No-cloning bound and secure teleportation beyond Gaussian states

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Abstract

No-cloning theorem is a profound principle making quantum communication secure unlike classical protocols. While establishing the no-cloning bound (NCB) offers a crucial benchmark to assess ultimate security in communication, the NCB for non-Gaussian states was largely unexplored. We establish the NCB for a broad class of non-Gaussian states in an operationally motivated form and investigate its relation to quantum non-Gaussianity (QNG), a notion to rule out mixture of Gaussian states. We show that NCB decreases with QNG, but remarkably, the lower NCB does not mean an easier achievement of NCB in quantum communications. We study the minimum resource to beat NCB in quantum teleportation showing that more resource is required for states with higher QNG.

The no-cloning theorem states that unknown quantum states cannot be perfectly copied, implying that an eavesdropper cannot obtain information on quantum states without disturbing them. In quantum communication, Bob the receiver can be assured of security if the output fidelity is above NCB [1]. The NCB can be established by evaluating the highest fidelity achieved by the symmetric $1 \to 2$ cloning, which was studied for different classes of states in finite dimensions. In continuous-variable (CV) systems, the NCB for coherent states was widely studied [2], but the NCB for non-Gaussian states has not been addressed yet. It is of crucial importance to broadly explore CV quantum states beyond Gaussian regime aiming at an enriched capacity of quantum information processing. We here establish the NCB for quantum non-Gaussian states and study the resource requirement to beat the NCB for secure communication.

To our aim, we define a set $\mathcal{S}_{|\psi\rangle}$ of unknown input states as composed of states obtained by displacing a given state $|\psi\rangle$, where the displacement is completely unknown over the whole phase space. For instance, $|\psi\rangle = |0\rangle$ (vacuum state) gives the set $\mathcal{S}_{|0\rangle}$ composed of coherent states. We first consider the set $\mathcal{S}_{|\psi\rangle=|n\rangle}$ of displaced Fock states (DFSs) with $n = 1, 2, 3$. We show that the symmetric cloner yielding NCB is fully characterized by an ancillary two-mode state ρ_T, the resource state of cloner. The NCB for each n, $F_n^{\rm nc}$, can be evaluated by obtaining the largest possible value of $\frac{1}{2}\left\langle \sum_{j=1,2} \left[L_n(\hat{O}_j)\right]^2 \exp(-\hat{O}_j)\right\rangle_{\rho_T}$ over two-mode states ρ_T , where $\hat{O}_1 = \frac{\hat{p}_1^2+\hat{x}_2^2}{2}$ and $\hat{O}_2 = \frac{\hat{x}_1^2+\hat{p}_2^2}{2}$ with quadrature operators $\hat{x}_j$ and $\hat{p}_j$ of jth mode. For the coherent-state input, we obtain $F_0^{\rm nc} \simeq 0.6826$, as reported in [2]. By a full numerical calculation compared with semi-analytical approach, we obtain $F_1^{\rm nc} \simeq 0.5449$, $F_2^{\rm nc} \simeq 0.5145$, and $F_3^{\rm nc} \simeq 0.5033$ decreasing with n.

We further study the resource required to achieve the NCB in CV teleportation exploiting a two-mode squeezed state with squeezing parameter r as a resource state. We obtain the critical squeezing $r_n^{\rm c}$ to beat the NCB as $r_0^{\rm c} \simeq 0.3829$, $r_1^{\rm c} \simeq 0.6949$, $r_2^{\rm c} \simeq 0.8960$, and $r_3^{\rm c} \simeq 1.044$. The critical squeezing thus increases, which means it becomes harder to achieve secure teleportation, with n.

We have further conducted investigation more broadly for other class of non-Gaussian states including the superposition of Fock states and cat states, and investigated the behavior against the relative entropy of non-Gaussianity δ, a well-defined measure of QNG for pure states. We overall find that NCB decreases with δ but the required squeezing for secure teleportation increases with δ. To clarify the role of QNG beyond pure non-Gaussian states [3], we have also investigated mixed non-Gaussian states, e.g. mixture of Fock states, which broadly confirms the correlation between NCB and QNG.

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