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Systemic Instability of the Interbank Credit Market - A Contribution to a Resilient Financial System

Thomas Gries and Alexandra Mitschke

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by T. Gries[a)] and A. Mitschke[b)]
[a)]Department of Economics, Paderborn University, Warburger Straße 100, D-33098 Paderborn, Germany, thomas.gries@notes.upb.de
[b)]Department of Economics, Paderborn University, Warburger Straße 100, D-33098 Paderborn, Germany, alexandra.mitschke@upb.de

Abstract

Interbank markets have infrequently exhibited sudden interest rate spikes. These disruptions in a key financial market can undermine financial stability. Imperfections, such as moral hazard, financial frictions, and negative externalities have been suggested as potential explanations for interbank market disruptions. However, we still know very little how the interbank market works (Allen et al., 2018). We complement traditional stock analyses by modelling dynamics in the interbank credit market, focusing on the flow process between lending and borrowing institutions. In our theoretical model credit supply is restricted by the availability of stochastic liquidity inflows to lending institutions. Following a shock in the form of an increase in volatility of these liquidity inflows, a sequential flow adjustment process sets in. In "normal times" the flow dynamics remain smooth within a stable adjustment regime. However, a higher volatility can change the lending process, resulting in a bifurcation of the equilibrium. Defining interbank "market resilience" as the probability of remaining in the stable regime, we examine the impact of monetary policy tightening on interbank market stability. A change in the volatility of reserve flows, which is more likely when central banks tighten monetary policy, may threaten the resilience of interbank markets and increase the probability of the market to fall into a regime of unstable dynamics. Thus, we stress that tightening monetary policy could incidentally reduce interbank market resilience up to a potential market collapse, even in the absence of contagion phenomena.

JEL classification code: E44, E52, G11, G21
JEL keywords: Financial Markets, Interbank Lending, Monetary Policy

"We still know very little how the interbank market works, despite the existence of many recent studies on interbank market risk and interconnection." (Allen et al., 2018, p. 2)

1 Introduction

The smooth reallocation of liquidity within the interbank market (IBM) from banks with a surplus to banks with a deficit contributes substantially to financial system resilience (Gabrieli and Georg, 2014). In addition, the interest rate in the interbank credit market serves as operational target of monetary policy implementation, consequently affecting the whole economy via various transmission channels. Under normal circumstances, the smooth functioning of the IBM seemed to be reliable (e.g., Afonso et al., 2011). However, in an adverse scenario, the IBM can become dysfunctional, in the extreme case leading to a complete dry out of IBM liquidity and inducing central banks to intervene in order to re-establish normal conditions.

Motivated by the observation of infrequent and sudden interest rate spikes in the IBM without explanatory cause, we study the effect of the reserve flows on the resilience of the IBM. In the period between 2005 and 2008, the Federal Reserve Bank also increased policy rates, shortly before the Great Financial Crisis (GFC) materialized. Is a monetary policy tightening necessarily related to a smooth adjustment process within the IBM? - Or can monetary policy tightening trigger an IBM instability, which reduces the resilience of the financial system?

This paper examines the risk of illiquidity in the IBM as a general market instability, indicating a substantial vulnerability of the financial system.[1] We identify determinants of financial system resilience and offer first insights in the avoidance of the adverse scenario in the IBM. In light of the pre-pandemic tendency to monetary policy normalization, we additionally shed light on the effect of monetary policy tightening on the resilience of the IBM.

