

ASPECTS OF FLEXIBLE VISCOELASTIC SUSPENSION MODELING FOR FRICTIONAL ROLLING CONTACT ANALYSIS USING ADAMS

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ABSTRACT

Tire-wheel assembly is the only connection between road and vehicle. Contacting directly with road within postcard size of contact area, it is mounted and guided by the suspension system. Therefore kinematics and compliances of suspension system greatly influence the frictional coupling of tire tread elements and road surface asperities by affecting pressure and sliding velocity distribution in the contact zone. This study emphasizes the development of a numerical methodology for frictional rolling contact analysis with focus on interaction of suspension system dynamics and tire-road contact using ADAMS. For this purpose a comprehensive flexible multibody system of the multi-link rear suspension is established, where both flexible and rigid bodies are modeled to allow large displacements with included elastic effects. To meet accuracy requirements for the high frequency applications, such as road excitations, the amplitude- and frequency-dependency of rubber-metal bushings is included. Furthermore the proposed flexible viscoelastic suspension model is enhanced by a Flexible Ring Tire Model (FTire), which describes a 3D tire dynamic response and covers any road excitations by tread submodel connected to road surface model. Concerning the verification and validation procedure numerous experiments are carried out to confirm the validity and the accuracy of both the developed submodels and the entire model. The devised approach makes it possible to investigate the influence of suspension system design on dynamical rolling contact and to evaluate tire tread wear. Therefore it can be a useful tool to predict frictional power distribution within the contact area under more realistic conditions.

Keywords: Kinematics and compliances; flexible viscoelastic suspension model; frictional rolling contact analysis; frictional power distribution.

INTRODUCTION

Automotive chassis forms the connection between the vehicle body and the road. Its main task is to guide and control the wheel on the road, thereby ensuring a robust contact between tire and road surface. By road excitations or driving dynamics of the vehicle, such as pitch, roll and yaw, tire undergoes generally a combined load of the longitudinal, lateral and vertical forces. The induced forces have to be supported on one side by the road surface within postcard size of contact area and on the other side by the suspension system connected to vehicle body. Based on the so-called unified theory of

rubber and tire friction [1] the total friction force in contact patch consists of a sum of different components, not all of which need to occur simultaneously. The individual components of the friction force are based on physical effects such as adhesion, hysteresis, cohesion and viscous friction. The adhesion and hysteresis parts make the greatest contribution to the rubber friction. By virtue of the viscoelastic properties, the rubber and tire friction differs of the classic solid friction by Coulomb. Here the friction coefficient unlike assumptions of classical friction laws is not constant. With increasing normal pressure it decreases and is also dependent on sliding velocity. Thus it increases with increasing sliding velocity at first, reaches a maximum and then decreases again. Moreover, there is a strong temperature dependence characterized by the rubber composition.

Due to high complexity of the wear process for elastomers with different wear mechanisms and interactions, a number of empirical wear laws based on experimental investigations have been established [2]. In connection to these wear laws the wear can be construed as the result of the friction process in the form of a proportional relationship between wear volume and frictional energy or frictional power. Whereby, that primarily depends on normal force or normal pressure, friction coefficient and sliding distance or sliding velocity. Instead of wear volume, the usage of indicators such as wear mass, wear height or wear depth is also widespread [2]. Often these wear indicators are referred to the duration or sliding distance due to the friction process, what leads to the corresponding wear rate. Contacting directly with surface asperities, the tire tread wears down caused by the unavoidable slippage of rubber tread elements. This results in degradation of the tire running performance and shortening the tire lifetime. As numerous studies have shown [3-5], the composition of tire wear represents a toxic compound and becomes a crucial source of air pollution. In addition to harmful substances tire wear also includes fine dust. The particulate matter can penetrate into the human lung bronchi and cause serious allergic or oncological diseases. Particularly with regard to the recent worldwide intensification of the environmental protection regulations new technical requirements and related test procedures, which would limit the emissions of tire dust and its carcinogenic substances, represent a great challenge to both tire and car maker.

