Time-efficient analysis of nonlinear peak bending behavior

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Abstract

A measurement method is presented that combines the advantages of the multisine measurement technique with a prediction method for peak bending behavior. This combination allows the analysis of the dynamic behavior of mechanical structures at distinctly reduced measurement durations and has the advantage of reducing high excitation impacts on the structure under test. After a brief presentation of the algorithm, the validity scope of the approach is investigated with emphasis on an exemplary error investigation.

1 Introduction

In mechanical engineering, the knowledge of the dynamic properties of structures is an important information. Among readily-available products, vibrations can be a quality factor, they can be important for the functionality of products, or lead to errors in operation. Furthermore, they may influence aspects of comfort and well-being of the customer and influence purchase decisions. Contrasting these importances, the growing demands of lightweight construction make technical structures more and more susceptible to dynamic excitation. Hence, reliable and effective methods to measure and analyze the dynamic behavior of structures play an important role in todays engineering disciplines.

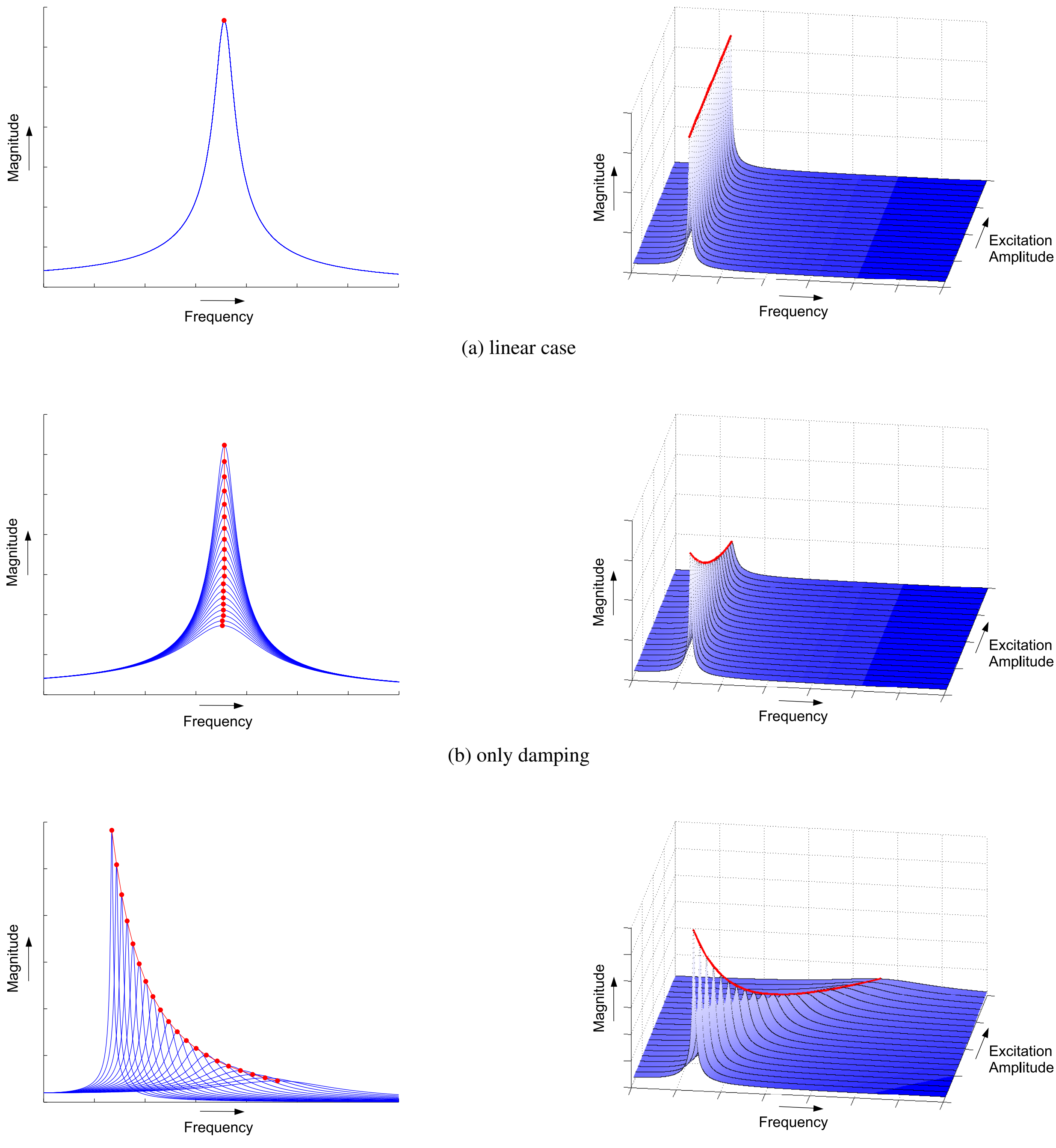
Classical vibration testing is a widely-spread tool that is thoroughly known and successfully used. The common approach is to measure the input-output-behavior of the structure using stepped sine or sweep (chirp) excitation signals. From these measurements, the frequency response function (FRF) of the system is derived by relating the input- and output-signal to each other; it is important to point out that this approach presumes a linear system behavior. Although these techniques are well known and generally reliable in terms of good results among near-linear structures, they can show significant errors in their results when used for structures with strong nonlinear behavior, as will be explained in this paper. Besides variances in the measurement results, these measurement techniques can be very time consuming.

