Measuring quantum coherence through statistical properties

Valeria Cimini[1], Ilaria Gianani[1], Marco Sbroscia[1], Jan Sperling[2], and Marco Barbieri[1]

[1] Dipartimento di Scienze, Universitá degli Studi Roma Tre, Via della Vasca Navale 84, 00146 Rome, Italy

[2] Integrated Quantum Optics Group, Applied Physics, University of Paderborn, 33098 Paderborn, Germany

Abstract

In quantum resource theory, the quantification of coherence of a system plays a fundamental role. An easy way to study this feature is to look at failures of classical restrictions to measurement outcomes. Here we experimentally measure the coherence of a single qubit state following the idea proposed in arXiv:1805.12404, (2018).

It is well know that quantum features such as entanglement, quantum discord and non-locality can be used as resources in many quantum information protocols. It is now clear that even the smallest deviation from classical properties can be useful and exploited in these tasks, offering an advantage compared to the classical counterpart. A convenient way to identify the quantum characteristics present in a state is to look at the difference of statistical behaviour of the system, compared to a classical scenario. This has been exploited in many experiments, e.g. by observing the photon-number statistics or the statistics of correlated quantum states. The presence of coherence is sufficient to cause significant differences in its statistical properties. This can be quantified by the violation of the law of total variances that is not fulfilled in the presence of quantum coherence. We demonstrate experimentally the violation of such law for an arbitrary mixed single qubit state ρ, by measuring the variance of a measurement $\hat{y}$ both on the original state (V_ρ), and in the instance that a measurement of the Pauli operator $\hat{z}$ is performed on the state, hence removing its coherence, obtaining in this case the variance V_σ.

$$\rho = \begin{bmatrix} 1-p & \sqrt{p(1-p)}\gamma \\ \sqrt{p(1-p)}\gamma^* & p \end{bmatrix}; \quad \hat{y} = \cos\theta \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} + \sin\theta \begin{bmatrix} 0 & e^{i\phi} \\ e^{-i\phi} & 0 \end{bmatrix}; \quad \hat{z} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

Experimentally, our state is encoded in the polarization degree of freedom of single photons obtained by parametric down conversion. The value of p can be changed moving a half wave plate (HWP) in the preparation stage of an angle α so that $p = sin^2 2\alpha$ while θ is four times the angle of the HWP in the measurement stage. In order to realize the two measurement schemes, we use a C-sign gate that allows to implement the measurement with or without the presence of coherence. One photon is used as the signal while the other acts as meter. The signal state will be influenced depending on which basis the meter is prepared and measured. The $\hat{y}$ measurement is easily implemented with a HWP and a polarizing beam splitter (PBS) in the measurement stage, when the gate is turned off and the meter state is measured in the $\{|0\rangle, |1\rangle\}$ basis. For the second scenario, we turn on the gate and then we repeat the $\hat{y}$ measurement. While the law of total variance would predict $V_\rho - V_\sigma = 0$, the presence of quantum coherence yields:

$$\Delta V = V_\sigma - V_\rho = 4\sqrt{(1-p)p}\gamma \sin\theta \left((2p-1)\cos\theta + \sqrt{(1-p)p}\gamma \sin\theta\right).$$

The experimental results, toghether with the theoretical trend of ΔV, are presented in Fig. 1.

In summary, we experimentally demonstrate the violation of the total variances law for a single qubit state in presence of coherence, showing that the amount of violation is dependent on the amount of coherence present in the state. This approch provides an easy way to quantifing the coherence of an arbitrary mixed state.

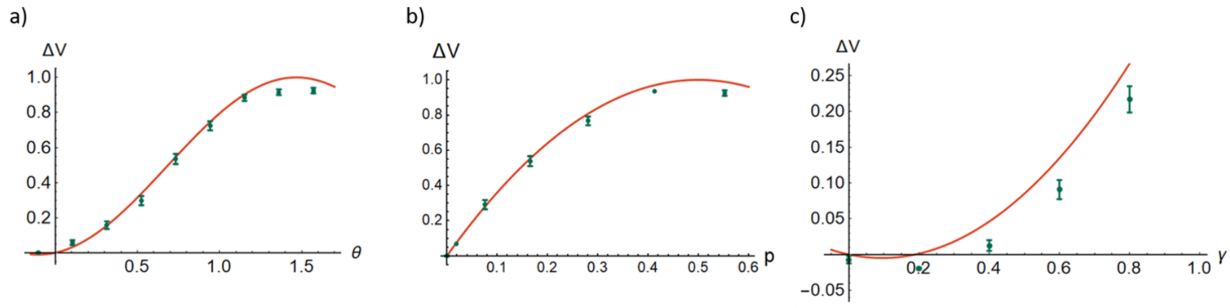

Figure 1: Results for a particular choice of data) ΔV in function of the measurement angle for $p = \frac{1}{2}$ and $\gamma = 1$ b) ΔV in function of the state parameter p for $\theta = \frac{\pi}{2}$ and $\gamma = 1$ c) ΔV in function of the state parameter γ for $\theta = \frac{\pi}{2}$ and $p \simeq \sin^2 \frac{\pi}{8}$.