In general, the amount of tire wear depends on many factors, such as material properties of tire rubber, construction of tire, environmental conditions and in particular tire operating conditions [6]. In the footprint tire behavior depends not only on the magnitudes of the lateral and longitudinal forces, but also on how they arise, what is characterized fundamentally by the automotive suspension system. As depicted in [7], the way the forces are generated, the combination of toe and camber angle change due to suspension dynamics, have great significance on the tire wear behavior. For evaluation tire wear performance owing to suspension or vehicle dynamics several approaches have been established. Traditionally by means of a prototype system experimental investigations are carried out either by outdoor tire wear testing along a specified course [8] or using indoor wear testing machine [9, 14], which reproduces selected steady-state outdoor load conditions. It is also possible to apply dynamic test conditions of load, slip and camber angle [8], but with respect to the test machine limits. These loads can be gained by vehicle outdoor maneuvers [8], by using an analytical calculation method [10] or by running a vehicle multibody simulation [7, 11]. However, these experimental methods are inconvenient to meet time and cost requirements at the earlier design stage. For that, numerical tire wear prediction tools have been developed, which in the main represent a virtual indoor wear testing procedure [12, 13]. It based on

a tire model with high level of detail, such as 3D patterned finite-element tire model [13] or physical rigid ring tire model [8]. To this, time histories of hub forces with slip and camber angle are applied, which obtained by vehicle's multibody simulation that reproduces the acquired working conditions of the vehicle. The problem here is mostly the used way of simple modeling for suspension, tire, road and interaction of these parts due to vehicle dynamics simulation, what lastly influences tire operating conditions and thus the calculation of frictional power and wear amount. On the other hand, this way does not cover the effective interaction between suspension system and tire and thus a proper understanding of actual tire operating conditions, especially considering kinematics and compliances of suspension design.

Therefore, the present paper illustrates a numerical methodology with focus on the interaction of suspension dynamics and tire-road contact using Automatic Dynamic Analysis of Mechanical Systems (ADAMS). The devised approach makes it possible to investigate the influence of suspension system design on the dynamical rolling contact and to predict the frictional power distribution within the contact area and the power loss at all. With the proposed numerical technique, frictional power oriented design of automotive suspensions is enabled, especially with regard to minimizing tire tread wear and improving vehicle energy efficiency for reduction of environmental pollution.

SUSPENSION MODELING

From kinematic point of view, an automotive suspension system has to allow a vertical degree of freedom between the wheel and the vehicle body due to driving dynamics and road excitations. The spatial wheel movement (kinematics) at steering and bounce is determined by the number and arrangement of hard points (topology) of suspension system [15]. At modern multi-link axles the wheel passes kinematically a screw motion, as a combination of a translational and rotational motion [16]. In general, kinematic motion is superimposed by elastic deformations (elastokinematics) of suspension parts and connecting elements such as rubber-metal bushings. These elasticities change the kinematic movement pattern and thus the attitude of the wheel, depending on the imposed forces and torques [17]. The wheel alignment, which results from suspension kinematics and elastokinematics, depicts a particular importance for the driving stability and tire-road rolling contact. The main objective of the elastokinematic design, it is to compensate the elastic deformations or to convert them in favorable compliances due to driving safety and comfort [15]. Today a chassis has high requirements in regard to driving safety, comfort and vehicle handling. Compliance with these demands often requires contrary actions to be taken, so that the suspension system finally represents only a compromise [18]. Meanwhile there are numerous different suspension concepts developed, which more or less meet the demands and desires. One is the independent multi-link suspension concept, which due to the large number of design parameters offers the best possibility to fulfill the complex kinematic, elastokinematic and dynamic requirements imposed on suspension systems of today's automobiles. In comparison with a solid axle or a twist-beam axle it allows each wheel on the same axle to move without affecting the wheel on the other side while bumping. Moreover it offers often the most freedoms to accomplish suspension design [19]. It is possible to set one parameter without affecting other parameters. But this type of vehicle suspension is expensive and complex in development. Figure 1 illustrates a multi-link suspension system placed at D-segment in European market car classification. The proposed study is based on this multi-link rear suspension system.