Our theoretical model adds to the general understanding of the operation of the IBM and associated financial vulnerabilities. Departing from portfolio equilibria in financial markets, we describe a dynamic adjustment process of banks' interbank lending for two different conditions. We show that under normal conditions, an unexpected, exogenous shock leads to a smooth adjustment process of the IBM to the new equilibrium. By contrast, under more volatile conditions, a bifurcation of the equilibrium changes the characteristics of the adjustment process, including a potential market instability. Hence, we show that under more volatile conditions, the probability of being in an unstable adjustment path increases. As we define market resilience as probability of remaining

[1] We note that a liquidity crunch is unlikely to occur in the current environment, given that the banking sector in the euro area is currently highly liquid and banks receive liquidity from the ECB at cheapest financing rates. However, we claim that a general understanding of the operation and stability of this market is important to safeguard financial stability.

in a stable regime of adjustment, we show that this probability decreases with higher volatility, and thus, market resilience declines. While we do not claim that volatility necessarily causes financial turmoil, which is a well-known phenomenon, we emphasize that higher volatility cannot only increase risk in the well-known sense, it may lead to a fundamental market instability up to a potential market freeze. Thus, we address another channel of potential IBM failure than normally discussed by contagion mechanisms.

Related Literature The IBM allows a bank to cover its individual need for liquidity and share risks (e.g., Bhattacharya and Gale, 1987). In the pre-crisis period, the IBM was generally regarded to operate perfectly (Bucher et al., 2017; Jakab and Kumhof, 2015), consequently leading to a rare literature on the operation of the IBM. For instance, Allen and Gale (2000) have shown that liquidity risk is not present, when the IBM is complete and aggregate shocks are absent.

The seizing up of the IBM in the financial and sovereign debt crises (see, e.g., Frutos et al. (2016) for a European focus on the crises), has led to a renewed interest of academic research in the functioning of the IBM. The resulting theoretical literature explains this illiquidity by moral hazard and financial frictions within the IBM, impeding an efficient risk-sharing process across the market. Examples for more recent theoretical models on IBM imperfections include asymmetric information about counterparty risk (e.g., Afonso et al., 2011; Freixas and Jorge, 2008; Heider et al., 2015), search costs for trading partners (e.g., Afonso and Lagos, 2015; Bech and Monnet, 2016; Vollmer and Wiese, 2016) or regulatory costs due to post-crisis newly implemented financial regulations (e.g., Bech and Keister, 2017; Bindseil, 2016; Jackson and Noss, 2015). These costs can hinder the smooth reallocation of liquidity across banks and as a result, can lead to liquidity issues, forcing the central bank to intervene.

Associated with the implementation of monetary policy, our paper is also related to theoretical models of banks' reserve management in the tradition of Poole (1968), who introduces uncertainty in a bank's reserve management. Using a stochastic model, Poole (1968) explains a single bank's profit-maximization under uncertainty and examines excess reserves, bank borrowing from the central bank and the central bank policy rate in an environment with scarce reserves (see also, e.g., Ho and Saunders, 1985). Following this first strand, academic research focused on micro-mechanisms of bilateral trading in the IBM (Afonso and Lagos, 2015; Bianchi and Bigio, 2017; Ennis and Weinberg, 2013). Finally, the introduction of unconventional monetary policy has induced a third strand to investigate the operation of the IBM under excess reserves (Armenter and Lester, 2017; Ennis, 2018; Martin et al., 2013, 2016; Williamson, 2019). We refer to the second strand, dealing with micro-mechanisms of trading in the IBM on a bilateral basis.

Besides excessive risk-taking of single institutions (e.g., Bhattacharya and Gale, 1987; Brunnermeier and Oehmke, 2013) as well as contagion and spillover effects (e.g., Allen and Gale, 2000; Freixas et al., 2000), the literature identifies amplification mechanisms as systemic risk with small shocks having large impacts. Examples are liquidity crises, market freezes and bank runs. In the first case, the liquidation of assets can lead to a market price decline, which induces further fire sales of assets and thereby a liquidity crisis (e.g., Shleifer and Vishny, 1992; Brunnermeier and Pedersen, 2009). Market freezes can occur when IBM imperfections like adverse selection (e.g., Flannery, 1996) or asymmetric information (e.g., Heider et al., 2015) hinder safe banks from borrowing in the IBM. Bank runs and panics build on the seminal work of Diamond and Dybvig (1983), showing that bank runs can be self-fulfilling and put the banks' liquidity at risk. In addition to these explanations for market freezes, our model shows that also an aggregate shock can hinder the smooth reallocation of reserves by impeding the planned portfolio adjustment process.

