Entanglement Witness for Hybrid Bipartite States Based on the Negativity Volume of the Wigner Function

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Abstract

We present a new entanglement witness for arbitrary (hybrid) quantum states based on the generalized Wigner function. It is shown that this witness can be also applied on quantum systems undergoing decoherence effects.

In the recent paper [1] Tilma et al. have derived a generalized Wigner function that can characterize both the discrete and continuous variable states at the same time, i.e., hybrid states. Due to its simplicity, one could expect that the negativity of the generalized Wigner function applied to the hybrid states can reveal their nonclassicality in analogy with the well-known Wigner function defined for the continuous variable states. Nevertheless, as further results have suggested the negativity of the Wigner function already related to the spin-1/2 systems can refer rather to the purity of the discrete variable states [2]. However, it is important to distinguish different origins of nonclassicality in hybrid states state [3].

In our contribution we demonstrate that the negativity volume of the generalized Wigner function of the hybrid bipartite states can be used as an entanglement witness of such states provided that it exceeds a certain critical value [4]. The negativity volume of the Wigner function is given by the integral $\mathcal{V} = \frac{1}{2}\int \Big(|W| - W\Big)\mathrm{d}\Omega$. Any mixed hybrid bipartite qubit-bosonic state which is separable can be represented as a convex sum of product states $\hat{\rho}_{sep} = \sum_i p_i \hat{\rho}_i^q \otimes \hat{\rho}_i^b$. For the hybrid bipartite qubit—bosonic states, it can be easily shown that the negativity volume of the Wigner function defined for separable hybrid sate $\hat{\rho}_{sep}$ satisfies the following inequality

$$\mathcal{V}(\hat{\rho}_{sep}) \leq \mathcal{V}_{cr} = \left(\frac{1}{\sqrt{3}} + \frac{1}{2}\right)\sum_i p_i \mathcal{V}(\hat{\rho}_i^b) + \frac{1}{\sqrt{3}} - \frac{1}{2}, \tag{1}$$

where $\mathcal{V}_{cr}$ stands for the critical value of the negativity volume for separable hybrid states. Then, we can say that for the given hybrid bipartite qubit–bosonic state $\hat{\rho}$ is entangled if $\mathcal{V}(\hat{\rho}) > \mathcal{V}_{cr}$.

Since the detection of the generalized Wigner function of hybrid bipartie states in the phase space is experimentally simpler than the reconstruction of the corresponding density matrix, our results thus present a convenient tool in the entanglement identification of such states.

[1] T. Tilma et al, *Wigner Functions for Arbitrary Quantum Systems*, Phys. Rev. Lett. **117**, 180401 (2016).

[2] M. A. Ciampini et al, *Wigner function reconstruction of experimental three-qubit GHZ and W states*, arXiv:1710.02460 (2017).

[3] J. Sperling et al., *Quantum correlations in composite systems*, J. Phys. B **50**, 134003 (2017).

[4] I. Arkhipov et al., *Negativity volume of the generalized Wigner function as an entanglement witness for hybrid bipartite states*, Sci. Rep. **8**, 16955 (2018).