Particle Filtering of Database assisted Positioning Estimates using a novel Similarity Measure for GSM Signal Power Level Measurements

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Abstract - In this paper we present a novel and statistically motivated similarity measure for database assisted positioning of GSM mobile terminals by evaluating signal power level reports which are transmitted regulary. Unlike a simple Euclidian distance measure, the proposed scheme incorporates inherent information about signal power level measurements requested by the serving base station but not reported by the mobile terminal. Furthermore we show how the Monte Carlo method of nonlinear post filtering using particle filtering helps to obtain better position estimates and surprisingly also helps to reduce the computational complexity. Results are presented for real field data.

1 Introduction

Location based mobile telephony services like automatic localisation of emergency calls or virtual travel guides are predicted to have a high market potential, making them attractive for content providers. Also network operators could benefit from location information e.g. for enhanced hand-over strategies or location-dependent billing. In particular the regulations put forward by the FCC on enhanced emergency calls [1] have incented major research efforts to obtain precise position estimates of the mobile terminal.

Whereas various other proposed localisation methods like triangulation based or GPS assisted approaches ask for extensive modifications of the network infrastructure or the terminal hardware, database assisted positioning (DAP) is compatible with existing terminals and the GSM network infrastructure. This pattern recognition approach utilises the location-dependency of signal parameters observed by the terminal such as signal power levels of neighbouring base stations, and compares the measured parameter values with those stored in a database, where a database entry consists of measured or predicted values along with the position coordinates. Additionally, DAP is applicable in the whole cellular network coverage area, even for indoor areas and in non line of sight scenarios which normally cause problems when using triangulation based or GPS assisted approaches.

However, DAP suffers from limited accuracy. The accuracy of better than 83m in 67% given in [3] seems to be quite optimistic since the model parameters have been optimised for the given test data. In [2] an accuracy better than 44m for 67% of all measurements has been reported. This excellent result, however, could only be achieved by exploiting map information in addition to the comparison with a database of measured signal power levels. In [4] it has been shown that the accuracy of the database approach can be improved by using robust metrics that are insensitive to variations of the measured signal power level due to fading or shadowing. Also post filtering using a state-space model of the terminal movement can significantly increase accuracy [5]. By using particle filtering for this purpose and evaluating data of wheel speed sensors within ABS units positioning accuracies similar to GPS were reported in [6].

The database assisted approach presented here is based on standard GSM system signalling. Signal power level measurement reports are evaluated which are regularly transmitted in the Radio Subsystem Link Control, e.g. to initiate hand-overs. Taking a pattern recognition approach, measured signal power levels are compared with a database (map) of measurement predictions of signal power levels using a novel similarity measure. Then particle filtering was adopted to smooth instantaneous position estimates over time.

After a brief introduction into the principle of DAP we will analyse the GSM signal power level measurement procedure and from this we will derive a statistically motivated similarity measure. The positioning accuracies obtained by this measure will be presented afterwards. In section 4 the principle of particle filtering will be presented and we will show how such post filtering can be applied to the domain of DAP. After presentation of experimental results, a conclusion ends this paper.

2 Database assisted Positioning

We are given a database of location dependent signal power levels. The database contains K entries of the form

$$T_k = (\ell_k, \mathbf{s}(\ell_k)); \qquad k = 1, \ldots, K. \tag{2.1}$$

Here, ℓ_k denotes the location of the k-th entry in 2-D coordinates, and $\mathbf{s}(\ell_k)$ is a vector of signal power levels, where each vector component corresponds to the predicted signal power level of a certain base station if the terminal is located at ℓ_k.

DAP faces two critical issues. The first one is the compilation of the database, which can be very cost-intensive. For cellular networks, however, these databases typically already exist, since wave propagation prediction is an essential element of the network planning process. In our case we used a database which contained predicted signal power levels of together 107 GSM-900 and GSM-1800 base stations of the Vodafone Network in the city of Stuttgart, Germany. These predictions where calculated using a COST-231 Model by our colleagues of the "Institut für Hochfrequenztechnik" at the University of Stuttgart. The database covers an area of 4500m $\times$ 4950m containing 223969 locations in a regular grid with an element spacing of 10m. Due to its size and for ease of maintenance the database should be stored in a central place in the network resulting in network-based position estimation.

