a^b)^T]^T$ and the corresponding state equation is

$$\mathbf{s}_{k_2+1}^{Acc} = \mathbf{F}^{Acc}\mathbf{s}_{k_2}^{Acc} + \mathbf{v}_{k_2}^{Acc}, \quad \mathbf{v}_{k_2}^{Acc} \sim \mathcal{N}(\mathbf{0}, \mathbf{Q}_{k_2}^{Acc}), \tag{11}$$

where $\mathbf{F}^{Acc}$ again includes the CA transition matrices depending on ΔT_2, and $\mathbf{v}_{k_2}^{Acc}$ is a zero mean white Gaussian noise of covariance $\mathbf{Q}_{k_2}^{Acc}$.

At times when no GPS measurements are available, only the accelerations in body frame with $\mathbf{z}_{k_2}^{Acc} = (\mathbf{z}_a^b)_{k_2}$ are used for updating. When a new GPS measurement arrives the measurement vector is augmented to $\mathbf{z}_{k_2}^{Acc} = [(\mathbf{z}_a^b)^T, (\mathbf{z}^{GPS})^T]_{k_2}^T$:

$$\mathbf{z}_{k_2}^{Acc} = h^{Acc}(\mathbf{C}_b^n(\mathbf{u}_{k_2}), \mathbf{s}_{k_2}^{Acc}) + \mathbf{w}_{k_2}^{Acc}, \quad \mathbf{w}_{k_2}^{Acc} \sim \mathcal{N}(\mathbf{0}, \mathbf{R}_{k_2}^{Acc}), \tag{12}$$

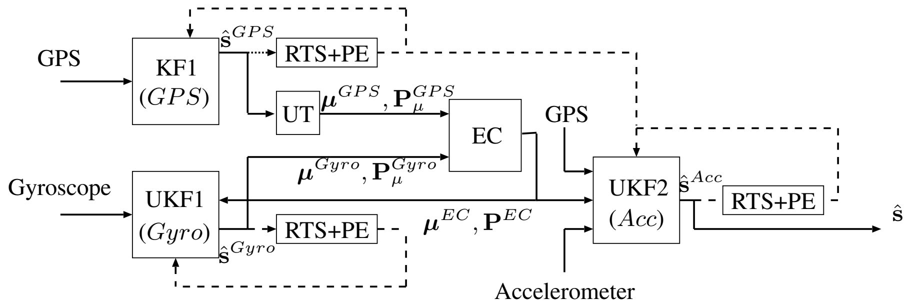

Fig. 1. Multi-stage Kalman filter architecture (solid lines) with extensions for parameter estimation (dotted lines).

where $h^{Acc}(\mathbf{C}_b^n(\mathbf{u}_{k_2}), \mathbf{s}_{k_2}^{Acc})$ denotes the coupling between Euler angles and state vector variables and the measured accelerations in body frame. Some entries of the measurement covariance of UKF2 $\mathbf{R}_{k_2}^{Acc}$ are the same as those of covariance $\mathbf{R}_{k_1}^{GPS}$ of KF1 and are therefore calculated only once in the E-step.

State estimates of each filter stage $f \in \{GPS, Gyro, Acc\}$ can be further improved by backward smoothing over a block of the last N values with the 'Rauch-Tung-Striebel' (RTS) algorithm [7]:

$$\mathbf{s}_{k|N}^f = \mathbf{s}_{k|k}^f + \mathbf{\Lambda}_k^f(\mathbf{s}_{k+1|N}^f - \mathbf{s}_{k+1|k}^f), \quad (13)$$

$$\mathbf{P}_{k|N}^f = \mathbf{P}_{k|k}^f + \mathbf{\Lambda}_k^f(\mathbf{P}_{k+1|N}^f - \mathbf{P}_{k+1|k}^f)(\mathbf{\Lambda}_k^f)^T, \quad (14)$$

where $\mathbf{\Lambda}_k^f = \mathbf{P}_{k|k}^f(\mathbf{F}_k^f)^T(\mathbf{P}_{k+1|k}^f)^{-1}$ is the smoother gain.

