A DIGITAL SYNCHRONIZER FOR LINEARLY MODULATED SIGNALS TRANSMITTED OVER A FREQUENCY-NONSELECTIVE FADING CHANNEL

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Abstract

This paper presents a digital carrier recovery structure which allows coherent detection on frequency-nonselective fading channels. Basically, the synchronizer estimates the multiplicative distortion introduced by the channel. It is shown that the proposed structure is clearly superior to a PLL and that it is well suited for a fully digital realization. A detailed synchronizer design and simulation results are presented for the land-mobile radio channel. This includes a novel scheme for a fully digital frequency offset estimation and correction.

I. Introduction

It is well known that coherent detection schemes are superior to differentially coherent or noncoherent schemes in terms of power efficiency. However, carrier recovery, necessary for coherent detection, is the key problem on a fading channel with its amplitude fluctuations and random frequency modulation. If the fading is rapid enough this precludes any phase-locked type of carrier recovery [1]. Thus, though desirable, reliable coherent communication with suppressed carrier modulation is often considered unfeasible. However this conclusion being based on a PLL as the carrier recovery unit should not be generalized. The PLL is just not a suitable structure for this application.

An alternative is the usage of a pilot tone for carrier tracking [e.g. 2]. However, the drawback of this method is that part of the transmitter power and bandwidth must be spent on the carrier thus degrading the signal-to-noise ratio for data detection. Another proposed alternative is to depart from coherent detection and to use differential detection since it is assumed that the detection loss due to phase jitter on the carrier reference exceeds the signal-to-noise ratio penalty associated with differential detection. However, differential detection suffers in the same way from phase variations of the channel as the coherent scheme does. Actually it can be shown that the (optimal) coherent detection presented here almost always outperforms differential detection [3].

In a previous paper [4] we systematically found the principal structure of the optimal carrier recovery by deriving the maximum a posteriori detector of the transmitted symbol sequence. The objective of this paper is to demonstrate the suitability of the synchronizer for application in the specific fading environment of a frequency-nonselective land-mobile radio channel. In section III we design the carrier recovery and compare it with a phase-locked loop. In many situations an unknown time-varying frequency offset may be present. In section IV the synchronizer structure is extended to cope with this situation. This results in a novel scheme for a fully digital carrier recovery in the presence of fading and frequency variations. In section V a simple method for elimination of the symbol phase for MPSK modulation is described.

II. Channel Model and Receiver Structure

We consider linearly phase modulated signals transmitted over a frequency-nonselective Rayleigh fading channel. The received signal, in complex baseband notation, is

$$r(t) = h(t)\exp\left[j\int_0^t \omega_d(\tau)d\tau\right]\sqrt{E_s/T}\sum_{i=0}^{\infty} a_i g(t-iT) + \tilde{n}(t) \qquad (1)$$

where a_i is the i-th transmitted symbol and $g(t)$ is the signal pulse. With the normaliza-

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tion $\int g(t)g^*(t)dt = T$ and $|a_i| = 1$, E_s denotes the energy per symbol. T is the symbol period and $\tilde{n}(t)$ is complex additive white gaussian noise with doublesided spectral density N_0. The fading distortion h(t) has unity average power and consists of two independent and identically distributed zero mean gaussian noise processes $h_I(t)$ and $h_Q(t)$, i.e. $h(t)=h_I(t)+jh_Q(t)$. Here, $\omega_d(t)$ represents a time-varying frequency offset (in rad/s) of the demodulated complex baseband signal. This frequency offset can be due to oscillator inaccuracies or instabilities which cause a difference between transmitter and receiver oscillator frequencies.

In the receiver the incoming signal is filtered by a matched filter with impulse response $g^*(-t)$. Sampling the matched filter output at t = kT and using a normalization to simplify later calculations gives the following sufficient statistic representation of the underlying continuous-time signal

$$z(k) := \frac{1}{\sqrt{E_s/T}} \frac{1}{T} \int r(t)g^*(t-kT)dt$$

$$= h(k)\exp\left[j\int_0^{kT} \omega_d(\tau)d\tau\right]a_k + n(k)$$

$$=: h_s(k)a_k + n(k) \qquad (2)$$

Here we assumed perfect bit timing, absence of intersymbol interference, and that the variation of $h(t)\exp(j\int\omega_d(\tau)d\tau)$ during a length of the signal pulse is negligible. Violation of the latter assumption results in SNR degradation because of nonoptimality of the matched filter[1]. n(k) is complex white gaussian noise with variance $R = N_0/E_s$. Here z(k) is denoted as received signal sample. Fig. 1 shows the model of the transmission system when using a state space description of the fading distortion and modelling the frequency variations by a random walk in frequency

$$\Omega(k+1) = \Omega(k) + w_\Omega(k) \qquad (3)$$

with

$$\Omega(k) = \omega_d(kT)T \qquad (4)$$

[1]E.g. for unfiltered PSK, the SNR degradation at the matched filter output is less than 0.25 dB for an offset of $\Omega = \omega_d T = \pi/4$.

