Introduction to Quantum Computation, UPB
Summer 2020, Assignment 3

To be completed by: Friday, May 15

1 Exercises

1. (a) Let $A \in \mathcal{L}(\mathbb{C}^d)$ be Hermitian. Prove that if for all $|\psi\rangle \in \mathbb{C}^d$, $\langle\psi|A|\psi\rangle \geq 0$, then A has only non-negative eigenvalues. (Hint: Start by taking the spectral decomposition of A, and then make clever choices for $|\psi\rangle$.)

   (b) Let $A \in \mathcal{L}(\mathbb{C}^d)$ be Hermitian. Prove that if A has only non-negative eigenvalues, then for all $|\psi\rangle \in \mathbb{C}^d$, $\langle\psi|A|\psi\rangle \geq 0$. (Hint: Write $|\psi\rangle$ with respect to the eigenbasis of A.)

2. Let $|\psi\rangle = |-\rangle \in \mathbb{C}^2$. Suppose we measure in the Z basis $B = \{|0\rangle\langle 0|, |1\rangle\langle 1|\}$. What are the probabilities for each possible measurement outcome, and the corresponding post-measurement states?

3. Consider the teleportation protocol we saw in class. Does it still work if we replace the use of the entangled Bell state $|\phi^+\rangle$ with the unentangled state $|00\rangle$ (i.e. Alice and Bob share the state $|00\rangle$)? How about if we use $\sqrt{2/5}|00\rangle + \sqrt{3/5}|11\rangle$ instead of $|\phi^+\rangle$?