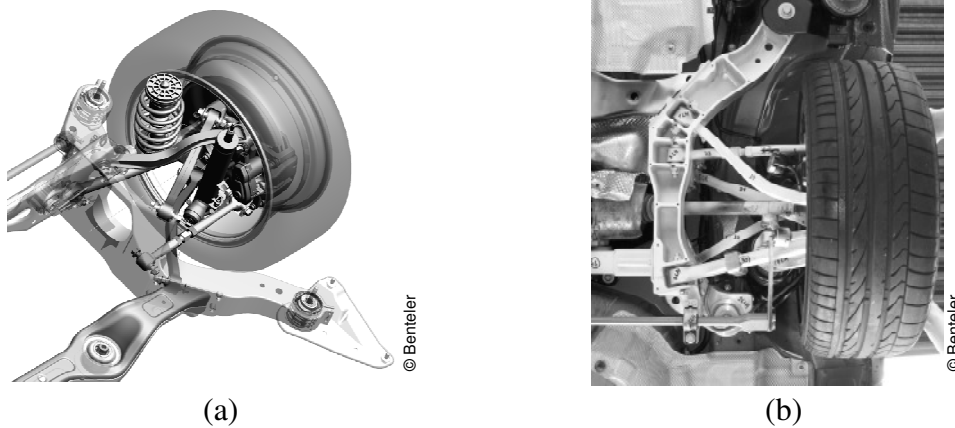
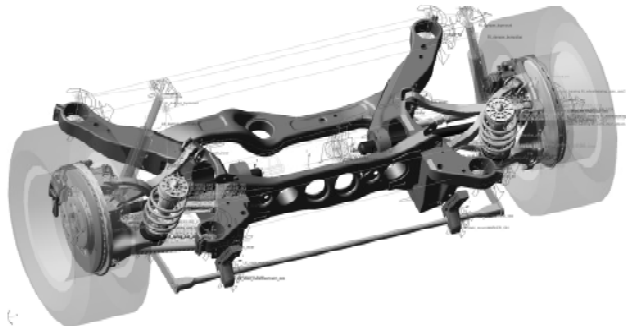


Figure 1. CAD model (a) and prototype (b) of the multi-link rear suspension system

It represents a conception of five spatially arranged linkages connecting the wheel carrier with the subframe by means of the rubber-metal bushings. The upper link-plane consists of two crossed links, front camber link (FCL) and rear camber link (RCL). The lower links, rear lower arm (RLA) and trailing link (TRL), form a virtual intersection behind the wheel center. This results a kingpin axis, which is located behind the wheel center. The kingpin axis represents a virtual steering axis and corresponds to the instantaneous screw axis of the wheel carrier performing rotation. A detailed description and calculation of the kingpin axis is carried out in [16]. The toe link (TOL) is placed in front of the wheel center. The suspension concept comprises a separated arrangement of coil spring and shock absorber, which are mounted to wheel carrier and to vehicle body. Moreover, the shock absorber includes an elastomer bump stop and rebound stop. Another elastomer spring is placed inside the coil spring and provides an additional progressive suspension rate. For preventing body roll a tubular anti-roll stabilizer bar is installed. The subframe is supported by four rubber-metal bushings.

In order to pass a detailed investigation on rolling contact and frictional power calculation, it is important to have an accurate model of the multi-link suspension system, as it essentially controls the attitude of the wheel and tire on the road, which in turn directly influences tire operating conditions. For solving problem formulations hereby arise, such as large displacement, great number of parts, complex multiple dependencies and non-linear system dynamics, an approach of numerical multibody system dynamics is most suitable. For that, ADAMS has ensured its widespread usage throughout the automotive industry and has been used for this study. It allows building a model of the technical system in a preprocessor through parts, which represent system components and markers for locations of acting forces and boundary conditions. The parts are constrained either by joints with initial conditions for displacement and velocity or force elements, which e.g. build elastic mounts. By means of motion constraint parts may be assigned directly an initial displacement or velocity. ADAMS numerically generates and solves the system equations as a representation of the technical system and its relevant mechanical characteristics [20]. With regard to the high level of detail through suspension modeling the total system has been decomposed in subsystems, which represent kinematic, elastokinematic and dynamic behavior. In accordance with that, the subsystems have been modeled in steps and composed by modular assembly and gradually enhancement to the overall flexible viscoelastic multibody system of multi-link rear suspension, as shown in Figure 2. The modular modeling concept allows different modeling approaches for each subsystem and on the

other hand to confirm the validity for the individual modeling level. Therefore, it offers the flexibility for the demanded requirements of accuracy and it is essential to the clarity of complex multi-link suspension system. In the following, main aspects of the flexible viscoelastic suspension modeling are described.