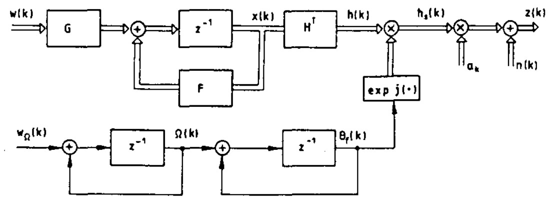

Fig.1: Discrete time model of the transmission system (w, w_Ω: white gaussian noise)

III. Carrier Recovery in the Absence of a Frequency Offset

First let us assume that $\Omega(k) = 0$ for all k. In [4] we derived the maximum a posteriori detector of the transmitted symbol sequence for this case. This derivation showed that carrier recovery equals estimation of the fading distortion h(k), a result that has been previously found by Kam and Teh [5]. If a linear state space model of the fading distortion exists then this signal model determines the Kalman filter as optimal state estimator [6] and thus as optimal carrier recovery unit. If not, a Kalman filter is not optimal. However it is still desirable since it allows recursive estimation.

Let us consider the land-mobile radio channel with its nonrational doppler spectrum [7]. Note that its cutoff frequency ω_D depends on the speed of the vehicle and thus varies randomly. We used a 2nd order signal model for the Kalman filter design. Its parameters $\omega_0 T = .05$ (3-dB cutoff frequency) and $\zeta = .7$ (damping factor) have been chosen such that the maximal error variance of the carrier recovery for an assumed cutoff frequency range of $.005 \le \omega_D T \le .09$ is minimized. Fig.2 shows the resulting bit error probability of QPSK for the worst case, i.e. for $\omega_d T = .09$ (corresponds to speed v = 180km/h for carrier freq. of 900MHz and 1/T = 10ksymbols/s). We used the fading simulator described in [8]. The calculation of the error rate is described in [3]. Fig.2 shows that the error rate is always less than for differentially coherent detection. In particular the irreducible error rate of differential detection is considerably larger. However part of this advantage is lost, if the factor 1-2 increase in error rate of the coherent receiver because of differential encoding of the information is taken into account. Considerable advantage of coherent compared to differential detection is only achieved for diversity reception or when coding together with interleaving is employed since then the error curves are steeper and thus the SNR loss due to

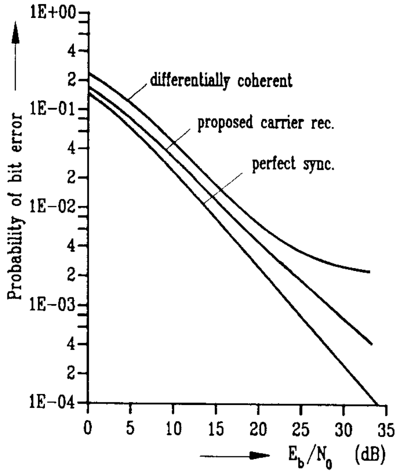

Fig.2: Bit error rate of QPSK on land-mobile radio channel with $\omega_D T = .09$ ($E_b = E_s/2$)

differential encoding is smaller [3].

The advantages of the described carrier recovery over a phase-locked loop are mainly due to the fact that the complex phasor h(k) is processed linearly, whereas a PLL, where the carrier phase $\Theta(k) = \arg\{h(k)\}$ is filtered, is an inherently nonlinear device. Therefore a "hang-up", which is a well-known problem of many synchronization circuits [9], cannot occur. For the same reason a threshold effect [9] does not exist. Note that the statistics of the carrier phase process $\Theta(k) = \arg\{h(k)\}$ depend on the amplitude of h: when $|h|$ is small the phase Θ changes more rapidly than when $|h|$ is large. Hence a phase tracking loop with constant bandwidth cannot operate optimally. The bandwidth of a phase filter would have to be adapted according to an estimate of $|h|$. For the filtering of the complex phasor h as proposed here this problem does not exist since the quadrature components $h_I(k)$, $h_Q(k)$ are independent processes.

Employing the complex phasor also yields information about the momentary transmission quality of the channel. It can be used in diversity reception for maximal ratio combining [10] or, when coding is used, to reduce the error rate by employing the channel quality measure in the decoding process. Further, note that the carrier recovery presented is easily implemented digitally since a simple fixed oscillator is used in the receiver and all synchronizer and detector operations are done using the sampled matched filter output (no VCO is needed).