To the best of our knowledge, the closest paper to ours is Bech and Keister (2017) who examine the impact of the newly implemented Basel III Liquidity Coverage Ratio (LCR) on interbank interest rates using a model of monetary policy implementation. Bech and Keister (2017) find that when central banks return to pre-crisis policy operations while the LCR is implemented, they might face substantial difficulties in controlling IBM interest rates. We also refer to liquidity risk with regard to the availability of central bank reserves but deviate from the model of Bech and Keister (2017) by not taking regulatory measures, such as the LCR into account. We differ from the theoretical analyses on IBM imperfections by showing that an IBM may be driven into illiquidity following a sufficiently large exogenous shock by introducing a stochastic adjustment process of interbank lending as financial market imperfection. The effect of an aggregate liquidity shock on the unsecured IBM is also analyzed, e.g., by Sarmiento (2019), whose regression results show that aggregate liquidity shocks can be mitigated by central bank liquidity injections. In addition, we investigate the effect of monetary policy normalization following the period of excess reserves in the IBM.

Furthermore, to our knowledge these models traditionally assume banks to act as financial intermediaries between savers and borrowers, which exposes them to liquidity risk due to maturity transformation. For instance, Bianchi and Bigio (2017) model an individual bank's liquidity management, where idiosyncratic liquidity risk can be reduced by holding large precautionary reserves, although reducing intermediation profits. While intermediation issues, such as bank runs, could still materialize in in our model, we shift the focus of liquidity risk from the private-banking sector link to the link between the banking sector and the monetary authority. Thus, we show the existence of another vulnerability in the financial system, which exists even in the absence of intermediation issues as sources of systemic risk.

The following section will shortly introduce the baseline of our theoretical model. Afterwards, Section 3 introduces a dynamic flow mechanism in the IBM, which explains how dynamic adjustment processes can affect the resilience of the IBM. Section 4 adds model implications for policymaking, and finally, Section 5 concludes.

2 The Baseline Model

We consider a very simplified model, where the domestic banking sector consists of a large number of competing investment banks. To provide insights on a market instability apart from payment shocks, we exclude traditional depositary institutions from our analysis. Consequently, banks do not hold (minimum) reserves as a back-up for deposit withdrawals. Instead, we refer to excess reserve holdings as precautionary liquidity buffer. Furthermore, we simplify their operation to three competitive markets, namely the central bank money market, the IBM, and an investment in a representative security in the asset market. To motivate banks' portfolio choices, the following subsections briefly present each market's characteristics, which determine the stable and unique portfolio stock equilibrium. We specifically refer to the euro area under normal conditions and refer the interested reader to Appendix A.1 for a more detailed description.

Central Bank Money Market The aggregate liquidity need of the banking sector is managed by the central bank, which provides and absorbs reserves to and from the IBM. The ECB uses mainly three interest rates to steer money market rates in the IBM. Key policy rate is the rate of the main refinancing operation (MRO), which is charged on longer-term central bank credits (denoted by i_R). To receive short-term funding, banks can refer to the marginal lending facility, which provides an overnight credit at higher costs (i_{LF}). Contrary to this, the deposit facility absorbs liquidity from the banking sector, where the central bank pays an interest rate (i_{DF}) on overnight deposits. Usually, the interest rates on these facilities determine a symmetric corridor around the MRO-rate, with the deposit facility acting as a floor and the lending facility as a ceiling ($i_{LF} > i_R > i_{DF}$).

