PAMM · Proc. Appl. Math. Mech. **14**, 293 – 294 (2014) / **DOI** 10.1002/pamm.201410134

Time-efficient analysis of nonlinear dynamic behavior

Christian Sprock[1,*] and **Walter Sextro**[1]

[1] Mechatronics and Dynamics, University of Paderborn, Germany

Many nonlinear mechanical oscillators show excitation-dependent behavior. In this paper, a new measurement approach is presented to analyze such structures. The main advantage of the presented method is the high efficiency, since measurement duration and loads to the structure are significantly reduced.

1 Introduction

Vibration testing is a widely-spread tool that is thoroughly known and successfully used in research and industry. It provides methods to gain knowledge of the dynamic behavior of technical structures; in general, the dynamic input-output-behavior of structures is described utilizing frequency response functions (FRFs). The common approach is to excite the structure with a force vector covering the desired bandwidth (signals with harmonic content or impulse type) and record the structure response to the forced vibration. From this data, the FRF is calculated [1, 8]. Most state-of-the-art measurement techniques are based on linear or pseudo-linear assumptions. In many cases, this leads to usable results, although it is known that every real world system contains more or less nonlinear elements in its dynamic behavior (e.g. friction).

Many mechanical oscillators exhibit excitation-amplitude dependency in their dynamic behavior. This type of nonlinearity usually appears in a stiffening or weakening behavior of the structure that is dependent on the excitation amplitude (force or displacement), typically combined with amplitude-dependent damping characteristics. In general, these systems behave linear as long as they are excited by small force vectors. On increasing force amplitude of the excitation signal, the amplitude dependency dominates the dynamic behavior, which leads to a shift of the resonance peak(s) to higher or lower frequencies compared to the 'linear' ones at low input levels. The resulting shift of the resonance peaks can be described by a peak bending curve (PBC), sometimes also referred to as backbone curve [5, 6].

To get an entire description of such behavior, it is necessary to analyze the frequency behavior in dependency of the excitation amplitude level. As a consequence, a change from the classical input-independent concept of single FRFs to characteristic diagrams is worthwhile; ignoring the amplitude dependency would lead to inaccurate descriptions of the dynamic behavior [6]. In order to obtain these characteristic diagrams, several frequency response measurements at different excitation amplitude levels have to be carried out. As sweep or stepped sine techniques would be used for that, this method is very time consuming. Besides that, another drawback of this approach is that the structure under tests needs to be exposed to high loads.

2 Gathering and processing the dynamic behavior

In this paper, a new approach is presented to determine the characteristic diagram of the above structures exhibiting excitation-amplitude dependent behavior. The approach is based on the fact that a nonlinear system NL can always be split into a linear base system L, superimposed by nonlinear distortions NL' as shown in figure 1, see [3, 4].

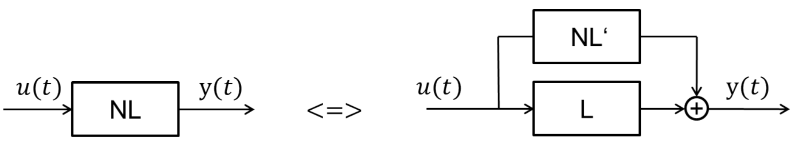

Fig. 1: Segmentation of nonlinear system into linear base system and nonlinear distortions

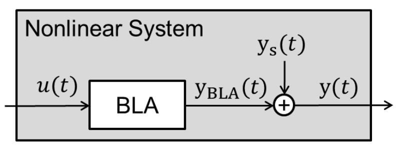

Fig. 2: Basic principle of the BLA

Settling to this idea and assuming that the system only has weak nonlinear behavior, a linear description can be found that minimizes the least square errors between the true output of the nonlinear system and the output of the linear model [4], see figure 2. This approximation is called the best linear approximation (BLA) of the nonlinear system [3]; it is dependent on the input signal u(t). The assumption of weak nonlinear system behavior means no limitation concerning the presented approach, since only low input level measurements are required, see later.