IV. Carrier Recovery in the Presence of an Additional Time-Varying Frequency Offset

Now we consider the case that $\Omega(i) \neq 0$. In this section we assume absence of modulation ($a_k = 1$ for all k). It is well known that a frequency offset causes a phase error in a first order PLL, if a stable point of operation exists at all [9]. Then a second order PLL has to be used. The synchronizer proposed here must also be extended to avoid an increase of the error variance. A joint estimation of $\Omega(k)$ and h(k), however, leads to an extended Kalman filter [6] since the state space model with state vector $x_s^T = (x^T; \Omega, \Theta_f)$ is nonlinear, see fig.1. To avoid the computational load associated with this solution (filter gain cannot be calculated off line) we use another approach. For the design of the filter we first assume that the frequency offset is known. Then a simple but very effective way to estimate it, is derived.

i) Filter Design for known Frequency Offset

For a given frequency offset the signal model for $h_s(k)$ is linear. Fig. 1 suggests that a frequency offset could be incorporated into a state space model for $h_s(k)$ by a time-varying output matrix

$$h_s(k) = \exp(j\Theta_f(k))H^T x(k) \qquad (5)$$

However, introducing the state variable

$$x'(k) = x(k)\cdot\exp\left[j\sum_{i=0}^{k-1}\Omega(i)\right] \qquad (6)$$

the following state space description can be found

$$\begin{aligned} x'(k+1) &= F\cdot\exp(j\Omega(k))x'(k) + Gw'(k) \\ &=: F(k)x'(k) + Gw'(k) \end{aligned} \qquad (7)$$

$$h_s(k) = H^T x'(k) \qquad (8)$$

where w'(k) is white gaussian noise with the same variance as w. In contrast to the first approach (5), this signal model becomes time-invariant if the frequency offset is constant, and the dimension of the state vector (x'^T, Ω) is reduced by one. Further, using this signal model for the design of the Kalman filter has the significant advantage that the gain L(k) of the filter is independent of the frequency offset Ω. Thus it can be calculated off line in the same way as for a linear Kalman filter [6].

ii) Estimation of the Frequency Offset

Here we are searching for an estimate $\hat{\Omega}$ to replace F(k) in (7) by $\hat{F}(k) = F\cdot\exp(j\hat{\Omega}(k|k))$. This problem can also be solved in a very effective way by using a Kalman filter. Since the frequency offset is only measurable as a phase difference we need a second order signal model with the state vector $(\Theta_s;\Omega)$. Here $\Theta_s = \arg\{h_s(k)\}$ represents the phase fluctuations on the channel due to the frequency offset and the fading. They are modelled by a simple "second order" random walk which proved to be very robust (see fig. 3):

$$\Theta_s(k+1) = \Theta_s(k) + \Omega(k) + w_{\Theta s}(k) \qquad (9)$$

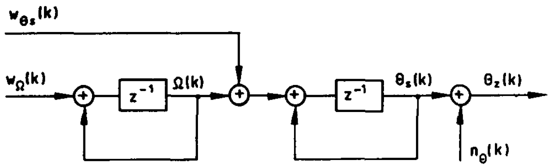

Fig.3: Approximate description of received phase process in a state space model

Here, the phase changes due to fading are approximated by the white gaussian noise term $w_{\Theta s}(k)$ with variance [11]

$$E[w_{\Theta s}^2] = \frac{1}{2}(1 - \rho_h^2(1)) \qquad (10)$$

where $\rho_h(i)$ is the autocorrelation function of the fading

$$\rho_h(i) = \frac{E[h(k+i)h^*(k)]}{E[|h(k)|^2]} \qquad (11)$$

Further it can be shown [11] that the measurement equation

$$\Theta_z(k) := \arg\{z(k)\} = \arg\{h_s(k) + n(k)\}$$
$$=: \Theta_s(k) + n_\Theta(k) \qquad (12)$$

is biasfree and that the noise term n_Θ is white and approximately gaussian. With the signal model, eq. (3), (9), and (12) a Kalman filter estimating Θ_s and Ω is determined. However it must be taken into account that the phase error can only be measured modulo 2π. So a nonlinearity must be introduced in the innovation sequence of the filter. Using the Kalman equations [6] the estimate of the frequency can be found to be

$$\hat{\Omega}(k+1|k) = \hat{\Omega}(k|k)$$
$$= \hat{\Omega}(k|k-1) + L_\Omega(k)g(\Theta_z(k) - \hat{\Theta}_s(k|k-1)) \qquad (13)$$

where g(.) denotes the above mentioned nonlinearity. Here $L_\Omega(k)$ is the second component of the Kalman gain vector $\tilde{L}^T = (L_{\Theta s};\ L_\Omega)$. Using this frequency estimate in the signal model (7), (8) the estimator shown in fig.4 results (M = 1 corresponds to absence of modulation).

