

Quantum-limited time-frequency multiparameter estimation through mode-selective photon measurement

Vahid Ansari¹, John M. Donohue¹, Jano G. López¹,
Jaroslav Řeháček², Zdeněk Hradil², Bohumil Stoklasa², Martin Paúr²,
Benjamin Brecht¹, Luis L. Sánchez-Soto^{3,4}, and Christine Silberhorn¹

¹ Integrated Quantum Optics, Paderborn University, Warburger Strasse 100, 33098 Paderborn, Germany

² Department of Optics, Palacký University, 17. listopadu 12, 771 46 Olomouc, Czech Republic

³ Departamento de Óptica, Facultad de Física, Universidad Complutense, 28040 Madrid, Spain

⁴ Max-Planck-Institute für die Physik des Lichts, Staudtstrasse 2, 91058 Erlangen, Germany

Abstract

Projective measurements onto complex optical modes enables quantum-limited precision in parameter estimation, free of uncertainties arising from overlapping intensity profiles. Here we extend these methods to the time-frequency space and demonstrate multiparameter estimation of the centroid, the separation, and the relative intensities of two incoherent light sources with quantum-limited precision.

When attempting to find the distance separations between two incoherent light emitters, it is conventional to take a full image in order to extract the parameter of interest. The smallest precisely measurable separation, however, is limited by the point-spread function on the image plane, with a vanishingly small amount of information available for smaller and smaller separations especially when photon shot noise is the dominant noise source, e.g. in astronomical observations. This limit can be formalised by the Fisher information, which quantifies the amount of information accessible per photon detection and is directly associated with Cramér-Rao lower bound (CRLB). For direct intensity imaging, the Fisher information drops to zero for object separations smaller than the spread of the optical field, sometimes referred to as Rayleigh's curse, which limits the usefulness of photon counting for metrology. Surprisingly, when one calculates the quantum Fisher information [1] (i.e., optimised over all possible measurements), the associated quantum CRLB maintains a fairly constant value for any separation of the sources. This shows the potential for parameter estimation beyond Rayleigh's curse.

In this work, we extend these techniques and show that mode-selective measurement can be utilised to estimate separations in time and frequency well below the spread of the source light [2]. We find that mode-selective measurements are capable of estimating the separation accurately in a regime where intensity-only measurements would be ineffective. We implement spectrotemporal mode-selective measurements through the quantum pulse gate (QPG), a sum-frequency generation process in a group-velocity engineered waveguide. By shaping the strong QPG pump pulse and measuring the upconverted photon, the QPG implements a projective measurement onto the spectrotemporal mode defined by the QPG pump [3]. For the Gaussian pulses we consider here, the optimal measurement is the Hermite-Gaussian basis. Measuring only the first two Hermite-Gauss modes allows us to estimate the sub-bandwidth spectral separation between two incoherently mixed pulses, well below the CRLB for conventional intensity detection. Furthermore, we go beyond single-parameter estimation and show how our technique can be used for simultaneous multiparameter estimation of the centroid, the separation, and the relative intensities of two incoherent sources [4], in both frequency and time. This is possible by performing optimal detection, in this case, extending our projective measurements to higher-order Hermite-Gauss modes and their superpositions. By outperforming standard intensity-only detection, our quantum multiparameter estimation scheme can bring benefit to many practical scenarios.

- [1] M. Tsang, E. Nair, and X. M. Lu. Quantum Theory of Superresolution for Two Incoherent Optical Point Sources. *Phys. Rev. X*, 6(3):031033, 2016.
- [2] J. M. Donohue, V. Ansari, J. Řeháček, Z. Hradil, B. Stoklasa, M. Paúr, L. L. Sánchez-Soto, and C. Silberhorn. Quantum-Limited Time-Frequency Estimation through Mode-Selective Photon Measurement. *Phys. Rev. Lett.*, 121(9):090501, 2018.
- [3] V. Ansari, G. Harder, M. Allgaier, B. Brecht, and C. Silberhorn. Temporal-mode measurement tomography of a quantum pulse gate. *Phys. Rev. A*, 96(6):063817, 2017.
- [4] J. Řeháček, Z. Hradil, B. Stoklasa, M. Paúr, J. Grover, A. Krzic, and L. L. Sánchez-Soto. Multiparameter quantum metrology of incoherent point sources: Towards realistic superresolution. *Phys. Rev. A*, 96(6):062107, 2017.