B. M-Step: Covariance Estimation

As the precision of the GPS measurements are time-variant due to the changing satellite constellation and changing radio propagation characteristics, and as the vehicle kinematics depend on the driving conditions, the covariance matrices of both measurement and process noise of all filters are reestimated to account for these changes. To allow for later online processing the input data are segmented into blocks of N samples, where N has to be chosen as a trade-off between reliable parameter estimation and low latency. Within each block the covariances can be reestimated using the results (13), (14) of the E-step [4]:

$$\begin{aligned}\hat{\mathbf{Q}}^f = \frac{1}{N}\sum_k(\mathbf{s}_{k|N}^f - \mathbf{F}^f\mathbf{s}_{k-1|N}^f)(\mathbf{s}_{k|N}^f - \mathbf{F}^f\mathbf{s}_{k-1|N}^f)^T \\ + \Big[\mathbf{P}_{k|N}^f + \mathbf{F}^f\mathbf{P}_{k-1|N}^f(\mathbf{F}^f)^T - \mathbf{P}_{k,k-1|N}^f(\mathbf{F}^f)^T \\ - \Big(\mathbf{P}_{k,k-1|N}^f)(\mathbf{F}^f)^T\Big)^T\Big] \quad (15)\end{aligned}$$

The use of an unscented transform can increase the performance compared to an analytical linearization. Instead of calculating the Jacobians of $h^f(\mathbf{s})|_{\mathbf{s}=\mathbf{s}_{k|N}}$, we apply the unscented transform to get the estimate $\tilde{\mathbf{H}}_k^f = E[h^f(\Delta\mathbf{s}_k^f)(h^f(\Delta\mathbf{s}_k^f))^T]$, where $\Delta\mathbf{s}_k^f = \mathbf{s}_k^f - \mathbf{s}_{k|N}^f$ and $h^f(\Delta\mathbf{s}_k^f)$ denoting the nonlinearity between the state and observation vector. This results in the observation covariance estimate

$$\hat{\mathbf{R}}^f \approx \frac{1}{N}\sum_k(\mathbf{z}_k^f - h^f(\mathbf{s}_{k|N}^f))(\mathbf{z}_k^f - h^f(\mathbf{s}_{k|N}^f))^T + \tilde{\mathbf{H}}_k^f. \quad (16)$$

A typical block size would be to estimate $\mathbf{R}_{k_1}^{GPS}$ over the last 30-40 sec. and the other covariance matrices every 2-6 sec.

IV. ON-LINE TV APPROACH

As the E-step involves the backward RTS-smoothing, which indeed is important to obtain good parameter estimates, we propose the following 'on-line' version of the above time-variant multi-stage Kalman filter (denoted TV-MS in the following), see also Fig. 2:

1) Forward filtering of data in block w using the currently available parameter estimates (E-step).
2) RTS-smoothing of the filter output of block $w-1$ (E-step): eqs. (13), (14).
3) Parameter estimation using the smoothed data of block $w-1$ (M-step): eqs. (15), (16).
4) If at least one of the following conditions is met:
   - Convergence of the log-likelihood,
   - Forward filtering of block w is finished,
   - Maximum no. of iterations is reached,

   then abort parameter estimation and goto 6. Else goto 5.
5) Forward filtering of the signals in block $w-1$ using the parameter estimates of step 3 and goto step 2.
6) Advance block index ($w := w+1$) and goto step 1, using the parameter estimates of step 3.

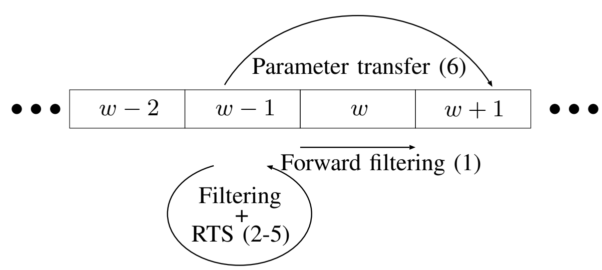

Fig. 2. Illustration of on-line TV-MS.

It should be pointed out, that usually the filtering and parameter estimation in steps 2, 3 and 5 is much faster than step 1, because all data of block $w-1$ are already cached.

V. EXPERIMENTAL RESULTS

A. Covariance Estimation

Fig. 3 shows the variance of the GPS position measurement in east direction. At time $t = 120$ sec. we simulate a rapid change of signal quality by reducing the variance from $\sigma_e^2 = 40$ to $\sigma_e^2 = 20$. The red (dashed) curve displays the estimate obtained from applying the proposed EM algorithm to reestimate the observation covariance matrix of KF1. The plot shows that the change in the variance is tracked very well, although only the observations of the last 40 sec. were used for estimation and although the initial value of $\sigma_e^2(0) = 32$ was chosen poorly. Fig. 4 shows how the measurement covariance

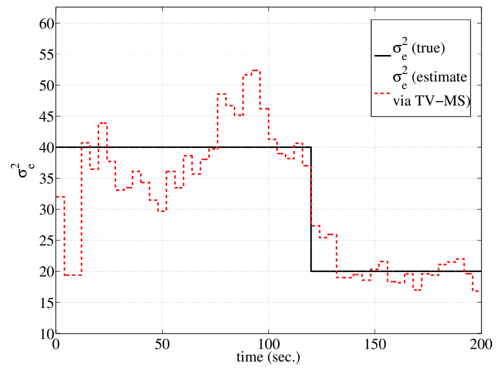

Fig. 3. Measurement covariance of position (east-direction).