The second issue is the necessity of an appropriate similarity measure. The similarity measure presented in this paper has been derived from a probabilistic model of the database entries and the signal power level measurements which was obtained by analysing the hand-over procedure which is part of the GSM Radio Subsystem Link Control.

2.1 Measurement Procedure

In order to guarantee stable operation and to initiate hand-overs if necessary, the serving base station periodically requires feedback about the signal quality received by the mobile terminal when operating in "dedicated mode". Therefore every 480ms a list (network neighbourhood list) of $N_B \leq 16$ neighbouring base stations is transmitted to the terminal to initiate the measurement of the signal power levels of their broadcast control channels (BCCHs). Additionally, the terminal will measure the signal power level of the serving base station. Let the resulting measurement vector of size $N_B + 1$ be denoted by $\boldsymbol{\gamma}(n) = (\gamma_0(n), \gamma_1(n), \dots, \gamma_{N_B}(n))^T$, where n is the discrete time index. It consists of the entries $\gamma_j(n)$, where $j = 0$ refers the serving base station and $j = 1, \dots, N_B$ denotes the measurement of the neighbouring base station. Without loss of generality, let these N_B measurements of neighbouring base stations be sorted in descending order. Clearly, $\gamma_1(n)$ is the measurement of the neighbouring base station with the largest measured signal power level, $\gamma_2(n)$ the measurement of the neighbouring base station with the second-to-largest level, etc. However, not the complete vector is reported to the serving base station, but only the signal power level measurement of the serving base station and the $N_R \leq 6$ largest signal power levels of the neighbouring base stations together with the corresponding base station identifiers. Note that N_R may change over time, however we ignore this in our notation.

Due to the GSM-internal representation of signal power level measurements in a meter called RXLEV, reported measurements can only represent values in the range of -110dBm to -48dBm in step sizes of 1dBm. Thus all measurements above -48dBm and below -110dBm are clipped before they are transmitted. These clipped values can be regarded as "above -48dBm" and "below -110dBm" respectively.

Let the resulting subvector of $\boldsymbol{\gamma}(n)$ which is reported to the base station be denoted by $\boldsymbol{\gamma}'(n) = (\gamma'_0(n), \gamma'_1(n), \dots, \gamma'_{N_R}(n))^T$. The first entry $\gamma'_0(n)$ refers to the serving base station and $\gamma'_j(n), j = 1, \dots, N_R$ refers to the possibly clipped measurement $\gamma_j(n)$ of the neighbouring base station, for which a signal power level was measured and ranked on j-th position. Note that at different times n a different subset of base stations delivers the N_R largest values.

2.2 Field Data

A measurement campaign has been conducted by our colleagues from the University of Stuttgart. A team member equipped with a GSM-handheld (Siemens SL45) and a GPS unit both hooked up to a laptop walked four different routes each of a maximum length of about approx. 3km. Every 5 seconds on average the signal power level measured by the cellular phone was recorded together with the geographic coordinates obtained by the GPS unit, amounting to a total of 947 measurement vectors.

The left part of figure 1 shows the histogram of the recorded signal power level measurements. Clearly the effect of clipping at -48dBm can be observed in respect to excellent reception conditions.

The effect of reporting only the strongest signal power level measurements and, again, of clipping can be seen in the scatter plot presented in the right part of figure 1. It shows the difference between the measured and predicted signal power levels versus the predictions, where the predictions are obtained from the database. While the effect of clipping is seen by the fact that the data cloud is strictly upper bounded by a certain line with negative slope, the fact that only the strongest measurements are reported causes the average difference between measurement and prediction to be positive, since the measured signal power level tends to be larger than the predicted one (because a small measured signal power level may not have been reported). Indeed, for predictions not seriously affected by clipping, it can be clearly seen that the average of the measurement errors is positive for predictions.