Interbank Money Market Banks can lend and borrow reserves in the IBM to cover their individual liquidity needs. In accordance with Hauck and Neyer (2014), we assume the IBM to be characterized by two types $i = d, s$ of banks, according to the banks' demand or supply of liquidity in the IBM. Banks of type s supply excess reserves in the IBM, whereas banks of type d demand liquidity in the IBM. Each single bank is randomly assigned to its type, while Brousseau and Manzanares (2009) claim that its main business activities are a key indicator for the bank's liquidity situation. Banks of type s provide credits (C_s) at the IBM credit rate (i_C). The associated credit demand of borrowing banks is denoted by C_d.Compared with the short-term central bank money market, banks can usually borrow at lower costs in the IBM ($i_{LF} > i_C$). Furthermore, in "normal times", the overnight interbank credit rate has generally remained close to the longer-term central bank rate, although averaging slightly above the MRO-rate ($i_{IB} \geqslant i_R$). This positive spread between the overnight interbank credit rate and the longer-term central bank rate is of highest interest, indicating a non-perfect adjustment of IBM rates to monetary policy rates.

Asset Market Besides the markets for reserves, both banks can invest in a representative asset of government bonds (B) with i_B denoting the associated rate of return. We further assume this bond to add to portfolio risk.

Portfolio Optimization Problem and Equilibrium Derivation The asset characteristics define the portfolio optimization problem of lending bank s (see Appendix A.2 for a simplified balance sheet of bank s). We assume bank s to hold a longer-term credit from the central bank before learning its liquidity surplus. Afterwards, it lends this excess liquidity in the IBM at a higher IBM rate i_c or, alternatively, invests in the bond market. The optimal asset holdings of bank s define a stationary stock portfolio equilibrium, in which bank s has determined its optimal credit supply in the IBM ($\tilde{C}_{s,t}$).

Whenever an exogenous shock occurs, the adjustment process of bank s's portfolio, embedded in the three markets, would set in. An increase in the monetary policy rate, which is interpreted as the effect of monetary tightening, leads to an increase in the interbank credit volume. In the following, we deviate from the standard analysis of a unique portfolio stock equilibrium and focus on the adjustment path of the liquidity supply provided by lending bank s in the interbank credit market.

3 Flow Mechanism in the Interbank Market

We now introduce a new, periodic flow mechanism, which may set in following an exogenous shock, guiding the economy from the already determined, initial stock equilibrium ($\tilde{C}_{s,t}$) to a new stationary stock equilibrium ($\tilde{C}_{s,t+\tau}$) as could be described in traditional portfolio choice models (see, e.g., Christensen and Krogstrup, 2018). The time span between the two equilibria, i.e., the adjustment period, is assumed not to be instantaneous but rather to take a few periods until the flow process terminates in the new stock equilibrium.

Following an increase in the main monetary policy rate (i_R), the arbitrage opportunities in the markets for liquidity change, with bank s now willing to lend a higher volume. Bank s has taken a cheap one-week loan from the central bank, which is charged at the MRO-rate (i_R). The bank uses this liquidity to lend it in the IBM at the higher rate of the EONIA (i_C) for several times, potentially lending six times overnight in the IBM until the MRO loan matures after one week.

Theoretically, an immediate adjustment of the interbank credit interest rate to the new policy rate could clear markets instantaneously. However, institutional properties of the IBM hinder a momentarily adjustment to the excess demand of interbank credits and slow down the adjustment process. For instance, the IBM is an OTC market, where credit contracts are negotiated on a bilateral basis (e.g., Vollmer and Wiese, 2014; Bianchi and Bigio, 2017). However, generally, bank s is now willing to expand its credit provision in the interbank credit market accordingly ($\dot{C}_s(t) > 0$), until it reaches its new equilibrium stock of interbank credit supply ($\tilde{C}_{s,t+\tau}$).