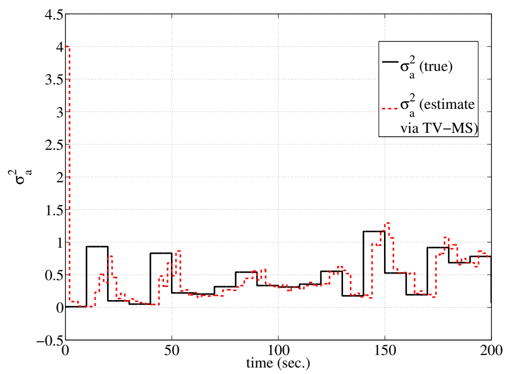

Fig. 4. Measurement covariance of acceleration.

of the first acceleration component of UKF2 is tracked. Here, we also assume a wrong initialization value, which does not seem to matter, as it is corrected very fast by reestimation.

B. Localization Performance

In order to study the localization performance vehicle trajectories were generated with a piecewise linear kinematic model, where the switching between the two models CV and CA was done according to the transition probability matrix $\mathbf{\Sigma} = \begin{pmatrix} 0.9168 & 0.0832 \\ 0.2328 & 0.7672 \end{pmatrix}$. These values of $\mathbf{\Sigma}$ had been trained on real driving data.

Time-varying observation and process noise covariances were employed in the data generation, where variances were drawn from a normal distribution whose standard deviation was 0.4 times the mean. For example, the GPS position measurements were generated by adding to the true position value observation noise, whose RMS value was drawn from a normal density with mean 35 m and standard deviation $0.4 \cdot 35$ m, limited to positive values.

Fig. 5 compares the true value of the yaw angle φ (black) with the estimated yaw angle computed by the unscented transform following the GPS filter KF1 (blue) and the filtered gyroscope data at the output of UKF1 without EC and feedback to its prediction step (green). While the filters were initialized with the true value, the drift of the gyroscope as described in section II can be clearly seen. Although the GPS

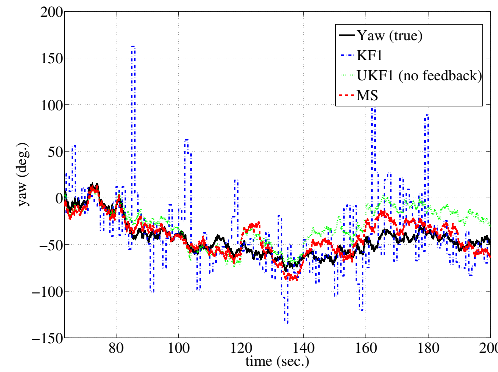

Fig. 5. Effect of estimator combination.

measurements seem to be not so good, their combination with the UKF1 output in the EC module and the feedback of the output to UKF1 delivers estimates at the EC output of higher quality (red). The reason is the weighting of the individual estimates by their error covariance matrices, which accounts for the estimator quality in an optimal way.

Fig. 6 shows the cumulative density functions (CDF) of the position estimates of different filtering approaches, where all approaches assumed constant observation and process noise covariances, which had been set to the average value of those underlying the data generation. As can be seen, a GPS Kalman filter or a dead reckoning using GPS data as nodes alone perform very poorly, while the MS Kalman filter achieves slightly better results than the IMM. Note that for the results given here the MS approach did not include the RTS smoothing, as this is only used for parameter estimation. As a kind of performance upper bound results are given for an IMM which incorporates backward filtering and smoothing between forward and backward estimates on the complete trajectory of length 500 sec. (batch IMM). This offline batch method is of course not applicable for online processing in a real car environment as it is unable to deliver instantaneous position estimates.