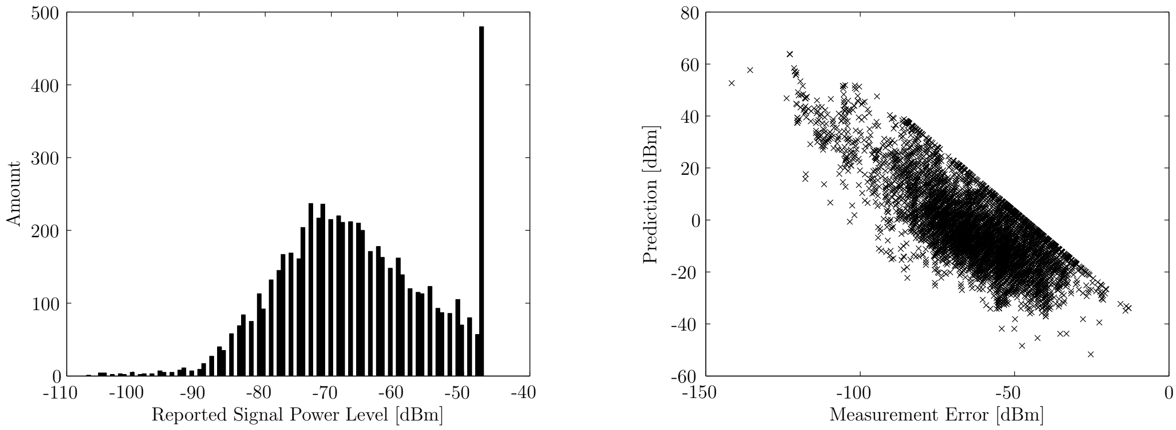

Figure 1: Histogram of recorded signal power level measurements (left) and scatter plot of deviations of measurements from predictions versus the predictions (right).

3 Similarity Measures

Regarding DAP as a pattern recognition problem, the location of that entry of the database is chosen as an estimate of the terminal position, which maximises the probability of having given the reported vector:

$$\hat{\ell}(n) = \underset{\ell_k}{\operatorname{argmax}}\, p\left(\boldsymbol{\gamma}'(n)|\mathbf{s}(\ell_k)\right). \tag{3.2}$$

Thus the required similarity measure is equal to the probability of the reported N_R signal power level measurements having emerged from the predictions of the regarded position in the database. In order to calculate this likelihood, the density function $p\left(\boldsymbol{\gamma}'(n)|\mathbf{s}'(\ell_k)\right)$ must be known. Assuming independent and identically distributed (i.i.d.) measurements, the probability of the reported measurement vector can be split up into the product of the probabilities of its N_R separate measurements:

$$p\left(\boldsymbol{\gamma}'(n)|\mathbf{s}(\ell_k)\right) = \prod_{i=0}^{N_R} p\left(\gamma_i'(n)|s_i(\ell_k)\right), \tag{3.3}$$

where $s_i(\ell_k)$ is the corresponding prediction of $\gamma_i(n)$ for position ℓ_k.

The required density functions $p\left(\gamma_i'(n)|s_i(\ell_k)\right)$ can be obtained by analysing the described measurement procedure, beginning with the measurement of the N_B base stations. In lack of evidence for any other distribution we assume an additive error model for these measurements, where the error is normally distributed:

$$\begin{aligned} p\left(\gamma_j(n)|s_j(\ell_k)\right) &= p_{e_j}\left(\gamma_j(n) - s_j(\ell_k)\right) \\ &= \mathcal{N}(\gamma_j(n) - s_j(\ell_k); \mu, \sigma^2). \end{aligned} \tag{3.4}$$

Here, $e_j = \gamma_j(n) - s_j(\ell_k)$ is the prediction error for the j-th base station's signal power level measurement, and $\mathcal{N}(x; \mu, \sigma^2)$ denotes a Gaussian distribution with mean μ and variance σ^2, evaluated at x. The bias μ considers systematically prediction errors.

Now the probability, that $\gamma_1(n)$ is observed and that it is the maximum of all N_B measured signal power levels can be computed using order statistics [7]:

$$p\left(\gamma_1(n)|s_1(\ell_k)\right) = \sum_{i=1}^{N_B} \left(p\left(\gamma_i(n)|s_i(\ell_k)\right) \prod_{l=1,l\neq i}^{N_B} P\left(\gamma_l(n)|s_l(\ell_k)\right) \right), \tag{3.5}$$

where $P(\cdot)$ denotes the cumulative probability function. Note that in this first step, clipping is disregarded by assuming $\gamma'_i(n) = \gamma_i(n)$. Then equation 3.5 is the probability of having observed $\gamma'_i(n)$: $p\left(\gamma_1(n)|s_1(\ell_k)\right) = p\left(\gamma'_1(n)|s_1(\ell_k)\right)$. Similarly, the likelihoods of the other reported signal power levels ranked second until rank N_R can be obtained [8].