3.1 Adjustment Path to the New Equilibrium

Bank s has already provided interbank credits, resulting in the stock of interbank credits currently (at time t) recorded in its balance sheet ($\tilde{C}_{s,t}$). The net credit expansion path of bank s, as denoted by $\dot{C}_s(t)$, is assumed to be determined by three components.

$$\dot{C}_s\left(t\right) = -C_s\left(t\right) + \rho\left(C_s\left(t\right)\right)^{1-\alpha} + b\left(\tilde{C}_{d,t+\tau} - C_s\left(t\right)\right) \tag{1}$$

Knowing that the IBM is a very short-term market and consists of mainly overnight credits (see also Appendix A.1), we assume a daily return flow to reduce the current credit stock. Thus, at every point in time t (every day) a large number of these credits is repaid ($-C_s\left(t\right)$) by banks of type d. For simplicity, we assume that all credits are overnight credits and paid back every day. As we assume no defaults in normal interbank relations, the total number of credits issued on the previous day are repaid to 100%, resulting in an equivalent return flow.

However, bank s tries to expand its lending in the IBM for arbitrage reasons and starts in the easiest available way with a roll-over mechanism ($\rho\left(C_s\left(t\right)\right)^{1-\alpha}$). It revolves some part ($\rho \in [0,1]$) of the existing former borrower relationships on an ongoing basis, where it is assumed that the intensity of relationships depends on the central bank interest rate ($\rho(i_R)$). The parameter ρ refers to bank s's traditional relationships to borrowing banks that have developed over time (see, e.g., Cocco et al., 2009; De la Motte et al., 2010; Afonso et al., 2013, on relationship lending). The IBM is regarded to have a hierarchical structure, where liquidity is distributed via bank-pairwise relationships. While the reason for this structure is still open, Fricke and Lux (2012) suggest a comparative advantage of core banks in gathering and distributing information about their counterparties as a crucial factor. More recently, Bräuning and Fecht (2017) find that lending relationships reduce search frictions in the OTC market as well as uncertainty regarding counterparty credit quality. This relationship lending is the first source of credit expansion for bank s, since it knows its customers from previous credit relationships and can easily and quickly offer and expand credit. Due to the fact that partner institutions are already recorded in the trading books of bank s, it thereby minimizes its time effort with respect to search costs, transaction costs, etc. for the bilateral contracts. This information time is denoted by $(1-\alpha)$ with $\alpha < 1$. Consequently, the credit expansion process cannot take place instantaneously but is a time-consuming process. Starting with best-known relationships, the information advantage decreases with increasing credit provision, slowing the roll-over process down.

Moreover, bank s is willing to provide completely new credits within the IBM, provided credit demand in the new equilibrium ($\tilde{C}_{d,t+\tau}$) exceeds its current credit supply ($C_s\left(t\right)$) with credit demand in the new equilibrium depending on the central bank interest rate ($\tilde{C}_{d,t+\tau}(i_R)$). For this new credit provision $\left(b\left(\tilde{C}_{d,t+\tau} - C_s\left(t\right)\right)\right)$ a new customer acquisition is required in addition to the existing relationships recorded in the roll-over mechanism. While traditional relationships have already been established, a completely new credit extension is characterized by even higher adjustment obstacles, represented by the parameter b, where $b \in (0,1)$. A lower value of b indicates higher adjustment obstacles, such as search for trading partners, present in the IBM. It takes time to search for further trading partners (e.g., Afonso and Lagos, 2015; Vollmer and Wiese, 2016), to gather information about these new borrowers, including their counterparty risk (e.g., Afonso et al., 2011; Heider et al., 2015), as well as to establish the new trading partnership. Consequently, bank s cannot instantaneously provide more credit to the market.

These three components can be described together in the relationship above, which results in the total net credit expansion schedule as given in the following Equation (2) where $B = b\tilde{C}_{d,t+\tau}(i_R)$.

$$\begin{aligned}
\dot{C}_s(t) &= -C_s(t) + \rho\left(C_s(t)\right)^{1-\alpha} + b\left(\tilde{C}_{d,t+\tau}(i_R) - C_s(t)\right) \\
&= b\tilde{C}_{d,t+\tau}(i_R) + \rho\left(C_s(t)\right)^{1-\alpha} - (1+b)C_s(t) \\
&= B + \rho\left(C_s(t)\right)^{1-\alpha} - (1+b)C_s(t)
\end{aligned} \tag{2}$$

