

# Fundamental limits of discrete-modulation quantum-secure covert optical communication

Boulat Bash<sup>1,2</sup>, Christos Gagatsos<sup>2</sup>, and Saikat Guha<sup>1,2</sup>

<sup>1</sup> Department of Electrical and Computer Engineering, University of Arizona, Tucson, Arizona 85721, USA

<sup>2</sup> College of Optical Sciences, University of Arizona, Tucson, Arizona 85721, USA

## Abstract

We report on a fundamental difference between detectability of discrete modulation schemes over thermal-noise bosonic channels by quantum vs. classical adversaries: while discrete modulation is as effective in hiding data as the optimal Gaussian modulation against classical adversaries, it is strictly less effective against a quantum-enabled adversary. This impacts the design of quantum-secure covert communication systems as well as demonstrates yet another fundamental advantage of quantum detection techniques.

Covert, or low probability of detection/intercept (LPD/LPI), communication prevents transmission's detection by an adversary. This is a stricter security requirement than protection of transmission's content from unauthorized access provided by standard methods, e.g., quantum key distribution. *Square root law* (SRL) governs covert communication over additive white Gaussian noise (AWGN) [1] and bosonic [2] channels: no more than  $\propto \sqrt{n}$  covert bits can be transmitted reliably to the intended receiver in  $n$  uses of the channel; attempting to transmit more results in either detection by the adversary with high probability, or unreliable transmission. For optical channels,  $n$  is the number of available spatio-temporal-polarization modes. Even though the capacity of covert channel is zero, as  $n$  increases, SRL allows transmission of a significant number of covert bits even when the adversary is quantum-capable, provided he does not control some noise on his channel to transmitter [2].

SRL is the consequence of covertness requiring the mean transmitted photon number per mode to scale as  $\bar{n}_S = c/\sqrt{n}$ , with the SRL constant  $c$  characterizing the amount of transmissible covert data. Here we report on the impact of restriction to discrete modulation on  $c$  in communication over the thermal-noise lossy bosonic channel of transmissivity  $\eta$ , with  $\bar{n}_T$  mean photons injected per mode by a thermal environment. We compare the covertness of two modulation formats: 1) coherent state modulation using isotropic Gaussian prior,  $\mathcal{M}_G = \{|\alpha\rangle\}, p(\alpha) = \frac{e^{-|\alpha|^2/\bar{n}_S}}{\pi\bar{n}_S}\}$  and 2) binary coherent state modulation using equal prior  $\mathcal{M}_B = \{|\beta\rangle, |-\beta^*\rangle, p(\beta) = p(-\beta^*) = \frac{1}{2}\}$  with  $|\beta|^2 = \bar{n}_S$ .

When the adversary uses a homodyne receiver, he experiences an AWGN channel with noise power  $\sigma_T^2 = \frac{2\eta\bar{n}_T+1}{4(1-\eta)}$ . Then, maximum  $c_{AWGN} = 2\sigma_T^2$  [1, 3, 4]. While the optimality is proven using  $\mathcal{M}_G$  [3, 4], we can also attain  $c_{AWGN}$  using  $\mathcal{M}_B$  [1, Theorem 1.2]. Thus, it is as difficult to detect  $\mathcal{M}_B$  as it is  $\mathcal{M}_G$ , resulting in identical covert communication performance. Similar result holds for an adversary using a heterodyne receiver.

Surprisingly,  $\mathcal{M}_B$  fares worse than  $\mathcal{M}_G$  when adversary is given an arbitrary receiver. We show that the maximum is  $c_{bos} = \sqrt{2\eta\bar{n}_T(1+\eta\bar{n}_T)}$ , achieved by  $\mathcal{M}_G$ . However, maximum SRL constant achievable by  $\mathcal{M}_B$  is  $c_{BPSK} \leq \left( \frac{1}{2\eta\bar{n}_T(1+\eta\bar{n}_T)} + \frac{1}{1+2\eta\bar{n}_T} \log \left( 1 + \frac{1}{\eta\bar{n}_T} \right) \right)^{-\frac{1}{2}}$ . We note that we do not contradict that the Holevo capacity is achieved by  $\mathcal{M}_B$  at low signal-to-noise ratios [5], and, in fact, our calculations confirm this. Rather, we claim that a quantum receiver may make  $\mathcal{M}_B$  easier to detect than  $\mathcal{M}_G$ , requiring smaller  $\bar{n}_S$  and degradation in performance for  $\mathcal{M}_B$ . In addition to obvious implications for quantum-secure covert communication system design, this is yet another example of a classical-quantum gap in optical receiver performance.

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