2 Motivation: Excitation Amplitude Dependency of mechanical Oscillators

Many real-world mechanical oscillators exhibit excitation-amplitude dependency in their dynamical behavior. This type of nonlinearity origins in a stiffening or weakening behavior of the structure, typically combined with amplitude-dependent damping characteristics. The result of this is a shift of the resonant peaks that can be described by a Peak Bending Curve (PBC).

In order to illustrate the consequences of this kind of nonlinearity, the different influences of the nonlinearity on the dynamic input-output-behavior are isolated in figure 1. It is important to mention that the figures contain artificial data only in order to illustrate the effects. The plots on the left show the classical magnitude-over-frequency view of the FRF, the right plots show the FRFs at different levels of the input amplitude to the system. The full dynamic behavior is shown as a surface in the right part of the figure. The red dots mark the corresponding resonance peaks, that set up the Peak Bending Curve being shown as a red line.

For a linear system, the dynamic behavior is independent from the excitation amplitude, which leads to a FRF being the same for all inputs as shown in figure 1a. Among the observed class of systems, a typical nonlinear effect is the increase of the damping at higher excitation levels, which results in lower resonance



(a) linear case

(b) only damping

(c) damping and stiffening

Figure 1: Influence of input-dependent nonlinearities on resonance peaks

peaks as shown in figure 1b[1]. It can already be observed that the concept of one single FRF is not valid any more, since the input-output behavior of the observed structure depends on the input signal, so that a single FRF will not describe the full dynamic behavior any more.

The second important effect is the stiffening or weakening of the structure with increasing excitation amplitude, which leads to a higher or lower resonance frequency, respectively. The corresponding behavior is illustrated in figure 1c.

Looking at figure 1c, it is obvious that the classical concept of the FRF only is useful for linear and weakly nonlinear (or 'pseudo-linear') systems. If the nonlinear parts in the structure's behavior become significant, the use of the classical concept of (linear) FRFs to describe the structure's dynamics will end up in significant errors. The use of a characteristic diagrams should therefore be preferred, which include the dependency on the input signal.

In order to get this characteristic diagram, several measurements at different input amplitude levels commonly need to be carried out, and afterwards the different measurements are put together to get the full characteristic diagram. Typically, these measurements would be done by using stepped sine measurements or sweep signals with a slow frequency slope. Both kind of signals are state-of-the-art and will give proper results, but the measurement duration will be rather high. Besides the high time consumption, the structure under test is subject to high excitations levels, which might be a problem on sensible and / or expensive structures.

In this paper, a new approach is presented to obtain the full characteristic diagram of a structure without suffering the demonstrated drawbacks.

3 Transfer Behavior of nonlinear mechanical oscillators

The authors are developing a new method to determine the dynamic transfer behavior of mechanical oscillators with excitation-amplitude dependency. The presented method is intended to be used only for structures showing dynamic behavior that can be described by polynomials. Without limiting generality it can furthermore be assumed that such systems behave quasi-linear as long as they are excited with low input amplitudes; on increasing input force vectors, they deviate from this linear state and exhibit the peak bending behavior mentioned above. Settling to this assumption, the idea is to measure the quasi-linear region and to identify the peak bending curve. By merging these informations, the full characteristic diagram describing the dynamics of the structure under test can be derived.