Since these formulae are quite unwieldy, the densities are approximated by Gaussians with properly chosen means and variances, denoted by $\tilde{\mu}$ and $\tilde{\sigma}^2$ in the following. Utilising the reported signal power level measurements, we arrive at a first similarity measure, which is denoted by version "A":

$$p_A\left(\boldsymbol{\gamma}'(n)|\mathbf{s}(\ell_k)\right) = \prod_{i=0}^{N_R} p_A\left(\gamma'_i(n)|s_i(\ell_k)\right) = \prod_{i=0}^{N_R} \mathcal{N}(\gamma'_i(n) - s_i(\ell_k); \tilde{\mu}, \tilde{\sigma}^2). \tag{3.6}$$

This similarity measure can be enhanced by incorporation of clipping. In the following we show how to extract maximum information from a reported signal power level $\gamma'_i(n)$ clipped to a value meaning "above -48dBm".

While the density function of the measurement is known, see equation (3.6), it is not possible to calculate the exact likelihood of a clipped value, because the unclipped value unknown. The only fact known is that this value is greater than or equal to -48dBm. Nevertheless it is possible to calculate a minimum mean square error (MMSE) estimate of the likelihood due to the fact that the probability density function is known and bounded by -48 dBm. The MMSE estimate is the conditional expectation:

$$\hat{p}(\gamma'_i(n) > -48\text{dBm}|s_i(\ell_k)) = E[p(\gamma'_i(n)|s_i(\ell_k))|\gamma'_i(n) > -48\text{dBm}]. \tag{3.7}$$

To simplify notation, let $x \sim p(\gamma'_i(n)|s_i(\ell_k))$, i.e. x is a random variable drawn from the respective density. The a-posteriori probability density function of x given $\gamma'_i(n) > -48\text{dBm}$ is:

$$p(x|\gamma'_i(n) > -48\text{dBm}) = \begin{cases} K \cdot p(x) & \text{for} \quad x > -48\text{dBm} \\ 0 & \text{else} \end{cases} \tag{3.8}$$

where the normalisation constant

$$K = \frac{1}{\int_{-48\text{dBm}}^{\infty} p(\xi) d\xi} \tag{3.9}$$

ensures that the integral over the density is 1. $\hat{p}(\gamma'_i(n) > -48\text{dBm}|s_i(\ell_k))$ being zero for $x \leq -48\text{dBm}$ results from the known fact that the unclipped measured signal power level is above -48dBm. Thus a more reliable position estimation can be obtained by use of this enhanced similarity measure denoted "B":

$$p_B\left(\boldsymbol{\gamma}'(n)|\mathbf{s}(\ell_k)\right) = \prod_{i=0}^{N_R} p_B\left(\gamma'_i(n)|s_i(\ell_k)\right) \tag{3.10}$$

with

$$p_B\left(\gamma'_i(n)|s_i(\ell_k)\right) = \begin{cases} p_A\left(\gamma'_i(n)|s_i(\ell_k)\right) & \text{if } \gamma'_i \leq -48\text{dBm} \\ \frac{\int_{-48\text{dBm}}^{\infty} p_A^2(x|s_i(\ell_k))dx}{\int_{-48\text{dBm}}^{\infty} p_A(x|s_i(\ell_k))dx} & \text{else.} \end{cases} \tag{3.11}$$

Sofar we have not yet exploited all available information. Comparing the list $\boldsymbol{\gamma}(n)$ of N_B neighbouring base stations for which the serving base station requested a signal power level measurement with the smaller list $\boldsymbol{\gamma}'(n)$ of N_R base station's signal power level measurements reported by the terminal, we can draw the following conclusion: the unreported signal power level of a neighbouring base station must have been smaller than or equal to the smallest reported signal power level, since the signal power level of all N_B stations have been measured, but only the strongest N_R have been reported to the serving base station by the mobile.