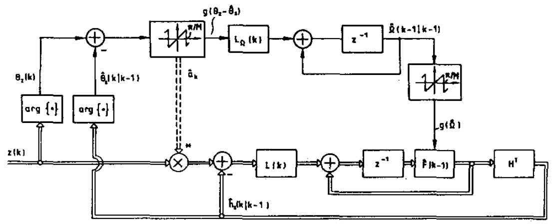

Fig.4: Digital filter for estimation of a complex phasor in the presence of a time-variant unknown frequency offset

This synchronizer is a nonlinear time-variant digital filter. Thus performance analysis must be mainly done by computer simulation. The stationary performance of the filter is very satisfying since, for not too low SNR (> 5 dB) the stationary error variance $\sigma^2 = E[|h_s(k) - \hat{h}_s(k|k-1)|^2]$ is not larger than the error variance of the linear filter in estimating h(k) for $\Omega = 0$.

To assess the acquisition performance we measured the cumulative distribution of the acquisition time of the filter on the land-mobile radio channel ($\omega_D T = .09$) after an initial frequency step at k = 0 (averaged over additive noise, fading, and initial phase of the filter). Here the acquisition time is defined as the time that the absolute value of the error in estimating both $\mathrm{Re}\{h_s(k)\}$ and $\mathrm{Im}\{h_s(k)\}$ is less than twice the stationary standard deviation $\sigma/2$ for ten successive estimates for the first time. Fig. 5 shows the result obtained from the simulation of 1000 acquisition processes. The Kalman gain sequence L(k) has been prolonged by a factor of 3 (i.e. two interpolating values inserted between sucessive values obtained by Kalman filter equations) to account for the initial frequency uncertainty. It can be seen that

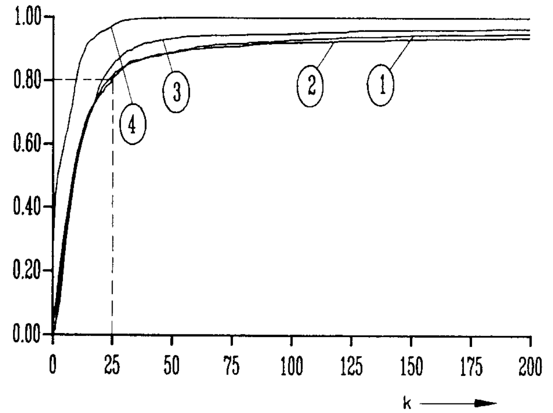

Fig.5: Cumulative distribution of acquisition time

1 estimator fig.4 (initial freq.step: Ω=.25

2 estimator fig.4 (Ω = 1.5)

For comparison:

3 extended Kalman filter (Ω = .25)

4 linear Kalman filter, no freq. step

the acquisition is either very fast (about 80 % of the acq. processes finish within k = 25 time steps, irrespective of the size of the frequency step) or the acquisition takes very long. The acquisition failures are a result of the periodic range of the frequency offset, which is unavoidable in a synchronizer working on a sampled input. This failure must be detected by a supervisor and reacquisition must be initiated. Since the whole synchronizer can be realized as a microprocessor program the supervisor couldbe part of that program.

V. Elimination of Symbol Dependence for MPSK

For MPSK modulation, i.e. $a_k \in \{\exp(j0),\ldots,\exp(j2\pi(M-1)/M)\}$, the symbol phase can be easily eliminated for carrier recovery. The nonlinearity $g(\theta_z - \hat{\theta}_s)$ must be simply substituted by a nonlinearity which performs a modulo $2\pi/M$ operation (instead of modulo 2π). This operation is equal to a decision directed elimination of symbol phase, but it is much easier to implement. E.g. if $\pi/M < |\theta_z - \hat{\theta}_s| < 3\pi/M$ then $\hat{a}_k = \exp(j2\pi/M)$ is detected. Then $2\pi/M$ must be subtracted from the input phase to remove the symbol dependence. This is done by the described nonlinearity. To remove the symbol dependance of the incoming signal of the lower part of the filter of fig. 4, z(k) is multiplied by $\hat{a}_k^*$ where $\hat{a}_k$ is obtained from the above modulo operation.

VI. Discussion

In this paper we presented digital carrier recovery structures which render coherent detection on fading channels feasible in many cases where carrier phase synchronization using a PLL fails. They can be easily implemented in digital hardware or as a microprocessor program. We designed and analyzed their performance for the land-mobile radio channel and introduced a new, simple structure for a fully digital carrier recovery in the presence of fading and additional frequency variations.

VII. References

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