* Corresponding author: e-mail christian.sprock@uni-paderborn.de, phone +49 (0)5251 / 60 1814, fax +49 (0)5251 / 60 1803

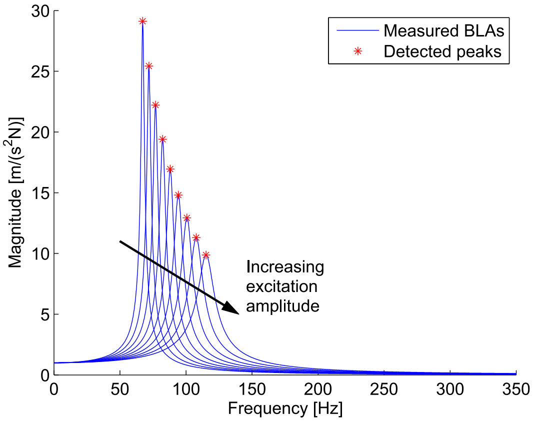

Fig. 3: Measured BLAs at low excitation levels

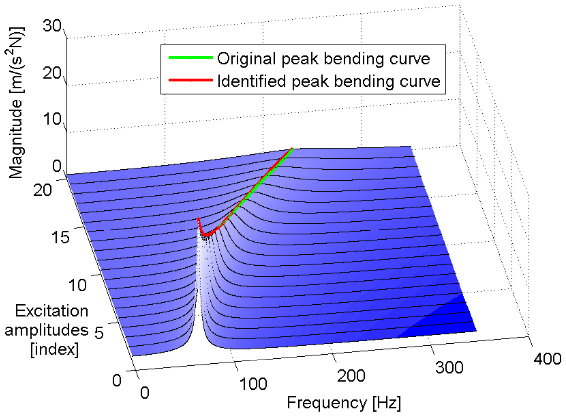

Fig. 4: Full characteristic diagram of dynamic system behavior

As described above, amplitude-dependent nonlinear systems show (pseudo-)linear behavior on low input amplitudes, and the nonlinear behavior shows up as deviations in the frequency of the resonance peaks. The locus of the resonance peaks at different excitation levels is described by the peak bending curve. The key idea of the proposed method is to measure the basic linear behavior and to identify the peak bending curve that corresponds to the nonlinear deviation of the system from its basic linear behavior. This information is merged to derive an entire description of the dynamics of the system.

The basic linear behavior of the system is measured in terms of some BLAs at different low excitation levels as shown in figure 3 (blue lines); the BLAs correspond to the FRFs in linear cases. Next, the resonance peaks (red stars) of these BLAs are identified. Based on a modal approach, they can be transferred into system poles, allowing to identify the peak bending curve in the Gaussian Plane. The system poles describe the resonance frequency and damping which are the quantities that lead to peak bending, making them well suited to describe the change in system behavior. Another advantage in using poles instead of resonance points is that the root locus normally has simple geometric shape, so it is easy to identify.

After the poles are determined, they are re-transfered into resonance points. Using the identified resonance peaks and the knowledge of the underlying linear base system, the full characteristic diagram can be extrapolated. For comparison, the actual behavior at the higher excitation amplitudes can also be simulated, since artificial data are used. The results are shown in figure 4: the black dotted lines are the best linear approximations at different excitation amplitudes, and the identified peak bending curve is shown in red. The green line shows the actual peak bending curve as it was simulated for the higher level excitations.

The example presented in the plots uses 9 BLAs for the algorithm and determines 11 additional amplitude levels. The identified resonance peaks were compared to the actual ones; the maximum relative error is 0.78%. The presented method requires approx. 3 minutes including all processing times. A stepped sine measurement was chosen as state-of-the-art method to benchmark the presented approach; with the same options (bandwidth, number of experiments) it required about 80 minutes, which corresponds to time savings of more than 90%.

3 Conclusions

The presented method utilizes the concept of the best linear approximation to construct a new method for measuring the dynamic behavior of nonlinear systems with input-amplitude dependency. The method is demonstrated on artificial test data and shows good performance. It significantly reduces measurement time at comparable accuracy, considering state-of-the-art methods. In our future work, the method will be tested on real world structures.

References

[1] K. G. McConnell and P. S. Varoto, Vibration Testing (John Wiley and Sons, Inc., 2008)
[2] A. Brandt, Noise and Vibration Analysis (John Wiley and Sons, Inc., 2011)
[3] J. Schoukens, R. Pintelon and Y. Rolain, Mastering System Identification in 100 Excercises (John Wiley and Sons, Inc., 2012)
[4] R. Pintelon and J. Schoukens, System Identification-A Frequency Domain Approach (John Wiley and Sons, Inc., 2012, 2nd ed.)
[5] D. Wagg and S. Neild, Nonlinear Vibration with Control (Springer, 2010)
[6] K. Magnus, K. Popp and W. Sextro, Schwingungen (Springer Vieweg, 2013, 9th ed.)
[7] K. Worden and G. R. Tomlinson, Nonlinearity in structural Dynamics (IoP Publishing Ltd, 2001)
[8] D. J. Ewins, Modal Testing - theory, practice and application (Research Studies Press Ltd., 2000, 2nd ed.)