Components of suspension model:

- 60 rigid bodies
- 16 flexible bodies
- 1 point mass
- 63 joints
- 50 force elements
- 138 Kelvin-Voigt models
- 40 viscoelastic models
- 2 tire models
- 1 road model

Figure 2. Flexible viscoelastic multibody system of the multi-link rear suspension

During the modeling process of suspension, it is important to take into account the unsprung mass and the weight distribution resulting from the individual suspension components, which lastly contribute to the forces occurring in the tire-road contact area and so influence the frictional power distribution. Therefore, it is essential to consider the mass and the moments of inertia of the pertinent components. Furthermore, the suspension system undergoes relatively large translational and rotational displacements, which are superimposed by effects of nonlinear elasticity and viscosity implied by utilized components and material properties. All of that outcomes in an inhomogeneous stiffness and damping distribution and has to be covered within the proposed multibody system for frequency bandwidth of interest.

Numerical Modeling of Elastic Effects

By reason of great importance of suspension alignment due to the driving conditions, not all of suspension components can be treated as ideal rigid bodies. Therefore, it requires the implementation of elastic bodies in multibody system. A standard technique for treating elastic structures is represented by the Finite Element Method (FEM) or Analysis (FEA), which discretizes the entire structure by small parts, called finite elements. This leads to a critical large number of degrees of freedom, which cannot be handled due to the multibody dynamics simulation. Thus, it makes necessary of using adequate reduction techniques [21], such as modal approximation, which tailors the modal basis and thus the number of degrees of freedom and capture the desired level of dynamic content.

Figure 3 illustrates the schematic way used in this study for generating flexible bodies on the example of an aluminum rear lower arm. First, the volume part with geometrical characteristics obtained through Computer Aided Design (CAD) has to be imported to a FEA and modal reduction capable program or directly to ADAMS/Flex, which supports the generation of flexible bodies based on a modal neutral file (MNF). This is an output file, which includes the characteristics of the flexible body, e.g. geometry, nodal mass and inertia, mode shapes, etc. Then the characteristic data of material have to be specified, so that an inertial rigid body is available. The next step is to create the mesh by setting its element type and order. By defining the interface nodes and the boundary conditions, such as attachment manner of the part, the further

interconnection possibility to the multibody system is obtained. Finally, it follows the numeric modal analysis of the FE part accompanied by the modal reduction technique based on the Craig-Bampton method [22], which delivers a flexible body in the form of MNF with a greatly reduced number of degrees of freedom, but capable to capture the inertial and compliance properties. In regard to the underlying multi-link rear suspension system, toe link, front camber link, rear camber link, trailing link, rear lower arm and wheel carrier on both sides as well as subframe, which connects the both sides, are modeled as flexible bodies using the described procedure.

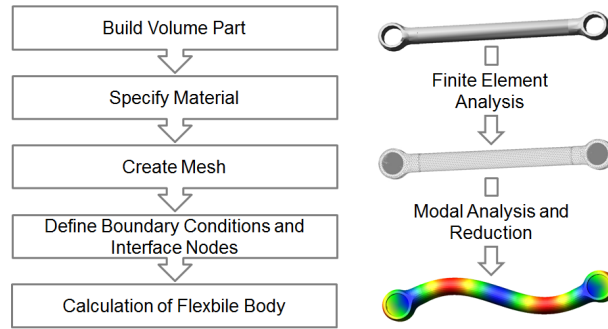


Figure 3. Schematic workflow for generating flexible body of the rear lower arm

Mostly, in multibody systems a spring is modeled as a massless force element acting along a line between two interface nodes, whereby the spring force depends on the product of the spring stiffness and the spring deformation. However, are some natural frequencies of the spring located within the frequency bandwidth of interest, its dynamic properties must be considered [23]. The reason for that undergoes the natural oscillations of the spring, which contribute to the overall dynamic system behavior. In regard to the high frequency excitations, e.g. through road unevenness, the coil spring module and the anti-roll bar module are based on the flexible body modeling approach.

