Many-particle interference to test Born's rule

Marc-Oliver Pleinert[1,2], Joachim von Zanthier[1,2], and Eric Lutz[3]

[1] Institute of Optics, Information and Photonics,
Friedrich-Alexander-University Erlangen-Nuremberg (FAU), 91058 Erlangen, Germany

[2] Erlangen Graduate School in Advanced Optical Technologies (SAOT), FAU, 91052 Erlangen, Germany

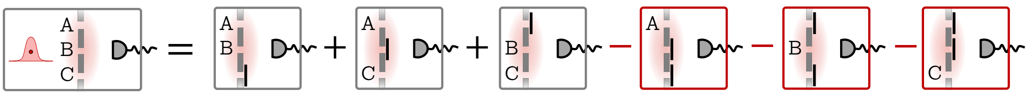
[3] Institute for Theoretical Physics I, University of Stuttgart, D-70550 Stuttgart, Germany

Abstract

Born's rule, one of the cornerstones of quantum mechanics, relates detection probabilities to the modulus square of the wave function. Single-particle interference is accordingly limited to pairs of quantum paths and higher-order interferences are prohibited. Deviations from Born's law have been quantified via the Sorkin parameter which is proportional to the third-order term. We here extend this formalism to many-particle interferences and find that they exhibit a much richer structure. In particular, we demonstrate that all interference terms of order $(2M+1)$ and greater vanish for M particle-interference. We further introduce a family of many-particle Sorkin parameters and show that they are exponentially more sensitive to deviations from Born's rule than their single-particle counterpart.

According to Born's rule the probability of a measurement outcome is given by the modulus square of the corresponding probability amplitude [1]. This fundamental postulate of quantum mechanics establishes a link between the (deterministic) mathematical formalism and experiment. It additionally introduces a random component into the theory. To date none of the attempts to replace the postulate by a derivation from underlying principles seems to be generally accepted. Experimental tests of Born's law are therefore crucial to assess its range of validity.

A direct consequence of Born's rule is that single-particle quantum interference originates from pairs of quantum paths [2]. Interferences of higher order than the second do not occur in quantum mechanics and the vanishing of the third-order term (which can be visualized in a triple slit setup, see Fig. 1) has been investigated in the context of a general quantum measure theory by Sorkin [2]. This led to the introduction of the so-called Sorkin parameter κ, which vanishes if Born's rule holds. An actual measurement of κ for single-particle interference has been conducted in 2010 with photons in a triple-slit experiment [3], where the Sorkin parameter was bounded by 10^{-2}.



$$I_3 := P_{ABC} - (P_{AB} + P_{AC} + P_{BC}) + (P_A + P_B + P_C) = 0$$

Figure 1: The single-particle interference pattern of the triple slit can be decomposed into the sum of the respective interference patterns of all comprised double and single slits. The third-order interference I_3 (as defined above), which quantifies 'new' genuine interference, is thus zero.

Quantum mechanics, however, is not limited to single-particle interference. It also allows for many-particle interference in the case of indistinguishable particles. In this study, we investigate general setups with greater number of slits, where many-particle interference, and hence higher-order correlations among the particles, can be observed. In this setting, we generalize Sorkin's idea to incorporate many-particle interference displaying a richer structure and far more nonzero higher-order interference terms. We derive higher-order sum rules that can be used to test Born's rule and introduce a family of many-particle Sorkin parameters, which have the benefit of displaying an exponentially increased sensitivity to deviations from Born's rule with respect to their single-particle counterpart. We thus expect them to stimulate new theoretical and experimental studies and more precise tests of quantum theory.

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[4] M.-O. Pleinert et al., *Many-particle interference to test Born's rule*, arXiv:1810.08221 (2018).