This information can be included in a similar way like clipped values are handled: Since these measurements are not known by the serving base station, it is possible to calculate their likelihoods by conditional expectations due to the fact that their distribution function is known and that these unreported measurements are bounded by the smallest reported measurement. This allows to extend our similarity measure by all unreported measurements resulting in similarity measure "C":

$$p_C\left(\boldsymbol{\gamma}'(n)|\mathbf{s}(\ell_k)\right) = \prod_{i=0}^{N_R} p_B\left(\gamma_i'(n)|s_i(\ell_k)\right) \prod_{l=N_R+1}^{N_B} p_C\left(\gamma_l(n)|s_l(\ell_k)\right) \tag{3.12}$$

with

$$p_C\left(\gamma_l(n)|s_l(\ell_k)\right) = \frac{\int_{-\infty}^{\gamma_{N_R}'(n)} p_A^2\left(x|s_l(\ell_k)\right)dx}{\int_{-\infty}^{\gamma_{N_R}'(n)} p_A\left(x|s_l(\ell_k)\right)dx} \tag{3.13}$$

3.1 Experimental Results

In a first step, these three versions "A", "B" and "C" of similarity measures are compared using the data from the measurement campaign described earlier. Measurements from three routes through Stuttgart were used as training data to estimate means and variances of the Gaussians, and the data of the fourth campaign was used as test data.

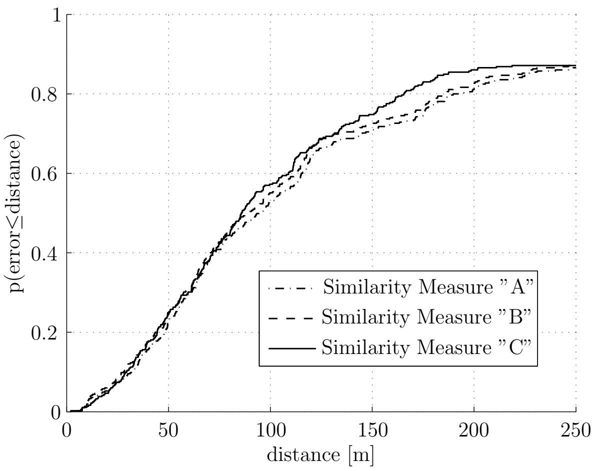

Figure 2: Positioning errors by instantaneous position estimates.

Figure 2 depicts the cumulative probability function of the position error for the three presented similarity measures. The simplest similarity measure "A" is marked as a dash-dotted line. Using this measure a positioning error of less than 100m was obtained in 52% of the cases and below

250m in 86% which does not meet FCC's requirements. The results of clipping compensation are shown as a dotted line. The positions estimates are slightly better since only 8.7% of all measurements were clipped. Therefore slightly more reliable results can be attained utilising the inherent information about unreported signal power levels.

In control experiments with synthetically generated measurement data and perfect knowledge of the list of neighbouring base stations the improvement compared to "A" by using similarity measure "C" were much higher. The somewhat disappointed improvements observed on the field data may in part be explained by the fact that the network neighbourhood list had originally not been recorded during the measurement campaign and could only partly be reconstructed afterwards.

4 Post Filtering

Since instantaneous position estimates disregard any constraints arising from the limited mobility of a mobile terminal post filtering using an appropriate state space model of the terminal movement and an appropriate measurement model is applied to raise accuracy.

The state space model of terminal movement consists of two equations. The state equation describes the terminal movement in a dynamical state model driven by a system noise vector $\mathbf{v}(n-1)$:

$$\mathbf{x}(n) = f_n(\mathbf{x}(n-1), \mathbf{v}(n-1)). \tag{4.14}$$

Here, $\mathbf{x}(n)$ is the state vector which in our case consists of the terminal position and velocity in Cartesian coordinates, and f_n is the function describing the process dynamics.

The second equation is the measurement equation, which describes how the measurement $z(n)$ depends on the state vector $\mathbf{x}(n)$:

$$\mathbf{z}(n) = h_n(\mathbf{x}(n), \mathbf{w}(n)), \tag{4.15}$$

where h_n is a possibly non-linear function and the vector $\mathbf{w}(n)$ is called measurement noise.

Our goal is to estimate $p(\mathbf{x}(n)|\mathbf{Z}(n))$, i.e. the a posteriori density of the system state at time n given the measurements $\mathbf{Z}(n) := \{\mathbf{z}(1), \mathbf{z}(2), \ldots, \mathbf{z}(n)\}$ in a recursive manner from $p(\mathbf{x}(n-1)|\mathbf{Z}(n-1))$. In principle this can be done by the following two equations:

$$p(\mathbf{x}(n)|\mathbf{Z}(n)) = \frac{p_L(\mathbf{z}(n)|\mathbf{x}(n)) \cdot p(\mathbf{x}(n)|\mathbf{Z}(n-1))}{c(n)}, \tag{4.16}$$

where $c(n) = p(\mathbf{z}(n)|\mathbf{Z}(n-1))$ is a normalization constant, and

$$p(\mathbf{x}(n)|\mathbf{Z}(n-1)) = \int p(\mathbf{x}(n)|\mathbf{x}(n-1)) p(\mathbf{x}(n-1)|\mathbf{Z}(n-1)) dx(n-1). \tag{4.17}$$

Here, $p_L(\mathbf{z}(n)|\mathbf{x}(n))$ is called likelihood function.

