CEWQO 2019

Topological aspects of discrete and continuous time quantum walks on one dimensional lattices

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Abstract

We consider three settings for the time dependence of gapped quantum walks in one dimension with additional discrete symmetries. First, there is the continuous time setting with a constant Hamiltonian. Second, we consider periodic driving, i.e., a continuous time setting where the discrete symmetry applies to the whole driving process. In the third scenario only the Floquet unitary operator at one driving period is considered, and taken as a discrete time dynamical system in its own right. In all three cases the classifying indices are given, and shown to be closely related to those of translation invariant systems. We show how to construct examples of all classes.

The topological classification of lattice systems with discrete symmetries has become one of the cornerstones of a theory of topological quantum matter. In the quantum optical context simulated systems [4,5] in optical lattices exhibit the same structures. In the present work we describe some natural variations in the setting which lead to distinct but related classification results. We focus on the one-dimensional lattice case, where all statements can be made rigorous, and the classification is usually in terms of a few integer values or parity valued indices.

As in all topological classifications, two objects, in our case, quantum dynamical systems, are considered equivalent, if they are connected by a continuous path of systems of the "same type". It is crucial which kind properties are included in this phrase. Typically, they are the following (1) *Unitarity* in the discrete time, *Hermiticity* of the Hamiltonian in the continuous time case (2) *Locality* in the sense that matrix elements of the Hamiltonian or of the one-step unitary decay with the distance between sites. (3) *Discrete Symmetries*, typically taken from the so-called tenfold way. This involves a combination of reflection symmetries, of either unitary or antiunitary Wigner type, with or without time reversal. (4) A *spectral gap* around points invariant under the symmetries, which are 0 in the Hamiltonian case and ± 1 in the unitary case. Often the interest is in eigenvalues appearing at a boundary between bulks, in which case it is helpful to relax the gap condition to allowing eigenvalues of finite multiplicities. (5) *Translation invariance* would seem make the problem much easier, but we do *not* assume this, precisely because we want to include joined systems.

In the translation invariant Hamiltonian case, one then gets a classifying index either in the group $\mathbb{Z}$ or in $\mathbb{Z}_2 = \{0, 1\}$, which depends only on the symmetry type. All indices described below will be in this group.

We now consider the three settings for time dependence mentioned in the abstract. Dropping translation invariance gives an index pair, of which one describes the asymptotic index on the right and the other on the left. Going to unitaries (Scenario three) one gets an additional index classifying local perturbations, which are not gentle (i.e., cannot be deformed away)[1]. Finally, going to scenario two [3], i.e. considering the whole driving process as subject to symmetries, the half-time evolution operator carries the classification. In this case, we get 5 indices, of which 4 can be interpreted in terms of the Floquet operator and its counterpart in a different time frame, and 1 classifies additional shifts.

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