CEWQO 2019

Optical Resolution at the Quantum Fisher Information Limit

M. Paúr[1], J. Řeháček[1], Z. Hradil[1], B. Stoklasa[1], J. Grover[2], A. Krzic[2] and L. L. Sánchez-Soto[3,4]

[1] Department of Optics, Palacký University, 17. listopadu 12, 771 46 Olomouc, Czech Republic

[2] ESA—Advanced Concepts and Studies Office, European Space Research Technology Centre (ESTEC), Keplerlaan 1, Postbus 299, NL-2200AG Noordwijk, Netherlands

[3] Departamento de Óptica, Facultad de Física, Universidad Complutense, 28040 Madrid, Spain

[4] Max-Planck-Institut für die Physik des Lichts, Staudtstraße 2, 91058 Erlangen, Germany

Abstract

Optical resolution can be advantageously reconsidered from the point of view of quantum estimation theory. For optimised measurements reaching the level of Quantum Fisher Information the super-resolution is saturable as will be demonstrated on several examples of precise optical detection schemes.

There are many problems where optics and quantum theory overlap, one of them being the fundamental limit upon the resolution. The spatial resolution of any imaging device is restricted by diffraction, which causes a sharp point on the object to blur into a finite-sized spot in the image. This intrinsic blurring is encoded in the point-spread function, which hinders to distinguish two neighbourhood points- an effect known as "Rayleigh curse". The same problem can be reconsidered from the perspective of quantum estimation theory, as done by Tsang and coworkers [1] showing that corresponding quantum Fisher information can be constant in this case as demonstrated experimentally is several experiments, see for example [2]. When only light intensity at the image plane is measured on the basis of all the traditional phase insensitive techniques such as CCD detection, the Fisher information falls to zero as the separation between two sources decreases in accordance with Rayleigh curse.

More general conditions for attaining the ultimate resolution for multi-parameter estimation of two point sources will be addressed in the talk. As will be shown, the constant quantum Fisher information is a consequence of the simple model adopted for parameter estimation. When more parameters such as relative intensities or coherence are assumed the Quantum Fisher Information is no more constant. For the general case of unequally bright sources, the amount of information related to separation falls to zero, but there is always a quadratic improvement in an optimal detection in comparison with the intensity measurements. Importantly such a bound can be saturated for carefully designed detection. Quantum-inspired imaging techniques can be further extended to shaping of point-spread function in order to achieve linear scaling of Fisher information [3] or adopted in time-frequency domain for resolving the optical pulses beyond the separation given by their optical bandwidth [6].

[1] M. Tsang, R. Nair, and X.-M. Lu, *Quantum Theory of Superresolution for Two Incoherent Optical Point Sources*, Phys. Rev. X, **6**, 031033 (2016).

[2] M. Paúr, B. Stoklasa, Z. Hradil, L. L. Sánchez-Soto, and J. Řeháček, *Achieving the ultimate optical resolution*, Optica **3**, 1144 (2016).

[3] M. Paúr, B. Stoklasa, J. Grover, A. Krzic, L. L. Sánchez-Soto, Z. Hradil, and J. Řeháček, *Tempering Rayleigh's curse with PSF shaping*, Optica **5**, 1177 (2018).

[4] J. Řeháček, Z. Hradil, B. Stoklasa, M. Paúr, J. Grover, A. Krzic, and L. L. Sánchez-Soto, *Multiparameter quantum metrology of incoherent point sources: Towards realistic superresolution*, Phys. Rev. A **96**, 062107 (2017).

[5] J. Řeháček, Z. Hradil, D. Koutný, J. Grover, A. Krzic, L. L. Sánchez-Soto, *Optimal measurements for quantum spatial superresolution*, Phys. Rev. A **98**, 012103 (2018).

[6] J. M. Donohue, V. Ansari, J. Řeháček, Z. Hradil, B. Stoklasa, M. Paúr, L. L. Sánchez-Soto, Ch. Silberhorn, *Quantum-limited time-frequency estimation through mode-selective photon measurement*, Phys. Rev. Lett. **121**, 090501 (2018).