In the case of a linear dynamical model and white Gaussian system and measurement noise the solution to this set of equations is the well-known Kalman filtering. In our case we can assume a linear state equation, but the measurement equation is definitely non-linear, since the measurements, i.e. the signal power levels depend in a non-linear fashion on the terminal position. Therefore particle filtering, which is able to cope with a non-linear system model, has been employed [9, 10].

4.1 Particle Filtering

Particle filtering is a sequential Monte Carlo method where densities are approximated by a weighted set of samples drawn from them rather than by moments. It operates on the true measurements, the signal power levels. If Kalman filtering would be used, the signal power level measurements would first have to be reduced to an instantaneous position estimate which is obtained by determining the maximum of the likelihood function, see equation (3.2). By doing so, a lot of information is lost, e.g. a hypothetical position with just a little smaller likelihood than the position winning the argmax-operation, would be completely disregarded.

This is not the case when using particle filtering. Here, the a-posteriori density $p(\mathbf{x}(n)|\mathbf{Z}(n))$ is approximated by a set of N_S samples $\mathbf{x}^j(n), j = 1, 2, \ldots, N_S$ with according weights $w^j(n)$ drawn from a so-called proposal density. These samples, which are called particles, represent hypothesised terminal states. In a first processing step of every iteration, all weights are updated by evaluating the new measurement using the likelihood function $p_L(\mathbf{z}(n)|\mathbf{x}^j(n))$:

$$w^j(n) := w^j(n-1) \cdot p_L(\mathbf{z}(n)|\mathbf{x}^j(n)) \tag{4.18}$$

Normalisation of the weights allows to calculate the MMSE estimate of the target's state is obtained as the weighted sum of all particles:

$$\hat{\mathbf{x}}(n) = \frac{\sum_{j=1}^{N_S} w^j(n) \cdot \mathbf{x}^j(n)}{\sum_{j=1}^{N_S} w^j(n)} \tag{4.19}$$

In a second step, resampling is performed in order to avoid degeneration of the system by discarding unlikely hypotheses. This step can be executed adaptively triggered by the effective weight $N_{\text{eff}} = \frac{1}{\sum_{m=1}^{N_S} (w^j(m))^2}$ having fallen below a given threshold N_T. Then resampling with replacement can be done where particles are sampled with a probability equal to their weights for sampling importance resampling (SIR) particle filtering.

In the third step, all particles are propagated using the state space model in order to model the a-priori density for processing the next measurement.

4.2 Particle Filtering for DAP

As mentioned earlier, for DAP we used a state vector which consists of the terminal position and its velocity in two-dimensional Cartesian coordinates. Further, a linear state equation was used where the system noise term corresponded to a random acceleration. The measurement vector is the vector of reported signal power levels: $\mathbf{z}(n) = \boldsymbol{\gamma}'(n)$.

The required likelihood function $p_L(\mathbf{z}(n)|\mathbf{x}^j(n))$ is the presented similarity measure "C" which delivered best results for instantaneous estimates. By use of this similarity measure the reported measurement vector is compared with the predictions on the particle's positions which are attained by bilinear interpolation of the 4 nearest locations in the database, if the hypothesised position was not on the grid for which predictions are stored.

A critical issue is the initialisation of particle filtering. Our designed filter is initialised by placing the particles on those locations of the database entries that achieve highest similarity for the first measurement.

