# Probability representation of photon states and tomography

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**Abstract**

The description of quantum states by fair probabilities (alternatives of the wave function) is reviewed. The probability representation of systems of parametric interacting quantum oscillators describing electromagnetic fields, where the corresponding photon quantum states are identified with tomographic-probability distributions (optical and symplectic), is presented. The case of f-oscillators is considered. New entropic-information inequalities describing photon states are obtained.

The transition probabilities between parametrically excited electromagnetic-field mode states are expressed as nonlocal overlaps of tomographic-probability distributions describing the states in terms of Frank–Condon factors. The relation with the Wigner-function formalism is elucidated. Pure quantum states, e.g., of photons in standard formulation of quantum theory are identified with wave functions or state vectors in a Hilbert space. Since such a state description differs radically from classical description of states, the attempts to find other representations like the phase-space representation provided the identification of states with Wigner functions, Husimi functions, and Glauber–Sudarshan functions, which are the quasidistributions on the phase space.

It turns out that the development of quantum-tomography technique of measuring quantum states from the viewpoint of theoretical studies gave the possibility to introduce [1] the description of states by fair probability distributions like symplectic tomographic probability distributions or symplectic tomograms. The tomograms $w(X, \mu, \nu)$ are the probability densities of the homodyne quadrature $X$ measured in an ensemble of the reference frames in the phase space $q$ and $p$, where the bases axes are rescaled ($q \to sq,\ p \to s^{-1}p$) and then rotated using local oscillator phase $\theta$, i.e., $\mu = s\cos\theta$, $\nu = s\sin\theta$, $X = \mu q + \nu p$. The measurable optical tomogram $w(X, \theta)$ is symplectic tomogram for $s = 1$.

The Wigner function and the density operator of photon states are determined by the tomographic-probability distributions; an analogous description was elaborated for qubit (spin-1/2) states [2–4], where the state is identified with three probability distributions $1 \geq p_1, p_2, p_3 \geq 0$, satisfying the inequality $\sum_{j=1}^{3}(p_j - 1/2)^2 \leq 1/4$. Three numbers $p_j$ are the probabilities to have the spin projection $m = +1/2$ on the axes $x, y, z$.

The aim of this talk is to review the probability description of photon states for the systems with nonstationary Hamiltonians, which are generic quadratic forms of the creation and annihilation operators. The fidelity, being the probability given by the Born rule $p_{12} = \mathrm{Tr}\,(\hat\rho_1\,\hat\rho_2)$, is shown to be expressed in terms of tomograms as $p_{12} = \dfrac{1}{2\pi}\displaystyle\int w_1(X_1, \mu, \nu)\, w_2(X_2, -\mu, -\nu) e^{i(X_1+X_2)} dX_1\, dX_2\, d\mu\, d\nu$, which for the pure stare is a new expression for Frank–Condon factors used in molecular spectroscopy. Also we describe the properties of the state of nonlinear f-oscillator introduced in [5] associated with photon fields by the probability distributions and obtain new entropic inequalities; one of them that can be checked experimentally reads

$$-\int w(X, \theta, t) \ln w(X, \theta, t)\, dX - \int w(X, \theta + \pi/2, t) \ln w(X, \theta + \pi/2, t)\, dX \geq \ln(\pi e).$$

We summarize our results by the following statement: Quantum states can be identified with probability distributions and quantum observables, with analogs of classical-like random variables.

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