CEWQO 2019
Invited

Measurement induced complex chaos in quantum protocols

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Abstract

Quantum informational protocols involve coherent evolution, measurement, and postselection of qubits. A typical example is entanglement distillation. The resulting conditional dynamics is nonlinear, in contrast to the usual evolution of both closed and open quantum systems. Already the simplest types of such protocols may result in rich, complex chaotic dynamics when applied iteratively.

State selective protocols, like entanglement purification, lead to an essentially non-linear quantum evolution, unusual in naturally occurring quantum processes. Sensitivity to initial states in quantum systems, stemming from such non-linear dynamics, is a promising perspective for applications. An important property of these iterated dynamical protocols is that initially pure quantum states remain pure throughout the evolution. For single-qubit systems, there is a one to one correspondence of the pure-state quantum dynamics to the iterated dynamics of quadratic rational maps with one complex variable [1]. For two-qubit systems, LOCC operations may lead to dynamics, where the evolution of entanglement is chaotic in the sense of crucially depending on infinitely fine details of the initial state [2]. We present an example illustrating that chaotic behaviour is a rather generic feature in state selective protocols: exponential sensitivity exists for all initial states in an experimentally realisable optical scheme.

We prove that any complex rational polynomial map, including the example of the Mandelbrot set, can be directly realised with measurement induced, iterated dynamics of qubits. In state selective protocols, one needs an ensemble of initial states, the size of which decreases with each iteration. We prove that exponential sensitivity to initial states in any quantum system has to be related to downsizing the initial ensemble also exponentially. Our results show that magnifying initial differences of quantum states (a Schrödinger microscope) is possible; however, there is a strict bound on the number of copies needed [3].

We propose a cavity quantum electrodynamical scenario for implementing a Schrödinger microscope capable of amplifying differences between nonorthogonal atomic quantum states. The scheme involves an ensemble of identically prepared two-level atoms interacting pairwise with a single mode of the radiation field as described by the Tavis-Cummings model. By repeated measurements of the cavity field and of one atom within each pair, a measurement-induced nonlinear quantum transformation of the relevant atomic states can be realized. The intricate dynamical properties of this nonlinear quantum transformation, which exhibits measurement-induced chaos, allow for an approximate orthogonalization of atomic states by purification after a few iterations of the protocol [4].

As an example for applications, we consider the task of deciding whether an unknown qubit state falls in a prescribed neighborhood of a reference state. We assume that several copies of the unknown state are given and apply a unitary operation pairwise on them combined with a postselection scheme conditioned on the measurement result obtained on one of the qubits of the pair. The resulting transformation is a deterministic, nonlinear, chaotic map in the Hilbert space. We derive a class of these transformations capable of orthogonalizing nonorthogonal qubit states after a few iterations. These nonlinear maps orthogonalize states which correspond to the two different convergence regions of the nonlinear map. Based on the analysis of the border (the so-called Julia set) between the two regions of convergence, we show that it is always possible to find a map capable of deciding whether an unknown state is within a neighborhood of fixed radius around a desired quantum state [5].

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