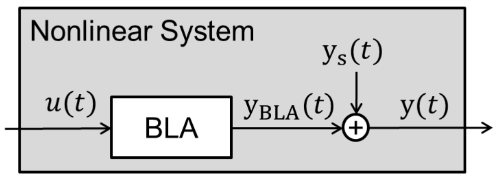


Figure 2: Basic principle of the best linear approximation (BLA)

The low-level measurements are performed using the concept of the Best Linear Approximation (BLA): For systems having weak nonlinear behavior, a linear approximation can be found that minimizes the least square error between the true output of the nonlinear system and the output of the linear model [3], see figure 2. This linear approximation describes the transfer behavior of the structure for a certain input signal. Changing the excitation amplitude will result in a different BLA. It is important that the input- and output signals should not contain any DC signals.

[1]It should be pointed out that figure 1 displays the transfer behavior, so that the output is related to the input

The Best Linear Approximation can directly be measured using a multisine technique. Multisines are a class of excitation signals consisting of a sum of harmonically related sines, so that the structure under test is excited simultaneously with a set of different frequencies. The time domain characteristics of such signals are specified through the phase design of the different frequency lines. Since many frequencies can be measured simultaneously, multisine measurements can save lots of time. In addition to that, transient terms only show up at the very beginning of the measurement, whilst at stepped sine measurements they are present at each frequency step.

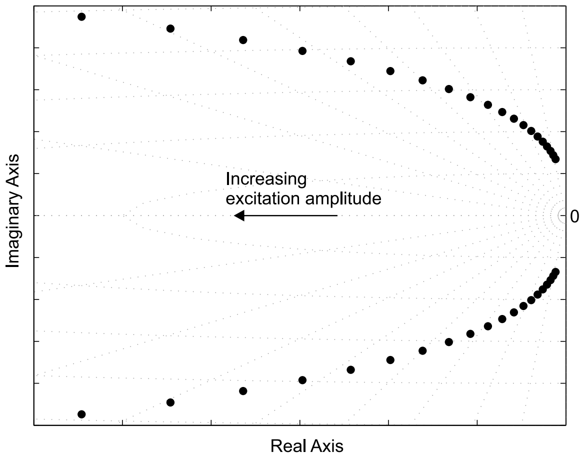


Figure 3: Locus of the system's poles in the Gaussian Plane

The resonance peaks are identified from the Best Linear Approximations utilizing a specialized peak detection algorithm. Based on a modal approach, the resonance peaks can be transferred into poles. When these are drawn in the Gaussian plane as shown in figure 3, the locus of such structures generally has simple geometric shape, which makes the determination of additional poles at higher excitation amplitudes easy. Since the poles contain information about resonance frequency and the corresponding damping, they are an ideal choice to describe the peak bending behavior that is induced by these quantities. In our work, we use an autoregressive model of third order, with the model parameters being calculated by the covariance method.

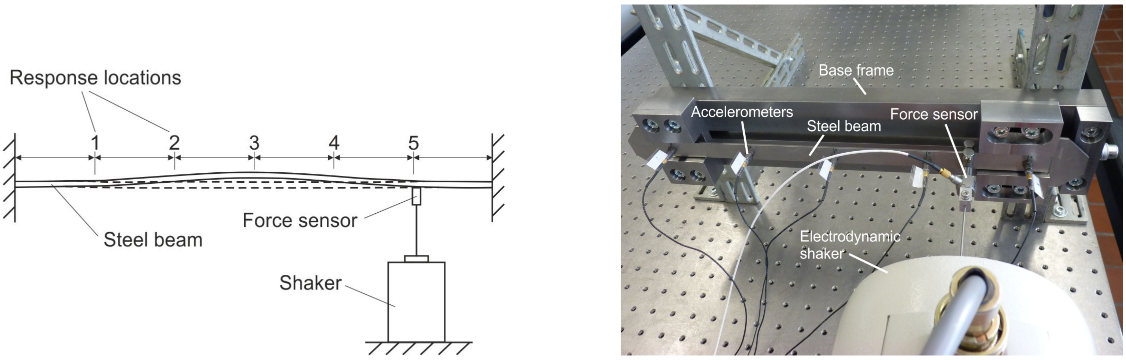
The identified system poles at higher input amplitudes are thereafter re-transferred into resonance peaks; in combination with the prior-known ones they describe the peak bending curve. Together with the measured Best Linear Approximations, the full characteristic diagram can be extrapolated.

A more detailed description of the approach is given in [10].

