Analysis of Different Operation Modes for Inertia Motors

M. Hunstig, T. Hemsel, W. Sextro
University of Paderborn, Mechatronics and Dynamics, Paderborn, Germany

Abstract:
Piezoelectric inertia motors, also known as “stick-slip-drives”, use the inertia of a body to drive it by means of a friction contact in small steps. While these steps are classically assumed to involve stick and sliding, the motors can also operate in a “slip-slip” mode without any phase of stick. In this contribution, a one degree of freedom model of an inertia motor is used to analyse the motor characteristics. Appropriate performance indicators for the slider motion are defined: Start-up time, steady state velocity, and smoothness of motion. Inertia motors reach their theoretical maximum velocity with ideal signals containing harmonics of very high frequencies. With frequency-limited real signals, high motor velocity can only be achieved in “slip-slip” mode. The maximum motor velocity is reached by superposition of two sinusoidal signals, other signal shapes can improve smoothness of motion. The results help motor designers to choose the appropriate mode of operation and the best drive parameters for their individual applications.

Keywords: Piezoelectric Motor, Inertia Motor, Stick-Slip, Slip-Slip, Optimization

Introduction
Originally developed for fine positioning applications in the laboratory, piezoelectric inertia motors found application in several other fields in the last years, mainly in miniaturised consumer goods like digital cameras for mobile phones [1–4]. This was facilitated by the fact that inertia motors have a simple construction and are controlled by a single driving signal, which allows for low production costs and simplifies miniaturization.

Inertia motors make use of the inertia of a body to drive it by means of a friction contact in a series of small steps. Motors of this type are also known as “stick-slip-drives” because these steps are classically regarded to be composed of a phase of static friction between the driving and driven part and a phase where the two parts slide on each other.

Even though the first piezoelectric inertia motors were developed in the mid-1980s [5–7], the fact that these motors can also successfully operate without phases of static friction has gained wider recognition only in the last years. This mode of operation with the parts continuously sliding and only the direction of relative motion changing is also known as the “slip-slip” mode. Some authors [4, 8, 9] have described inertia motors operating in both stick-slip and slip-slip mode. But the principal advantages and disadvantages of the two operation modes and how to use them advantageously are still unclear. This contribution aims to give answers to these questions.

Investigated Setup
Figure 1 shows the model which is the basis of the following analysis. The displacement \( x_R(t) \) of a rod is given. A slider of mass \( m_S \) hangs below the rod. The contact force \( F_C \) between rod and slider results from the gravitational force \( F_g \) and an external force \( F_{ext} \), both assumed to act on the centre of gravity \( C \) of the slider. The friction force \( F_f \) between rod and slider is modelled using a Coulomb friction model with the coefficients of static and dynamic friction \( \mu_s \) and \( \mu_d \). The equation of motion of the slider is

\[
m_S \ddot{x}_S = F_f - m_S g \sin \gamma,
\]

and in the regime of sliding friction the friction force is described by

\[
F_f = \mu_s F_C \operatorname{sgn}(\dot{x}_R - \dot{x}_S).
\]

In the investigations described below, horizontal operation \( (\gamma = 0) \) is assumed.

Ideal Excitation Signals for Maximum Velocity
In many applications of inertia motors, especially in consumer applications, high velocity is the primary design goal.

For movement in positive direction of \( x_S \) the maximum acceleration of rod and slider that can be
reached without leaving the regime of stiction is defined by

\[ a_{S,max} = \frac{\mu_s F_c}{m_S}. \]  

(3)

The movement cycle for maximum velocity in stick-slip mode is as follows: The slider accelerates with \( a_R = a_{S,max} \) until the rod reaches its maximum position \( x_{R,max} \). Ideally, it then instantly returns to \( x_R(t) = 0 \) and the slider starts to slide and is continuously decelerated. The next acceleration phase of the rod is started directly after it has returned to \( x_R(t) = 0 \), so the slider remains in continuous motion with alternating phases of stiction and sliding as depicted in figure 2(a). For such signals the fundamental signal frequency is determined as

\[ f_0 = \frac{a_R}{2 \pi R_{S,max}}. \]  

(4)

If discrete steps of the slider are desired, a phase of rest can be introduced after each acceleration phase, allowing the slider to decelerate down to \( x_S(t) = 0 \) as shown in figure 2(b). A more detailed derivation of these continuous and discrete modes of operation under the assumption of limited rod acceleration is presented in [10].