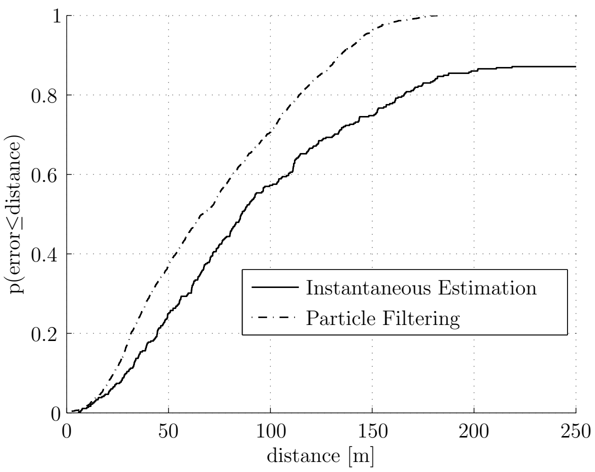

Figure 3: Positioning errors by post filtered position estimates.

Surprisingly, after initialisation particle filtering needs much less computing resources than the computation of the instantaneous estimate (3.2). This is due to the fact the time intensive search for the maximum is avoided that needs to access every entry of the database.

4.3 Experimental Results

Post-filtering was applied to the field data. We employed a SIR particle filtering using $N_S = 450$ particles with resampling triggered on a an effective weight N_{eff} of below $N_T = 90$. Since particle filtering belongs to the family of random algorithms a route was estimated ten times to reduce influences caused by randomisation.

The solid line in figure 3 repeats the results for instantaneous location estimates using similarity measure "C". The dotted line shows the results obtained from post filtering using particle filtering.

Applying particle filtering, a positioning error of less than 100m was obtained in 70% of the cases and below 300m in 99% which meets FCC's requirements. Compared to the results reported by Laitinen et al. [2] mentioned in the introduction, these results are modest. However, it should be noted that no additional information like a-priori knowledge about allowable positions is used and that the proposed scheme is fully compatible with existing terminals and network infrastructure.

5 Conclusions

In this paper, a novel similarity measure for database assisted positioning was presented. This measure was derived by analysing the GSM measurement protocol. While leaving the procedure of reporting measurements unchanged and thus being compatible with existing terminals and base stations, inherent information about clipped and unreported signal power level measurements can be considered while leaving the protocol unchanged. Furthermore we have shown how nonlinear post filtering through particle filtering can be carried out to obtain more reliable position estimates in reduced calculation time.

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References

[1] FCC, Wireless 911 Requirements, URL: `http://www.fcc.gov/911/enhanced/`

[2] H. Laitinen, J. Lähteenmäki, T. Nordström, "Database Correlation Method for GSM Location", *Proc. IEEE Veh. Technol. Conf.*, Rhodes, Greece, May 2001

[3] D. Zimmermann, J. Baumann, M. Layh, F. Landstorfer, R. Hoppe, G. Wölfle, "Database Correlation for Positioning of Mobile Terminals in Cellular Networks using Wave Propagation Models", *Proc. IEEE Veh. Technol. Conf.*, Los Angeles, CA, September 2004

[4] Dr. H. Schmitz, Dr. M. Kuipers, Dr. K. Majewski, Dr. P. Stadelmeyer, "A New Method for Positioning of Mobile Users by Comparing a Time Series of Measured Reception Power Levels with Predictions", *Proc. IEEE Veh. Technol. Conf.*, Jeju, Korea, April 2003

[5] D. Catrein, M. Hellebrandt, R. Mathar, "Location Tracking of Mobiles: A Smart Filtering Method and its Use in Practice", *Proc. IEEE Veh. Technol. Conf.*, Milan, Italy, May 2004

[6] F. Gustafsson, F. Gunnarsson, N. Bergman, U. Forssell, J. Jansson, R. Karlsson, P.-J. Nordlund, "Particle Filters for Positioning, Navigation and Tracking", *IEEE Transactions on Signal Processing*, vol. 50, no. 2, pp. 425-437, 2002

[7] A. Papoulis, S. U. Pillai, "Probability, Random Variables and Stochastic Processes with Errata Sheet", 4th ed., New York, NY, McGraw-Hill Education, 2002

[8] H. A. David, H. N. Nagaraja, "Order Statistics", 3rd ed., New York, NY, Wiley-Interscience, 2003

[9] A. Doucet, N. de Freitas, N. Gordon, "Sequential Monte Carlo Methods in Practice", Springer, June 2001

[10] M. Sanjeev Arulampalam, S. Maskell, N. Gordon, T. Clapp, "A Tutorial on Particle Filters for Inline Nonlinear/Non-Gaussian Bayesian Tracking", *IEEE Transactions on Signal Processing*, vol. 50, no. 2, pp. 174-188, 2002