4 Error analysis and validity scope of the method

In order to evaluate the presented approach and to investigate the validity scope and possible errors, a lab demonstrator was set up as shown in figure 4. Figure 4a illustrates the principle, and figure 4b shows the actual setup in the lab. The idea for the demonstrator is taken from [7], which also contains a theoretical description of its dynamic behavior. The demonstrator belongs to the class of Duffing-type oscillators, that draw on the work of G. Duffing [8] and are commonly used examples for the kind of nonlinear oscillators treated in this paper.

The demonstrator consists of a steel beam representing the structure under test, that is preloaded by two retainers bolted to a heavy frame. The broadened ends of the steel beam are sitting in small grooves, so that their position is fixed without inducing dissipation by clamping effects (e.g. friction). One of the retainers

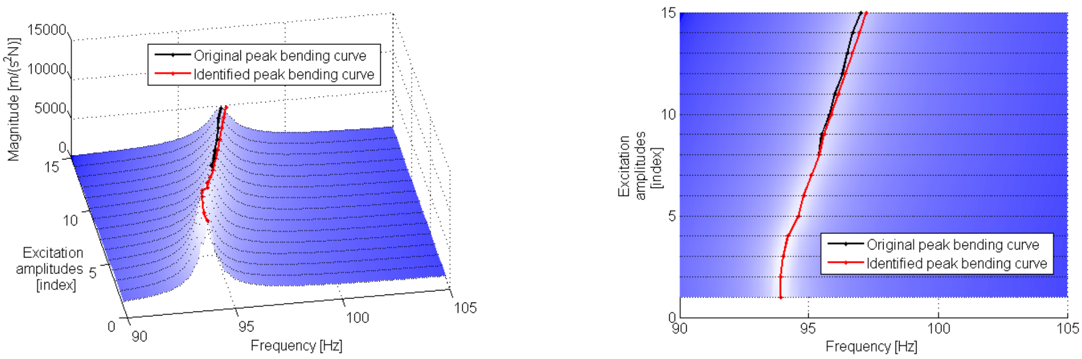


(a) Principle of demonstrator (b) Setup in the lab

Figure 4: Lab demonstrator showing Duffing-like behavior

bears a mechanism that allows changes in the applied preloading; the preloading is measured by a strain gauge. The steel beam is excited by an electrodynamic shaker. The force vector that is impinged to the structure is measured by a force sensor (input signal), and the system answer is measured by one of the accelerometers that are attached at different positions along the beam (output signal).

In a first step, the beam was measured at 15 different equispaced excitation amplitude levels; hereinafter, one of these measurements at a certain excitation level will be referred to as a data set. In order to evaluate the presented algorithm, the first 9 of these measurements (beginning at low excitation levels) were used as an input to the algorithm, and the 6 additional data sets at higher input amplitude levels were identified. The identified data sets were then compared to the measured ones (which are taken as reference) in order to evaluate the performance of the algorithm. The result is shown in figure 5: Figure 5a shows the 3-dimensional characteristic diagram, and figure 5b is the birds-eye view of the same diagram. The maximum deviation between the measured and the identified Peak Bending Curve is 0.23Hz, which corresponds to a relative error of 0.24%.



(a) Side view (b) Top view

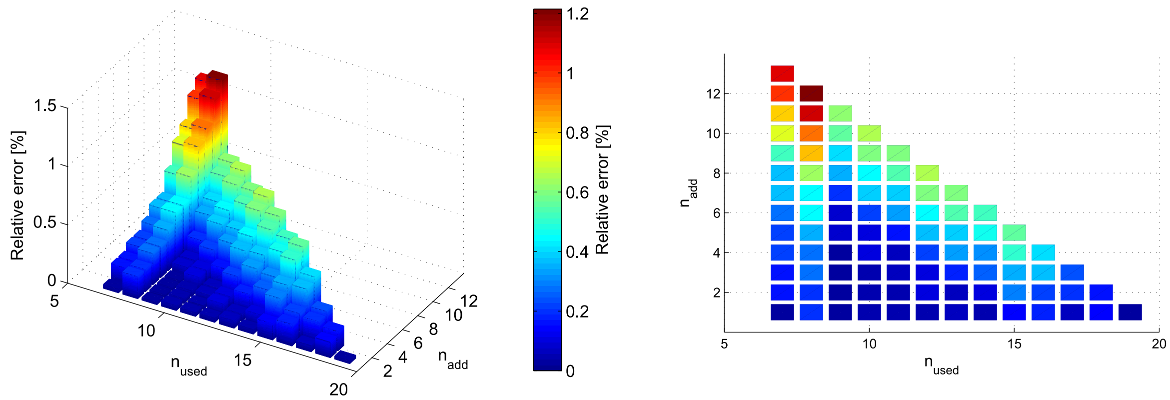
Figure 5: Characteristic diagram of lab demonstrator