![Fig. 2: Rod position and slider velocity for (a) continuous operation and (b) discrete steps](image)

If the rod acceleration \( a_R \) is increased above \( a_{S,max} \), there is no stiction between rod and slider and the motor operates in slip-slip mode. Continuous motion and discrete steps can be realized in this mode as well.

**Performance Indicators**

In many cases, an inertia motor requires several periods of the drive cycle to reach its maximum velocity. Figure 3 shows such a typical velocity increase. \( \bar{v}_S \) is the mean slider velocity in one period and the steady state velocity \( \bar{v}_{SS} \) is defined as \( \bar{v}_S \) after an infinite number of periods.

The start-up time \( t_{ss} \) is the time after which \( \bar{v}_S \) reaches 0.98 \( \bar{v}_{SS} \). It can also be defined for other percentages of \( \bar{v}_{SS} \). As \( \bar{v}_S \) is defined over complete periods only, \( t_{ss} \) is always a multiple of the drive cycle period.

![Fig. 3: Typical velocity increase at start-up](image)

During one period, the slider velocity ranges between the minimum velocity \( v_{\min} \) and the maximum velocity \( v_{\max} \). The smoothness of the slider motion can be defined as

\[ s = 1 - \frac{v_{\max} - v_{\min}}{2 v}. \]  

(5)

Steady state velocity, start-up time and smoothness of the slider motion are three indicators for the motional performance of an inertia motor.

**Performance with Ideal Excitation Signals**

A comparison of the modes with continuous slider movement shows that \( \bar{v}_S \) rises with \( a_R \), see figure 4. There is a jump in the curve at \( a_R = a_{S,max} \) due to the sudden change of the slider acceleration resulting from the assumption \( \mu_d \neq \mu_s \). If \( \mu_d = \mu_s \), there is no jump, but still a kink at the same position. The maximum velocity is limited in stick-slip mode because \( a_R \) must not exceed \( a_{S,max} \).

![Fig. 4: Change of steady state velocity and rod displacement characteristic with rod acceleration \( a_R \)](image)

In stick-slip mode, \( \bar{v}_S \) is always reached in the second period (cp. figure 2(a)) while in slip-slip mode the slider needs many periods to reach its steady state velocity. Due to this significant difference between the two modes, a jump at \( a_R = a_{S,max} \) is observed in figure 5 which shows \( t_{ss} \).

![Fig. 5: Change of start-up time \( t_{ss} \) with rod acceleration \( a_R \)](image)

Figure 6 shows the increase of the smoothness \( s \) of the slider motion with increasing \( a_R \). It smoothly converges towards the unreachable value of 1 for high rod acceleration.
In summary, higher velocities can be reached and the slider movement is smoother in slip-slip mode at the cost of a longer start-up phase and the requirement of higher frequencies compared to stick-slip mode.

**Frequency-Limited Real Excitation Signals**

Fourier analysis can be used to determine the harmonics of periodic signals and to reconstruct these signals by summation of their harmonics. The ideal excitation signals presented above can only be reproduced perfectly with an infinite number of harmonics, but no real actuator can do this.

Frequency limited signals are derived from the ideal signals using Fourier series of finite length $n$. Figure 7 shows resulting characteristics of displacement, velocity and acceleration for different values of $n$.