3.1.1 Illustrative Example

Start-Equilibrium-Period (t)

We start-off from the portfolio equilibrium of bank s in period t. Given the vector of equilibrium interest rates in the current period, bank s provides its optimal amount of credit supply, $\tilde{C}_{s,t}$ in the IBM. This stock is our point of departure; we therefore equate it with 100%

$$\tilde{C}_{s,t} = 100\% = 1$$

End-Equilibrium-Period ($t + \tau$)

In the next period, the central bank raises its policy rates, the MRO (i_R) and the marginal lending facility, by 5 basis points (bp) each. The MRO increases from 1% to 1.05% for a one-week credit. Following that, bank s desires to reach a new end-equilibrium, which it will reach after τ adjustment periods, i.e., in the period $t + \tau$. First, bank s takes at the next possible date a loan from the central bank, which it has to repay at the higher rate of 1.05% after one week. So far, the interbank overnight interest rate has remained at the t-period equilibrium level of $i_{C_t} = 1.1\%$. But with the new policy rate, bank s expects the interbank interest rate also to rise to the new equilibrium level of $i_{C_{t+\tau}} = 1.15\%$. In order to keep its portfolio choice relations and profits stable, bank s desires to gain profit from arbitrage opportunities. To do so, it would like to provide a higher stock of overnight interbank credits, receiving the higher rate of 1.15% every day when its IBM borrower repays its loan. This would lead to a weekly yield of $(1.15\% \cdot 6$ days$-1.05\%)\cdot$amount of credits provided. In addition, borrowers now demand a higher amount of overnight credits in the IBM, because the currently still applying equilibrium rate of $i_{C_t} = 1.1\%$ is still cheaper than borrowing overnight at the 5 bp higher marginal lending facility of currently 1.55% from the central bank. And it is even cheaper in relation to the spread before. Therefore, bank d demands a higher amount of interbank credits in the IBM.

In order to meet this excess demand for credits in the IBM, bank s would like to expand its credit provision by 10% in addition to the so-far existing stock. Thus, the new equilibrium stock of credit supply in the end-period is assumed to take an arbitrarily chosen number of 110%.

$$\tilde{C}_{s,t+\tau} = 1.1\tilde{C}_{s,t}$$

Consequently, bank s has to expand its credit supply by 10% as compared to the equilibrium in the start-period. How does bank s reach this new equilibrium? This is a sequential adjustment process over time, which is illustrated by the following Table 1.

Table 1: Exemplary Sequential Adjustment Process

period	inflows to bank s	outflows of bank s	stock $\tilde{C}_{s,\ period}$
$t+1$	$+1\tilde{C}_{s,t}$	$-1.02\tilde{C}_{s,t}$	$1.02\tilde{C}_{s,t}$
$t+2$		$-0.02\tilde{C}_{s,t}$	$1.04\tilde{C}_{s,t}$
$t+3$		$-0.02\tilde{C}_{s,t}$	$1.06\tilde{C}_{s,t}$
$t+4$		$-0.02\tilde{C}_{s,t}$	$1.08\tilde{C}_{s,t}$
$t+5$		$-0.02\tilde{C}_{s,t}$	$1.10\tilde{C}_{s,t}$

First, we assume bank d to repay all of the overnight credits already taken from bank s, which receives a liquidity inflow of $1\tilde{C}_{s,t}$ on the next day. However, bank s desires to expand its credit provision by 10%, reaching a new end-equilibrium stock of 110%.

