Relativistic light-matter interaction of an atom

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Abstract

We study the interaction of a hydrogen-like atom with the quantised electromagnetic field including the often neglected Röntgen term. This term arises from electronic charge currents already in the atomic rest frame since we do not consider a pointlike atom, but rather one with spatial extension naturally induced by the atom's wave function. We show that with the correct prescription the observable predictions are Lorentz invariant. In particular, the results should be helpful in the study of arbitrary relativistic atomic trajectories in the presence of light, including possible boundary conditions that yield Casimir-Polder forces.

In previous literature [1], a covariant description of the simplified scalar version of light-matter interactions has been given. Here, we want to characterize realistic light-matter interactions in a similar fashion: Ensuring the invariance of physical predictions, in particular in highly relativistic scenarios.

We will consider to that end a hydrogen-like atom that moves on an arbitrary relativistic trajectory in external quantised electric and magnetic fields $\hat{\boldsymbol{E}}$ and $\hat{\boldsymbol{B}}$. We shall start in the rest frame of the atom which presents itself as the natural frame to describe the interaction. For if we assume the dipole approximation, one can expect two terms to contribute: One is the dipolar coupling with the electric field, and the other one, often neglected, is the interaction of the charge current created by the electronic motion (the nucleus' motion will be of much less importance in the rest frame) with the magnetic field. The interaction Hamiltonian in the atom's rest frame will be cast as

$$\hat{H}_I = \hat{\boldsymbol{d}} \cdot \hat{\boldsymbol{E}} + \frac{\hat{\boldsymbol{P}}}{2m} \cdot \left(\hat{\boldsymbol{d}} \times \hat{\boldsymbol{B}}\right) + \left(\hat{\boldsymbol{d}} \times \hat{\boldsymbol{B}}\right) \cdot \frac{\hat{\boldsymbol{P}}}{2m}, \tag{1}$$

where m is the electronic mass, $\hat{\boldsymbol{P}}$ is the momentum operator of the electron and $\hat{\boldsymbol{d}} = e\hat{\boldsymbol{x}}$ is the dipole moment. A similar form of the Hamiltonian in the lab frame has been found in for instance [2, 3, 4], where it was assumed however that the atom is an electric dipole without accounting for the naturally smeared charge distribution which can be shown to follow from the atomic orbitals. Indeed, by going to position representation and expanding in the energy eigenbasis of the atom, the wave functions Ψ of the atom result in non-pointlike charge distributions. The dipole moment has then a spatial smearing

$$\boldsymbol{F}(\boldsymbol{\xi})_{kl} = \boldsymbol{\xi}\Psi_k^*(\boldsymbol{\xi})\Psi_l(\boldsymbol{\xi}), \tag{2}$$

where k, l denote the energy states, and $\boldsymbol{\xi}$ are the spatial coordinates in the atomic rest frame. Therefore, one should expect that the Röntgen term already appears in this frame.

Since the time evolution generated by the Hamiltonian cannot depend on coordinate changes, i.e.

$$\hat{U} = \mathcal{T}\exp\left(\frac{-\mathrm{i}}{\hbar}\int \mathrm{d}^n\xi \mathrm{d}\tau \hat{h}_I(\tau, \boldsymbol{\xi})\right) = \mathcal{T}\exp\left(\frac{-\mathrm{i}}{\hbar}\int \mathrm{d}^n x \mathrm{d}t \hat{h}_I(t, \boldsymbol{x})\right), \tag{3}$$

we have to impose restrictions on the Hamiltonian density $\hat{h}_I$ in order to guarantee Lorentz invariance. We will show as an example that transition probabilities are indeed Lorentz invariant if we impose (3).

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