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# Quantum limits to the superresolution of multiple point sources

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**Abstract**

The resolution of two incoherent point sources is—from a quantum estimation point of view—a well studied physical problem. For realistic purposes such as microscopy more complex descriptions involving multiple sources will be necessitated. We find that there is a limit to the parameters which can be estimated when resolving multiple point sources in a 1D configuration through the multi-parameter quantum Cramér-Rao bound.

Rayleigh's criterion sets a limit to the ability of an imaging system to resolve two point sources. However, this limit is not fundamental, but it arises from the fact that our imaging techniques depend on photon counting. Recent works have shown that this criterion can be beaten with the use of quantum measurements [1]. The fundamental limit can be evaluated through calculation of the quantum Cramér-Rao bound. For the separation of two sources this bound is found to be finite for small separations.

However, many applications of interest involve a large number of different sources, in which case calculations of the precision an imaging system can achieve are far more demanding. For this reason, analytical results have been calculated only for the order of magnitude behaviour of the QFI, which show that not all the parameters can be estimated [2, 3].

We consider the problem of estimating the $N$ positions of $N$ incoherent and weak sources aligned in a line. Firstly, we calculate analytically the precision for an approximate state in limit that the separation between the sources vanishes. We find that the rank of the QFI matrix is two, which means that only two parameters can be estimated. Secondly, we make a numerical calculation and obtain the QFI for the complete state. We find that, even in the case where the whole state is considered, the rank of the QFI matrix is still two in the sub-diffraction limit. This behaviour is explained by the fact that the rank of the quantum state also decreases as the separation of the sources becomes smaller. This behaviour is illustrated in Figure 1 which shows the eigenvalues of the state and the QFI, when only three sources are considered, as the separation of the sources increases.

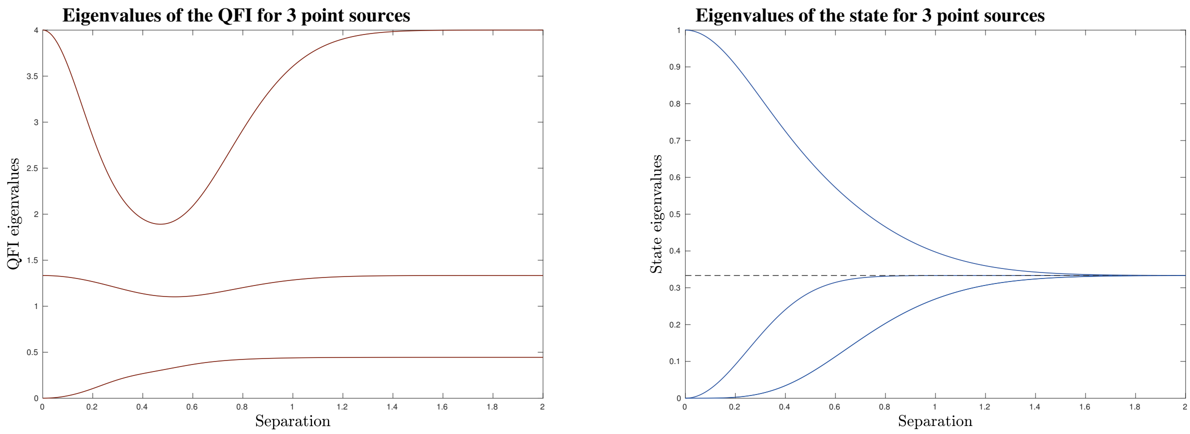


Figure 1: The eigenvalues of the density matrix (a) and the QFI matrix (b) in the case of 3 sources. (Normalised separation in the x axis).

[1] M. Tsang, R. Nair and X.M. Lu, *Quantum Theory of Superresolution for Two Incoherent Optical Point Sources*, Phys. Rev. X **6**, 031033 (2016).

[2] M. Tsang, *Quantum limit to subdiffraction incoherent optical imaging*, Phys. Rev. A **99**, 012305 (2019).

[3] S.Zhou and L. Jiang, *Modern description of Rayleigh's criterion*, Phys. Rev. A **99** 013808 (2019).