Experimental demonstration of Hardy's paradox using three-qubit states

Artur Barasiński[1,2,3], Antonín Černoch[2], Kateřina Jiráková[1], Karel Lemr[1], and Jan Soubusta[1,2]

[1] RCPTM, Joint Laboratory of Optics of Palacký University and Institute of Physics of Academy of Sciences of the Czech Republic, 17. listopadu 12, 771 46 Olomouc, Czech Republic

[2] Institute of Physics of Academy of Sciences of the Czech Republic, Joint Laboratory of Optics of PU and IP AS CR, 17. listopadu 50A, 772 07 Olomouc, Czech Republic

[3] Institute for Theoretical Physics, University of Wrocław 50-204 Wrocław, Poland

Abstract

The aim of our research is to show validity of probability conditions characterising Hardy's paradox. We have prepared a particular type of three-qubit entangled state, for instance general Greenberger-Horne-Zeilinger ($gGHZ$) state or W state. Our goal is to prove genuine three-partite nonlocality of these states. Practically, we realised qubits as polarisation states of two photons and spatial mode of one photon.

It has been suggested by Bell that there are quantum systems exhibiting measurement correlations that contradict local realistic theories [1]. His approach was based on statistical evaluation of measurement results and gave rise to famous inequalities. Analysis of the outcomes revealed that they are incompatible with every hidden variable theory. In 1992 Lucien Hardy devised an experiment which also demonstrated non-locality of quantum mechanics. In the original suggestion, there was used only two qubit system [2], however we broadened the concept and prepared specific three-way entangled states, like $gGHZ$ states (defined as $\cos\theta|000\rangle+\sin\theta|111\rangle$, $0 \leq \theta \leq \pi/4$) or W states (defined as $1/\sqrt{3}(|001\rangle+|010\rangle+|100\rangle)$), to verify satisfaction of Hardy-type conditions for them. Nonlicality of $gGHZ$ states has been recently addressed experimentally [3] showing violation of 3 particular Bell inequalities.

Simplified outline of Hardy's paradox is as follows [4] (the notation is also taken from there): there is a three-partite system denoted $n \in \{1,2,3\}$. Further, assume that each of local observers, e.g. the k-th one, performs a measurement of two observables $\{a_k, b_k\}$. Measurement outcomes may reach only values $\{0,1\}$. The paradox is given by a set of joint probabilistic conditions that cannot be satisfied all at once:

$$P(0_n|a_n) > 0, \qquad P(0_n|b_k a_{\bar{k}}) = 0,\ \forall k \in n, \qquad P(1_{k'} 1_k 0_{\bar{k}\bar{k}'}|b_{k'} b_k a_{\bar{k}\bar{k}'}) = 0,\ \forall k \in n \backslash \{k'\}, \tag{1}$$

where $\bar{k} = n\backslash\{k\}$ and $\bar{k}\bar{k}' = n\backslash\{k,k'\}$. For quantum systems, in our case genuine entangled state ϱ, we have to specify basis kets $\{|a_i\rangle, |b_i\rangle\}$ for every photon i. We denote $|\bar{a}_i\rangle$ state orthogonal to $|a_i\rangle$. Let us further choose two noncommuting observables A and B (e.g. polarisation of photon and its spatial mode) with $\{|a_i\rangle, |\bar{a}_i\rangle\}$ and $\{|b_i\rangle, |\bar{b}_i\rangle\}$ being their eigenstates, respectively. The following relations has to be valid

$$\langle a_n|\varrho|a_n\rangle > 0, \qquad \langle b_k a_{\bar{k}}|\varrho|b_k a_{\bar{k}}\rangle = 0,\ \forall k \in n, \qquad \langle \bar{b_{k'}} \bar{b_k} a_{\bar{k}\bar{k}'}|\varrho|\bar{b_{k'}} \bar{b_k} a_{\bar{k}\bar{k}'}\rangle = 0,\ \forall k \in \bar{k}'\,. \tag{2}$$

We measured 6 linear independent projections $\{|a_1a_2a_3\rangle, |b_1a_2a_3\rangle, |a_1b_2a_3\rangle, |a_1a_2b_3\rangle, |b_1b_2a_3\rangle, |b_1a_2b_3\rangle\}$ spanning the Hilbert space.

Experimentally, we generated photons using a process of spontaneous parametric down-conversion in a cascade of two BBO crystals. Such process provides photon pairs correlated in polarisation. The third qubit originates from splitting one photon's path by beam displacer into two, imposing on it a spatial mode denoted $\{0,1\}$. More detailed description of the experimental set-up for creation $gGHZ$ states is provided in [3].

[1] J. S. Bell, Physics **1**, 195 (1964).

[2] R. Rahaman, M. Wieśniak, and M. Żukowski, *True multipartite entanglement Hardy test*, Phys. Rev. A **90**, 062338 (2014).

[3] A. Barasiński, A. Černoch, K. Jiráková, K. Lemr, and J. Soubusta, Experimental verification of time-order-dependent correlations in three-qubit Greenberger-Horne-Zeilinger-class states, (in process of publication).

[4] Q. Chen, S. Yu, Ch. Zhang, C. H. Lai, and C. H. Oh, *Test of Genuine Multipartite Nonlocality without Inequalities*, PRL **112**, 140404 (2014).