Modeling of Elastic and Viscoelastic Bushings

In general, an automotive cylindrical bushing used for connecting links consists of a carbon black filled rubber vulcanized between an inner and an outer steel cylinder. So it constrains three translational and three rotational degrees of freedom by its stiffness resulting from construction and material behavior. Hereby, viscoelastic material properties lead to a nonlinear behavior. Figure 4a illustrates the characteristic dynamic stiffness change from a measurement of bushing radial direction due to frequency and amplitude of the harmonic excitation, where dynamic stiffness represents the relationship of force and displacement. With regard to an accurate describing of dynamic behavior, it is important to consider the frequency and amplitude dependent characteristics of bushing in the time domain. So in multibody system simulation the rubber-metal bushing is modeled as a general force element, where the resulting forces and torques are represented by the mechanical equivalent systems for each direction respectively. Taking into account the model computation and parameterization effort, viscoelastic behavior is considered only for main loading directions of bushing, one radial direction for a symmetrical bushing and two radial directions for an asymmetrical bushing. For the remaining directions a standardized Kelvin-Voigt model is used, as a parallel set of spring and damper, shown in Figure 4b. The equivalent system for viscoelastic bushing characteristics is given in Figure 4c. It displays a parallel

combination of so-called Maxwell elements for the frequency dependence and a set of a nonlinear spring and a spring with a displacement-dependent spring coefficient for the amplitude and hysteretic effects. According to [24], the analytical description of the resulting force is given by Eq. (1):

$$F = F_{El} + \sum_{i=1}^n F_{Mw,i} + F_{Hys} \quad (1)$$

with
$$\dot{F}_{Mw,i}(t) + \frac{c_i}{d_i} F_{Mw,i}(t) = c_i \dot{x}(t), \quad F_{Mw}(0) = F_{Mw,0} \quad (2)$$

$$\dot{F}_{Hys}(t) + \vartheta \dot{x}(t) \operatorname{sgn}(\dot{x}) F_{Hys}(t) = c \dot{x}(t), \quad F_{Hys}(0) = F_{Hys,0} \quad (3)$$

where t time
 x, \dot{x} displacement and velocity
 c_i, d_i spring and damper coefficients of the i -th Maxwell-element
 ϑ, c parameters for describing spring with a displacement-dependent spring coefficient
 $F_{El}(x)$ quasistatic force-displacement spline

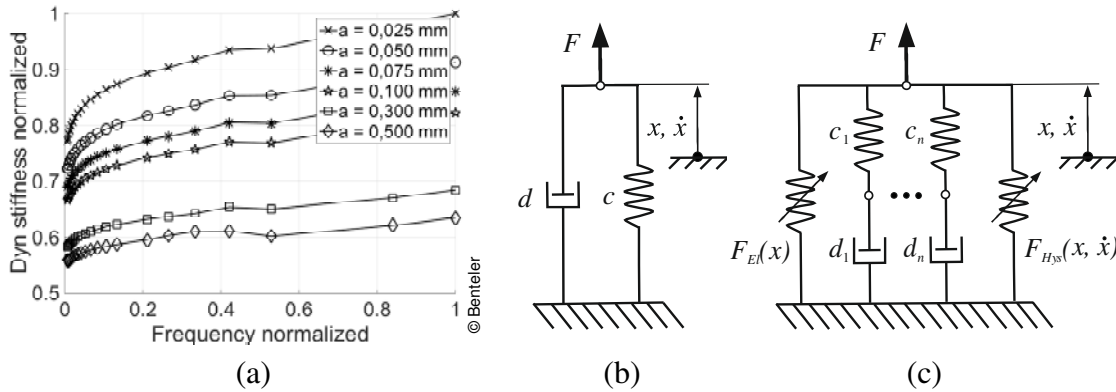


Figure 4. Dynamic stiffness due to excitation frequency and amplitude (a); Kelvin-Voigt model (b); equivalent system for viscoelastic bushing characteristics (c)

Modeling of Tire and Road

In order to calculate frictional power distribution, the proposed flexible viscoelastic suspension model has been enhanced by a commercial Flexible Ring Tire Model [25], which describes a 3D tire dynamic response. It is a combination of flexible tire structure submodel and tire tread submodel, which covers road excitations through discretizing the contact area by tread contact elements. The road model provides the synthesis of the surface roughness using an empirical approach based on the observed characteristics of measured profiles of roads [26, 27].