![Fig. 7: Rod displacement, velocity and acceleration for the ideal signal and for frequency-limited signals of orders 2 and 15](image)

While the displacement signal is approximated well already with low $n$, large oscillations are present in the frequency-limited velocity and acceleration due to the short phases where velocity or acceleration are infinite in the ideal signals. Except for a combination of very low $a_R$ and low $n$, the rod acceleration is effectively always above $a_{S,max}$ due to these oscillations. This means that there are no phases of static friction with significant length.

**Performance with Real Excitation Signals**

Figure 8 shows the steady state velocity reached by the simulated motor excited with frequency-limited signals with different $n$ and $a_R$. While there is no effective slider movement for $n = 1$, the steady state velocity rises with both $n$ and $a_R$. For high values of $n$, a further increase of $n$ yields only a small increase of velocity.

![Fig. 8: Steady state velocity with different $n$ and $a_R$](image)

The frequency of the highest harmonic present in the excitation signals is

$$f_{max} = n f_0 = n \frac{\Delta R}{\sqrt{2} a_{S,max}}.$$  

Real actuators have a maximum frequency up to which they can be practically operated, so $n$ and $a_R$ cannot be increased arbitrarily. For a given maximum frequency $f_{max}$ choosing a higher $n$ means to have a signal with more harmonics, but a lower fundamental frequency. Figure 9 illustrates this relation.

![Fig. 9: Three displacement signals with equal $f_{max}$](image)

Figures 10 (a) to (c) show steady-state velocity, start-up time and smoothness of the slider motion, respectively, reached with equal $f_{max}$ for different combinations of $n$ and $a_R$.

(a) Steady state velocity

![Fig. 10: Steady state velocity, start-up time and smoothness for different maximum frequencies](image)

Higher maximum frequencies lead to higher steady state velocity. For any given maximum frequency the highest steady state velocity is reached with $n = 2$. 

**ACTUATOR 2012**, Messe Bremen
The start-up time is generally larger for larger maximum frequencies. With any constant maximum frequency, increasing \( n \) decreases the start-up time.

The smoothness of the slider motion is generally larger for larger maximum frequencies. Increasing \( n \) slightly decreases the smoothness for constant maximum frequency.

**Influence of the Actuator Stroke on the Velocity**

The investigations described above are true independent of the actuator stroke \( x_{R,max} \). But this stroke has a large influence on the absolute value of the steady state velocity: It can be shown that if the fundamental excitation frequency follows equation (6), \( \bar{v}_n \propto \sqrt{x_{R,max}} \). This remains true for the ideal signals with limited rod acceleration [11]. To achieve high velocities it is therefore advisable to use actuators with large stroke. This theoretical result is supported by the fact that almost all inertia motors achieving high velocities (here defined as 20 mm/s or larger) use large actuator strokes achieved either by resonance effects [9, 12] or by mechanical amplification using compliance mechanisms [13] or bending actuators [14–16]. As far as known to the authors, only one motor [8] reaches such velocities with an un-amplified multilayer actuator.

**Conclusions**

This contribution showed that, if high velocity is desired, it is not recommendable to aim for classic “stick-slip” operation of inertia motors. In “stick-slip” mode these motors can be operated with signals containing only harmonics of relatively low frequencies and still reach velocities that are higher than the theoretical maximum for “stick-slip” operation.

With a given maximum signal frequency, the maximum motor velocity is always reached by superposition of two sinusoidal signals. This is beneficial as such a signal can use resonance amplification with passable complexity, demonstratad for example in [9, 12]. If other performance criteria are relevant in the application, the use of a different driving signal can make sense.

While the frequency-limited signals derived from the ideal excitation signals produce high velocities, it has not yet been determined whether these signals indeed produce the highest velocity. Whether other signal shapes containing the same harmonic frequencies can be more advantageous, especially regarding other performance criteria, is subject to further investigation.

As the knowledge about the advantages of “slip-slip” operation of inertia motors is growing and spreading, the authors expect that the number of dedicated “slip-slip” inertia motors will grow significantly in the near future, widening the field of application of inertia motors.

**References**


