Probing Rényi Entanglement Entropy via Randomised Measurements

Tiff Brydges*[1], Andreas Elben*[1], Petar Jurcevic[1], Benoît Vermersch[1], Christine Maier[1], Ben P. Lanyon[1], Peter Zoller[1], Rainer Blatt[1], Christian F. Roos[1],

[1] Center for Quantum Physics and Institute for Experimental Physics, University of Innsbruck & Institute for Quantum Optics and Quantum Information, Austrian Academy of Sciences, Innsbruck, Austria

Abstract

This work presents and experimentally demonstrates a new protocol for characterising entanglement in engineered quantum systems. Measuring the entropy of different partitions of a quantum system provides a way to probe its entanglement structure. This protocol measures the second-order Rényi entropy of the system, using statistical correlations between randomised measurements [1]. The experiments, performed on a trapped-ion quantum simulator, reveal the dynamical evolution of entanglement entropy of 10-qubit partitions of up to 20-ion strings - both in the absence and presence of disorder. The protocol represents a novel, universal tool for probing and characterising engineered quantum systems in the laboratory.

Entanglement is a key property to measure in engineered quantum many-body systems; for example, in order for quantum simulators and computers to provide an advantage over their classical analogues, they must generate large amounts of entanglement between their parts [2]. However, the development of new tools to probe entanglement in the laboratory is proving to be an outstanding challenge. Previous protocols to probe the entanglement structure of these systems, such as quantum state tomography or efficient tomographic methods (e.g. [3]), either scale unfavourably with the number of qubits, or place limitations on the quantum states which can be probed. An alternative way to probe the entanglement structure of these systems is through measuring the entropy of their partitions; in particular the second-order Rényi entropy, $S^{(2)}$. If the entropy of a part A of the system is greater than the entropy of the total system ρ, i.e. if $S^{(2)}(\rho_A) > S^{(2)}(\rho)$, then bipartite entanglement exists between A and the rest of the system [4]. Consequently, measurements of the entropy of the system provide information about its entanglement structure.

Previous work utilised collective measurements on identical copies of a quantum system, ρ, to obtain $S^{(2)}$, e.g. [5]. The protocol implemented here uses statistical correlations between randomised measurements to measure $S^{(2)}$ [1]. It not only scales more favourably than quantum state tomography, but imposes no a-priori assumption on the structure of the quantum state, and requires only a single copy of ρ. It can be implemented on any quantum platform with single-particle readout and control, applicable to systems with tens of qubits [6].

The experiments were performed on a trapped-ion quantum simulator, using $^{40}Ca^+$ ions confined in a linear Paul trap. Spin-spin Ising-type interactions evolve the ions under a long-range XY model, H_{XY} [6]. First, a ten-ion product state was time-evolved under H_{XY} for $\tau = 0, \ldots, 5$ ms and subsequent randomised measurements were performed to characterise the entanglement of the state. The results (see Fig. 1) show a rapid growth of entanglement in the system, with numerical simulations for the experimental parameters in good agreement with the data. Similar experiments were subsequently performed for 10-ion partitions of a 20-ion chain, with rapid increases in the entropy during a time evolution of 10 ms observed. In a final experiment, signifying a first application of the protocol to study dynamical properties of quantum many-body systems in connection with the concept of quantum thermalisation [7], a ten-ion product state was again time-evolved under H_{XY}, however in the presence of additional local, random disorder. The results demonstrated the strong diminishing effect of this disorder on the entropy growth rate at early times and the emergence of localisation.

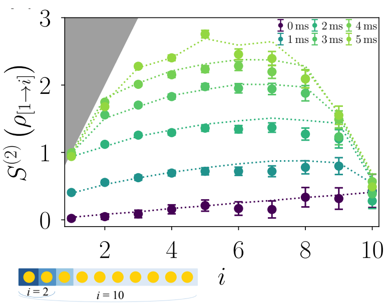

Figure 1: Measured growth of $S^{(2)}$ for a ten-ion system time-evolved under H_{XY}. Dotted curves are entropies derived from a numerical simulation [6].

This work demonstrates a new tool for measuring second-order Rényi entropies, providing a powerful method for both characterising engineered quantum systems and using them to tackle open questions in physics. The experiments, performed on ten-ion partitions of up to 20-ion strings, prove the overall coherent character of the system dynamics, revealing the growth of entanglement between its parts.

[1] A. Elben, et. al., *arXiv:1812.02624* (2018)

[2] G. Vidal, Phys. Rev. Lett. **91**, 147902 (2003)

[3] B. P. Lanyon, et. al., Nat. Phys. **13**, 1158 (2017).

[4] R. Horodecki, et. al., Rev. Mod. Phys. **81** 865-942 (2009)

[5] A. M. Kaufman, et. al., Science **353**, 794 (2016).

[6] T. Brydges, A. Elben, et. al., *arXiv:1806.05747* (2018).

[7] P. Calabrese, J. Cardy, Journal of Statistical Mechanics: Theory and Experiment **2005** P04010 (2005)