MODEL VERIFICATION AND VALIDATION

Prior to the frictional power analysis, it is necessary to verify and validate the simulation model. To confirm the validity and the accuracy, numerous experiments are carried out for both the developed submodels and the entire simulation model.

Therefore it is divided into three main parts, to ensure the validity of suspension's pure kinematics, elastokinematics and dynamical behavior. In order to the kinematics, reference is made to [16], where in previous work an equivalent mathematical model of the multi-link rear axle is built and compared to the rigid multibody system. Regarding the elastokinematics and the dynamics some of aspects are pointed out and described in the following.

To confirm the compliance effects of flexible body an experimental modal test for the aluminum rear lower arm is performed, as shown in Figure 5a. The test structure of rear lower arm is attached to tripod on vibration isolated platform by an elastic band to realize a "free-free" mounting. Attached by a stinger, shaker random and chirp excitations are used from two different sides of test structure to cover all modes up to 3200 Hz. By means of a Laser Vibrometer the response velocity of the test structure due to excitation has been taken at various measurement points (Figure 5b) and evaluated with LMS Test Lab. As shown in Figure 5c, the frequency response functions (FRF) due to the measurement setup #1 and #2 indicate an occurrence of five dynamic modes within the excitation frequency bandwidth. The 1st and 2nd mode are represented by simple bending shape, 3rd and 4th modes show double bending shape and 5th mode illustrates torsion shape.

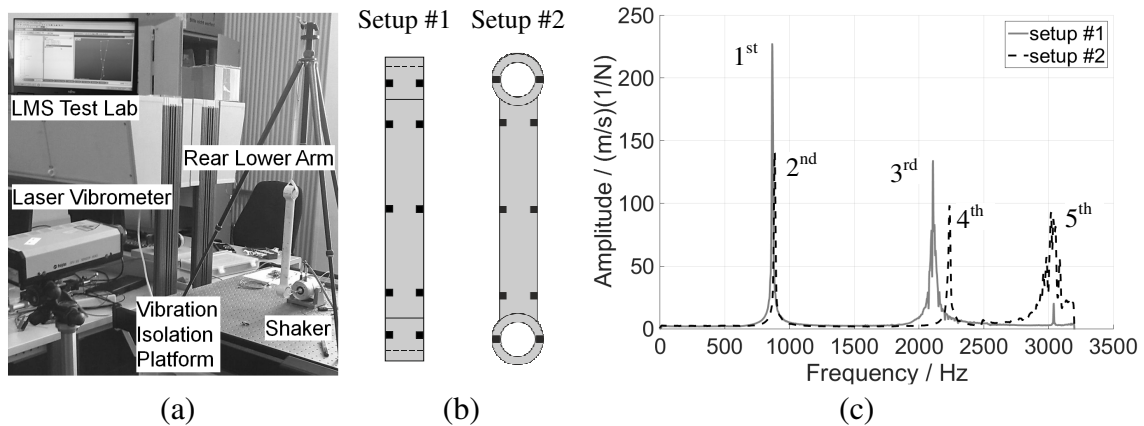


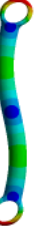




Figure 5. Experimental modal test (a); measurement points (b); FRF measurement (c)

A comparison with measured data in Table 1 shows a good match for the resulting behavior of rear lower arm, which is described by a flexible body. The maximum error in natural frequencies is less than 1%.

Table 1. Comparison of measured and calculated dynamic modes for the rear lower arm

Mode number:	1 st Mode	2 nd Mode	3 rd Mode	4 th Mode	5 th Mode
Mode shape:					
Measure / LMS:	865,92 Hz	882,74 Hz	2105,68 Hz	2236,99 Hz	3040,30 Hz
FEA model / ANSYS:	864,90 Hz	879,25 Hz	2102,20 Hz	2238,80 Hz	3068,30 Hz
Modal reduced model / MNF:	864,76 Hz	879,13 Hz	2101,79 Hz	2238,58 Hz	3060,79 Hz

By means of MTS Elastomer Test System quasistatic and dynamic investigations on radial direction of several cylindrical bushings carried out. About the bushing model to parameterize, it was reduced to an optimization problem, where the root-mean-square error between measurement and calculation was formulated as an objective function to be minimized. Figure 6a shows the comparison of measurement and simulation for quasistatic hysteresis of radial y - and z -directions of an asymmetrical bushing and Figure 6b and 6c represent its dynamic stiffness measured and calculated for radial y -direction. The comparison of dynamic stiffness due to frequency and amplitude exhibits a maximum error about 8% and an average error less than 2%.

