Comparison of semiclassical and fully quantum mechanical models of two and three level atoms interacting with a quantized cavity mode

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Abstract

Despite the simplicity of the interaction between atomic transitions and quantized cavity modes, the multiphoton structure of the spectrum is not well explored. In this work, we present results of numerical simulations of steady-state multiphoton cavity transmission spectra of strongly coupled two- and three-level atoms and a quantized optical cavity mode in parameter space where dipole and rotating wave approximations are valid (i.e., $\kappa\lambda \ll g \ll (\omega_a, \omega_c)$) using up to 30 dressed states. We show, in stark contract to the usual results of semi-classical models, existence of a very rich multiphoton spectral structures.

We consider a driven, Λ type three level atom one of whose transitions is resonantly coupled with a quantized cavity mode. The total Hamiltonian of the system has the form

$$H = \hbar\Delta_c a^\dagger a + \hbar\Delta_1\sigma_{31}\sigma_{13} + \hbar(\Delta_1 - \Delta_2)\sigma_{21}\sigma_{12} + \hbar(g\sigma_{31}a + \Omega\sigma_{32} + \eta + \text{H. c.}) \tag{1}$$

where $\Delta_i = \omega_{31} - \omega_l$ $(i = 1, 2)$ and $\Delta_c = \omega_c - \omega_l$ are the detunings of the atomic transitions and the cavity mode from the external driving frequency; and g and Ω are couplings between the atomic transitions and the cavity the free-space modes; and η the strength of the driving external. The density matrix of the system obeys the master equation

$$\dot{\rho} = -\tfrac{i}{\hbar}[H, \rho] + \tfrac{1}{2}\kappa\mathcal{L}_c\rho + \tfrac{1}{2}\sum_{j=1}^{2}\gamma_{3j}\mathcal{L}_j\rho \tag{2}$$

where κ and γ_{3j} are the cavity field and excited state decay rates and

$$\mathcal{L}_c\rho = 2a\rho a^\dagger - a^\dagger a\rho - \rho a^\dagger a \tag{3}$$

characterizes the cavity damping whereas

$$\mathcal{L}_a\rho = 2\sigma_{j3}\rho\sigma_{3j} - \sigma_{3j}\sigma_{j3}\rho - \rho\sigma_{3j}\sigma_{j3} \tag{4}$$

characterizes the atomic decay into lower states $|j\rangle$ $(j = 1, 2)$.

We compute the transition spectrum of two-level-cavity system (by setting $\Omega = 0$) as well as well as the three-level-cavity system (where $\Omega \neq 0$) at a various levels of driving field strength. Figure 1 (a) shows the transmission spectrum of the system under a various driving field conditions. The two sets of peaks on each side of Δ_c corresponding to multiphoton transitions between the ground state and higher level dressed states. Figure 1 (b) corresponds the results of semiclassical calculation under same conditions. On the other hand, Figure 2 (a) shows the transmission spectrum of the three-level atom-cavity system under moderate driving field conditions. Figure 2 (b) corresponds to the semiclassical calculation under the same conditions.

[1] M. O. Musa and H. Temimi, *Comparison of semiclassical and quantum models of a two-level atom-cavity QED system in the strong coupling regime*, submitted to Journal Modern Optics.

[2] M. O. Musa and H. Temimi, *Semiclassical and fully quantum mechanical models of a three level atom strongly interacting with a quantized cavity mode,* manuscript in preparation.