

# Modal approach to quantum temporal imaging

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## Abstract

We review the main results of the modal theory of temporal imaging, recently developed by us. We consider a simple quantum temporal imaging system on the basis of a time lens implemented by a parametric nonlinear optical process with a chirped pump. We illustrate this approach by an example of type-I non-collinear sum-frequency generation process, where the phase matching is limited by the temporal walk-off of the pump, signal and idler waves. Our theoretical treatment allows us to identify the criteria for designing imaging schemes with close to unity efficiencies.

Quantum temporal imaging is a technique for the manipulation of the time-frequency degrees of freedom of a quantum state and it is based on the space-time duality of optical processes [1]. By virtue of this duality a device able to imprint onto an incoming wavefront a quadratic phase modulation in time behaves as a time-lens. A time-lens can be implemented by means of a nonlinear frequency conversion process with a chirped pump.

We have recently applied the theory of quantum temporal imaging to squeezed light in the limit of infinite temporal aperture and perfect phase-matching [2, 3] and in the limit of finite aperture but still perfect phase-matching [4]. The conditions of applicability of the perfect-phase-matching approximation has been studied [5]. While the performances of a temporal imaging scheme can be easily quantified when these conditions are met for any conversion efficiency regime, as soon as one wants to include the limitations induced by the finite phase-matching the quantification of performances becomes a difficult task. In general the imaging scheme can be described by a unitary transformation

$$\hat{A}_{\text{out}}(\Omega) = \int d\Omega' \left[ h(\Omega, \Omega') \hat{A}_{\text{in}}(\Omega') + q(\Omega, \Omega') \hat{A}_{\text{vac}}(\Omega') \right], \quad (1)$$

that is characterized by two transfer functions  $h(\Omega, \Omega')$  and  $q(\Omega, \Omega')$ , where  $\Omega$  and  $\Omega'$  are sideband frequencies in the Fourier space. This transformation produces a scaled image of the input field  $\hat{A}_{\text{in}}(\Omega)$  that is limited by the presence of vacuum  $\hat{A}_{\text{vac}}(\Omega)$  entering in the process because of the finite temporal aperture of the time-lens and imperfect phase-matching. Note that the second transfer function  $q(\Omega, \Omega')$  describes a purely quantum effect and is zero in the classical formalism [1].

Despite the fact that for high conversion efficiency the transformation (1) does not have an explicit expression, a quantitative assessment of the performances (in terms of resolution and field-of-view) of the associated imaging scheme can be realized from the eigenvalues and eigenvectors of the two transfer functions, thus showing the interest of the modal approach for temporal imaging. We consider the example of a time lens implemented by a type-I non-collinear sum-frequency generation process and show that for the case of a chirped Gaussian pump the eigenvalues and eigenvectors of the transformation (1) can be obtained analytically. From this modal description is then possible to extract the resolution and the field-of-view. They can be expressed in terms of the experimental parameters of the systems such as the pump duration and chirp, the temporal walk-off between the pump, the signal and the idler waves. These results may find numerous applications in experiments dealing with manipulation and detection of temporally multimode squeezed light.

- [1] B. H. Kolner, *Space-Time Duality and the Theory of Temporal Imaging*, IEEE J. Quantum Electron. **30**, 1951 (1994)
- [2] G. Patera and M. I. Kolobov, *Temporal imaging with squeezed light*, Opt. Lett. **40**, 1125 (2015)
- [3] J. Shi, G. Patera, M. I. Kolobov and S. Han, *Quantum temporal imaging by four-wave mixing*, Opt. Lett. **42**, 3121 (2017)
- [4] G. Patera, D. B. Horoshko and M. I. Kolobov, *Space-time duality and quantum temporal imaging*, Phys. Rev. A **98**, 053815 (2018)
- [5] G. Patera, J. Shi, D. B. Horoshko, and M. I. Kolobov, *Quantum temporal imaging: application of a time lens to quantum optics*, J. Opt. **19**, 054001 (2017).