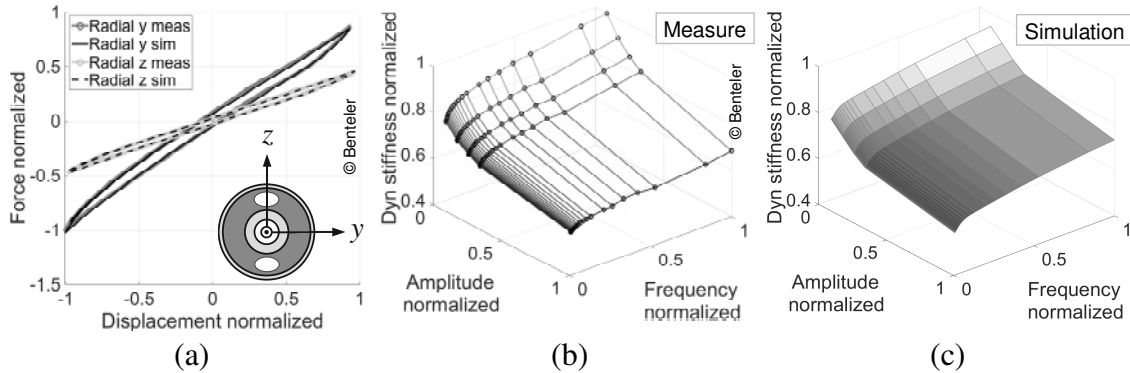


Figure 6. Quasistatic hysteresis (a); dynamic stiffness measurement (b); dynamic stiffness simulation (c)

Kinematics and Compliance (K&C) test rig used to confirm the elastokinematic characteristics of suspension system. For this, vehicle body clamped to the test rig platform, while forces and displacements applied through the wheel adapter respectively to the selected test, such as vertical bounce, longitudinal and lateral compliance. Corresponding load-, displacement- and orientation-parameters are measured. Figure 7a-c illustrates exemplary the comparison between K&C test data and simulation results for quasistatic toe change, wheel contact point displacement and suspension rate due to a vertical parallel and opposite test. To validate the dynamic behavior of the entire system an additional internal-force analysis is conducted, where the reaction forces on suspension links and damper travel measured due to the vertical step load change of the vehicle body considering tires, as presented in Figure 8a-b. Both elastokinematic and internal-force analysis provide a good confirmation for the proposed simulation model.

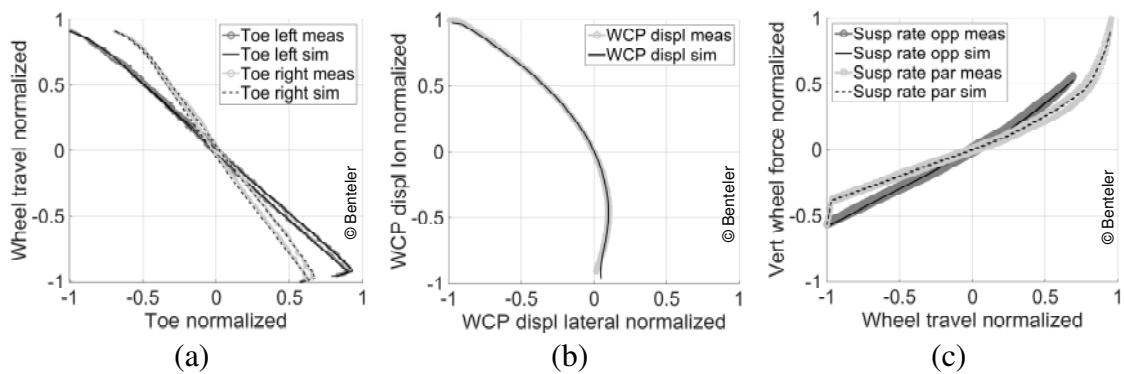


Figure 7. Toe angle (a); wheel contact point (WCP) - left (b); suspension rate - left due to quasistatic vertical suspension test (c)