In Fig. 7 we look at joint parameter estimation and tracking. It can be seen that the GPS and DR approaches benefit greatly from using time-variant (TV) covariance estimates delivered by the EM algorithm, while the IMM methods even degrade compared to using fixed values as in Fig. 6. The reason for the latter is probably that due to the high dimension of the state (27) and observation vector (9) in the IMM there are too many covariance parameters which cannot be estimated

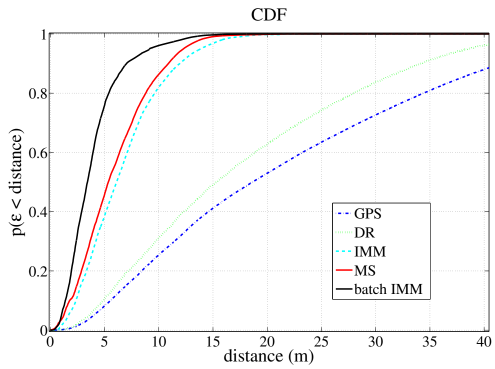

Fig. 6. Cumulative density functions assuming time-invariant parameters.

reliably. The time-variant MS approach is slightly better than the time-invariant MS method (see Fig. 6), and its resulting positioning accuracy almost reaches that of the batch TV-IMM algorithm.

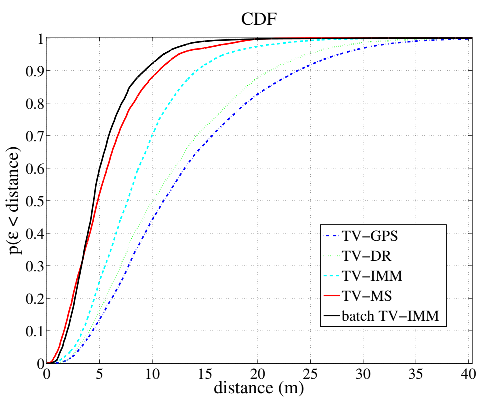

Fig. 7. Cumulative density functions for joint covariance parameter estimation and tracking.

C. Complexity

Although the multi-stage Kalman filtering approach seems to be computationally complex, its complexity is significantly lower than that of the IMM filter. Table I shows the mean elapsed times of the different processing steps in the two filtering approaches for processing a block of 10 sec. of measurement data. The simulations had been carried out with Matlab$^{\text{TM}}$ R2008 on a 2.33 GHz Quad-Core Xeon Processor with 4 GB RAM.

The IMM state vector is of size (27×1) and contains $[\mathbf{p}^T, \mathbf{v}^T, \mathbf{a}^T, \tilde{\boldsymbol{\mu}}^T, \dot{\tilde{\boldsymbol{\mu}}}^T, (\boldsymbol{\epsilon}_a^b)^T, (\dot{\boldsymbol{\epsilon}}_a^b)^T, (\boldsymbol{\epsilon}_\omega^b)^T, (\dot{\boldsymbol{\epsilon}}_\omega^b)^T]^T$, while the three state estimators of the MS filter have state vector dimensions of 9 (KF1), 12 (UKF1) and 15 (UKF2), respectively.

Data block of 10 sec.	TV-IMM	TV-MS
Forward filtering	15.5 sec.	6.5 sec.
Backward filtering + Smoothing	23.8 sec.	-
RTS	-	0.63 sec.
Covariance estimation	5.8 sec.	0.1 sec.

TABLE I
RUNTIMES OF MS AND IMM STAGES.

The table indicates that the forward filtering step of the IMM has a real-time factor of 1.5, while the backward filtering and smoothing asks for even 2.4 times real-time. On the other hand, the MS approach is well feasibly in real-time. The forward filtering requires about 6.5 sec., while the RTS-algorithm is much faster than the multiple-model backward filtering and smoothing of the IMM. Also the covariance estimation is much faster.

The higher elapsed time for the IMM is in part due to the higher dimension of the state vector. Compared to the other filter equations the calculations of the matrix inversions inside the Kalman filters are the computationally most expensive steps, as the complexity of an inversion is of order $\mathcal{O}(D^3)$, where $(D \times D)$ is the matrix dimension. Due to the smaller state dimensions of the MS method, these calculations can be done much faster than one inversion of a matrix of higher dimension (IMM). Note, that the EC requires one inversion, too. But this is only done, when a GPS measurement is available, see eq. (10).

Further, the backward filtering within the IMM algorithm is complex, as it needs a second Extended Kalman filter while the RTS of the MS approach is realized simply by eqs. (13), (14). The smoothing steps of the IMM are of higher complexity, too, because for each time step the state probabilities of all models have to renewed (Gaussian mixture densities are approximated by a single normal density).

VI. CONCLUSIONS

In this paper, a sensor fusion algorithm based on a multi-stage Kalman filtering combined with online parameter reestimation for a robust vehicle localization is proposed. While individual filters for GPS only or in combination with DR perform poorly, the proposed scheme turned out to be very effective. Even the IMM approach, which is of much higher computational complexity, provides worse estimation results. Further it is shown that the time-variant covariance matrices can be tracked with the block-wise EM algorithm, although there is a short latency. In future research we will investigate the use of this localization approach in the context of a car-to-car communication scenario.

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