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A trajectory in phase-space surpassing the Heisenberg - uncertainty limit

Jascha Zander[1], and Roman Schnabel[1]

[1] Institute of Laser Physics and Centre for Optical Quantum Technologies,
University of Hamburg, 22761 Hamburg, Germany

Abstract

The quantum uncertainty in a physical system is limited by the Heisenberg-Uncertainty-Relation (HUR), which contains the amount of information that can be extracted from two non-commuting observables. In this Experiment we demonstrate the EPR – Gedankenexperiment [1] by simultaneous measurement on a single gaussian wave packet, in principle to arbitrarily small precision. Consequently we demonstrate a simultanous measurement protocol for a very weak and time varying signal in the phase-space. Furthermore we show a trajectory within vacuum noise, with a resolution of the variance product of $\Delta^2\hat{X}\Delta^2\hat{Y} \approx 0.1$ over an extended period of time.

A well-established method for circumvent the limit of the HUR of a single observable is to utilize non-classical states to squeeze the imprecision, e.g. in the phase quadrature below its zero-point fluctuation, and therefore implies an increasing in the amplitude quadrature [2]. Of course this leads to an arbitrarily precise measurement of the phase at a certain time, while the amplitude is totally unknown at the same moment. However, for a joint measurement of two non-commuting observables another approach is required. Here we implement a sophisticated readout scheme, based on an EPR entangled two-mode system [3, 4].

Superimposing two orthogonal squeezed states on a beam splitter, one part (Subsystem A) is kept as a reference, while the other part (Subsystem B) undergoes two independent interactions U(t) and U'(t). Subsequently it carries two pieces of information encoded in two non-commuting observables. The recombination at a second beam splitter makes it possible to extract the information and detect the different observables on two balanced homodyne detectors.

The benefit of this scheme becomes clear, since the signal is now encoded in the difference of the amplitude quadrature's $\hat{X} = \hat{X}_{A,ref} - \hat{X}_B$ and the sum of the phase quadrature's $\hat{Y} = \hat{Y}_{A,ref} + \hat{Y}_B$ and those quantities are commute, $[\hat{X}_{A,ref} - \hat{X}_B, \hat{Y}_{A,ref} + \hat{Y}_B] = 0$. This is an important point, because $\hat{X}$ and $\hat{Y}$ are now indeed simultaneously precise defined, even though with respect to the reference frame. A simultaneous detection is not limited by the Heisenberg-Uncertainty-Relation anymore. The following uncertainty of a signal is then represented by a new relation [5]:

$$\Delta^2\hat{X}\Delta^2\hat{Y} \geq e^{-2r_A}e^{-2r_B}, \tag{1}$$

where r_A and r_B are the squeezing values of the input states. The achievable precision corresponds directly to the amount of squeezing and for infinite squeezing the uncertainty reduces to zero.

In conclusion we prove via the statistics of many such individual measurements on a large number of identical wave packets that, indeed, both quantities can be measured simultaneously with, in principle arbitrary precision. In this letter the goal will be to develop a more detailed physical picture of the Heisenberg-Uncertainty-Relation.

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