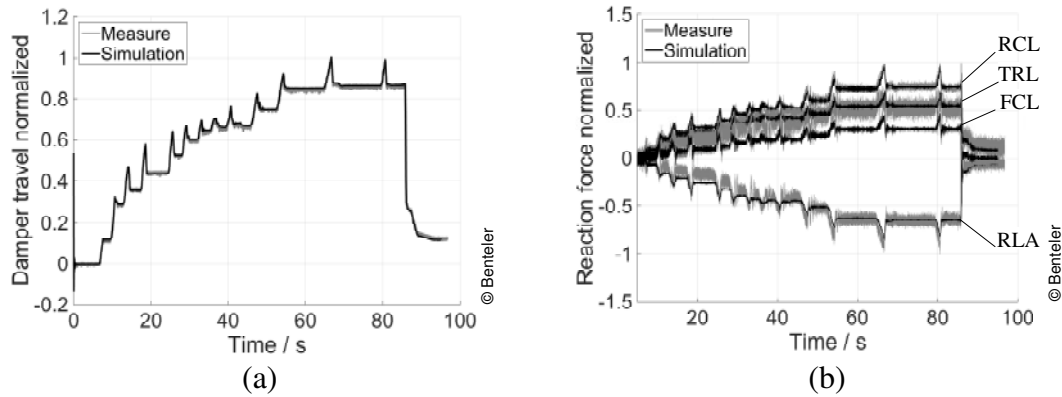


Figure 8. Damper travel - right (a); reaction forces - right (b) due to vertical vehicle test

FRICIONAL POWER DISTRIBUTION

With regard to the boundary conditions like prorated vehicle body mass and static wheel toe in and negative camber set up, simulations have been performed on a roughness road. The 3D road model approximately corresponds to the asphalt road with an International Roughness Index (IRI) of 3551 mm/km. The used exemplary FTire data set reflects a pneumatic tire for a passenger car with dimensions of 205/55 R16 without tread pattern. Therefore instantaneous frictional power is calculated for each tire contact element within the footprint due to its sliding velocity and frictional force considering dynamics of the rear multi-link suspension system. As expected, both height and sum of frictional power grow due to increasing wheel slip through increasing sliding velocity and sliding area, as illustrated in Figure 9a-c.

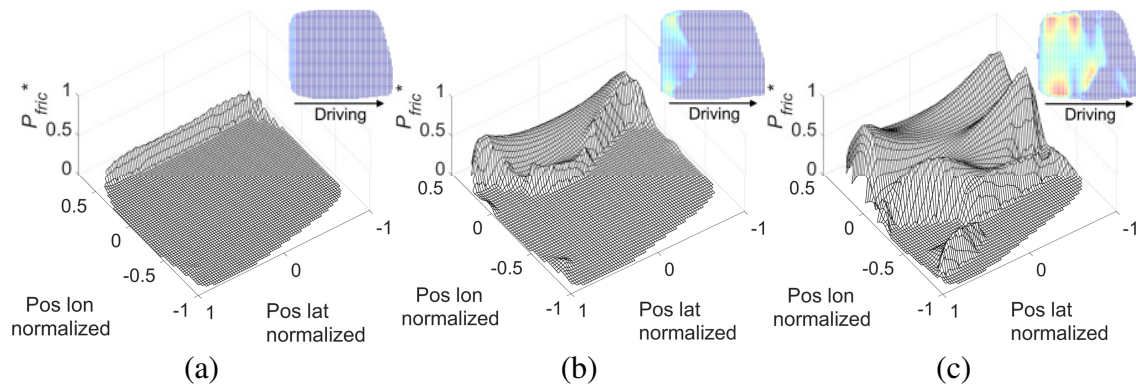


Figure 9. Instantaneous distribution of normalized frictional power due to driving mode with 1,5% - wheel slip (a); 3% - wheel slip (b); 8,5% - wheel slip (c)

CONCLUSION

The established methodology for flexible viscoelastic suspension modeling on an example of the rear multi-link suspension system in combination with an adequate tire and road models allows numerical calculation of frictional power distribution due to more realistic interaction of tire, road and suspension system. So it enables an evaluation of tire tread wear and a design study of an automotive suspension system, especially with regard to minimizing tire tread wear.

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