Quantum networks for multi-parameter metrology

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Abstract

We address the multi-parameter problem of estimating a set of linear functions of several parameters that have been encoded in a sensor-symmetric quantum sensing network. Relying on the fact that entanglement is known to play a relevant role in this scenario, we use the quantum Cramér-Rao bound to find the relationship between the geometry of the transformation defined by the linear functions and the amount of inter-sensor correlations of the network. Finally, we perform a full Bayesian analysis of some of our schemes to analyse whether the conclusions derived in this way are still valid in the regime of limited data.

Over the last few years quantum metrology has evolved from well understood single-parameter cases to multi-parameter scenarios with a vast set of unexplored possibilities in terms of new ways of enhancing our estimation protocols. One of the key ideas in this context is to exploit the type of correlations that these scenarios can offer and, in particular, to establish under which circumstances these correlations can be useful or detrimental. This was precisely the idea behind the quantum sensing network model introduced in [1, 2].

Within that framework it is possible to show that entanglement can play a crucial role whenever we are interested in estimating functions of the parameters that were originally encoded in the network in a local way, so that these functions can be seen as global properties. The goal of this work is then to explore how the nature of a given global property determines the type and amount of correlations that the probe state must have for the scheme to be optimal, and for the sake of simplicity, we make two assumptions: a) the state is sensor-symmetric, and b) the global properties are represented as a collection of linear functions.

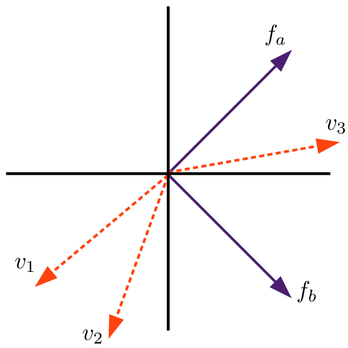

Figure 1: Space of linear functions for a two-parameter network. The solid arrows indicate the directions associated with a maximum amount of entanglement, while the dashed arrows are functions to be estimated.

Using the quantum Cramér-Rao bound first, we show the relationship between the correlations of this network and the geometry of the linear functions, finding that for this type of state the are two privileged subspaces in the space of linear functions that associated with a maximum correlation. On the other hand, since in our particular case the quantum Cramér-Rao bound is only meaningful asymptotically, we perform a Bayesian analysis following the methods introduced in [3] in order to check whether our conclusions are still valid in the regime of limited data as defined in [4]. Our results provide, therefore, a first look into two new regimes in multi-parameter metrology: the estimation of more than one global property and the performance of the protocol for a low number of trials.

[1] Timothy J. Proctor, Paul A. Knott, and Jacob A. Dunningham, *Multiparameter Estimation in Networked Quantum Sensors*, Phys. Rev. Lett. **120**, 080501 (2018).

[2] Timothy J. Proctor, Paul A. Knott, and Jacob A. Dunningham, *Networked quantum sensing*, arXiv: 1702.04271 (2017).

[3] Jesús Rubio, Paul A. Knott and Jacob A. Dunningham, *Non-asymptotic analysis of quantum metrology protocols beyond the Cramér–Rao bound*, J. Phys. Commun. **2**, 015027 (2018).

[4] Jesús Rubio and Jacob A. Dunningham, *Quantum metrology in the presence of limited data*, New J. Phys. in press (2019).