As a second step, the influence of the number of different excitation levels was investigated. For that purpose, five additional measurements at higher excitation amplitude levels were performed in order to cover the full range of possible excitation amplitudes that is viable on the lab setup. On lower levels, the pseudo-linear assumption holds, so that no lower level measurements are necessary. Excitation forces exceeding the upper limit of this range are not possible because of the area of operation of the electrodynamic shaker. On such higher levels, the shaker would exhibit significant nonlinear behavior itself, and strong retroactions between the structure under test and the shaker would occur. This would lead to unusable measurement results.

To investigate the resulting errors of the presented algorithm, a different number of low level measurements was used as input data to the algorithm, starting from 7 values which is the minimum number of data sets the presented algorithm requires[2]. Beginning from the seven data points, a rising number of additional datasets (each of them corresponding to an additional excitation amplitude measurement) is identified up to the total number of 20 sets (that is, 7 input data sets plus 13 identified ones). The identified data sets are then compared to the measured ones, which are taken as reference, and the maximum relative error between the resulting peak bending curves is calculated.

The same way, a rising number of additional data sets is identified, using 8 data sets as input to the algorithm, afterwards 9, and so on. As a result of this investigation, for every number of input data sets from 7 to 19, the errors for a varying additional number of identified data sets are derived. Using 19 data sets as an input, this investigation obviously can be done for 1 identified data set only, since only 20 measured data sets are available.

The results are shown in figure 6a: n_{used} is the number of data sets that were used as an input to the algorithm, n_{add} is the number of identified data sets, and the vertical axis displays the maximum relative error between the identified peak bending curve and the measured one. The relative error is also used for coloring the bars in order to improve the visibility. Figure 6b shows the bird's eye view of the same diagram.



(a) Side view (b) Top view

Figure 6: Errors of identified Peak Bending Curve for a different number of identified peaks

It can be observed that the general error level is rather low. The highest maximum deviation of 1.2% occurs in the case of 8 input data sets and 12 identified. In this case, the rate of unknown to known values is 1.5, meaning that for every data set that is fed into the algorithm, 1.5 additional data sets are derived.

It can also be seen that, as long as the ratio of n_{add} to n_{used} is less than 1.25, the resulting relative error in the demonstrated case will be 0.8% or less. Furthermore, for $n_{used} \geq 9$ input data sets the relative error narrows down significantly in the presented example.

The total duration to set up the full characteristic diagram also depends on the number of the acquired (input) data sets, but since the multisine method is much faster than a stepped sine or sweep signal measurement, it is significantly reduced in any case. In the 'worst' case of 19 acquired data sets and 1 identified set, the presented method will require less than 5 minutes including all calculation times. A stepped sine measurement with the same key data (frequency range and resolution, number of amplitude levels in the charateristic diagram) requires around 80 minutes. In the case of the highest error (8 input data sets, 12 identified ones) the presented method requires 2.3 minutes, whilst the stepped sine still requires approximately 80 minutes.

To summarize the above example, this means that in the range that would also be measured by classical

[2]Theoretically, the minimum number of data points would be 6, but using this number will result in numerical issues due to badly conditioned polynomials in the derivation of the autoregressive model.

approaches, the highest error of the presented method will be near 1.2%, but most errors will be 0.8% or less, whilst the duration of the measurements can be reduced by around 90%.

5 Conclusions

In this paper, a new approach to analyze the dynamic input-output-behavior of mechanical structures with nonlinear peak bending behavior is presented. The algorithm utilizes the concept of the Best Linear Approximation (BLA); to measure the BLA, multisine signals are used. The presented method identifies the Peak Bending Curve and enables the derivation of the full characteristic diagram without the need of measuring all supporting points.

The benefits of the presented approach are:

- The measurement durations are significantly reduced, since the multisine method is much faster than state-of-the-art methods that are typically used among mechanical oscillators.
- Measurements at high excitation amplitudes are no longer required, since the dynamic behavior of the structure under test at these levels can be identified.

After a short explanation of the basic idea, the method is exemplarily evaluated using a lab demonstrator. In this example, the dynamics of the demonstrator can be derived at maximum error rates of about 1.2%, whilst the measurement durations can be reduced by up to 96%, depending on the resolution of the characteristic diagram.

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