This expansion process is started in the easiest way, by referring to the former periods' relationships. It allows bank s to roll over a fraction of presumably 100% of credits of the previous day, leading to liquidity outflows of $1\tilde{C}_{s,t}$. So far, it has successfully restored its previous-period equilibrium credit supply ($C_{s,t+1} = 100\%$). However, bank s tries to extend its credit provision further on by gaining new customers. But new customers incur new risks and tasks to be performed. This process takes time and therefore allows bank s to expand its credit by no more than 2% per day. Furthermore, bank s desires to have liquidity outflows of $1.1\tilde{C}_{s,t+1}$ and, thus, it needs additional liquidity inflows of $0.1\tilde{C}_{s,t}$, which it has to gain from bank-external sources. Consequently, bank s can only reach a stock of interbank credits of $C_{s,t+1} = 102\%\tilde{C}_{s,t}$ in period $t+1$, which is below its target-value of $110\%\tilde{C}_{s,t}$. It will take 4 further days until bank s can reach its targeted credit expansion of $C_{s,t+\tau} = 1.1\tilde{C}_{s,t}$ in period $t+5$.

3.1.2 End-Equilibrium in the IBM

This stable dynamic process described by Equation (2) leads to the new stationary portfolio equilibrium. The total net credit expansion schedule can be illustrated by Figure 1, where the intercept is given by $B = b\tilde{C}_{d,t+\tau}$. The derivation of the characteristics of the credit expansion process is shown in detail in Appendix A.3.

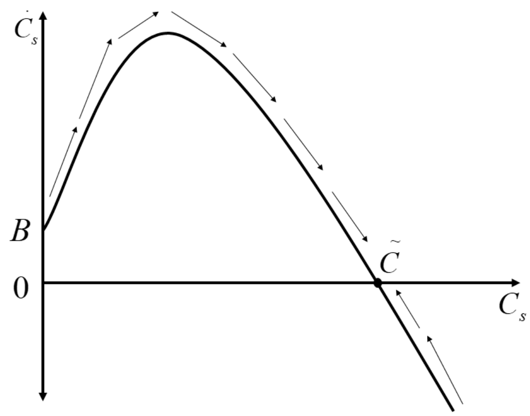

Figure 1: Credit Expansion Schedule

A qualitative analysis indicates a stable dynamic process under normal under normal conditions. We can also derive the stationary equilibrium at the end of the stable adjustment path when $\tilde{C}_{d,t+\tau} - C_s\,(t) = 0$ and explicitly state the new equilibrium credit supply (for a proof, see Appendix A.4):

$$\tilde{C}_{s,t+\tau} = \rho(i_R)^{(1/\alpha)} = \tilde{C}_{d,t+\tau} \tag{3}$$

In this case, the stationary equilibrium only depends on changes in the policy rate (i_R). It indicates a smooth adjustment process of the IBM to the new stable IBM equilibrium. Furthermore, with (3) we have also shown that the portfolio equilibrium is consistently described.

This process is the standard story of a smoothly adjusting financial market to the new equilibrium, a hypothetical idea of the IBM operation in the pre-crisis years. A standard general portfolio choice model, i.e., the equilibrium analysis, would end here. This solution indicates a stable adjustment process in the IBM, even when financial frictions, such as search costs or counterparty risks, are considered. However, according to the related literature (see, e.g., Hauck and Neyer, 2014), the lending process is anchored in a reallocation of reserves. Therefore, it is necessary to shed light on the dynamics when this constraint is taken into consideration.

3.2 Adjustment Path with Constraints

Bank s plans its credit expansion according to the above Equation (2), which is analogue to the stable adjustment path:

$$\dot{C}_s^P(t) = B + \rho\left(C_s(t)\right)^{1-\alpha} - (1+b)C_s(t)\,. \tag{4}$$

However, the reallocation of reserves in the secondary market for reserves, i.e., the IBM, (see, e.g., Affinito, 2013; Allen et al., 2009; Bräuning and Fecht, 2017), means that every credit outflow requires bank s to receive a respective reserve inflow (denoted by $F(t)$). Only if the reserve inflow is large enough, bank s can provide all credits described in the credit dynamics above ($F(t) \geqq \dot{C}_s(t)$). In other words, as reserves are necessary for credit provision, the reserve inflow can be a direct constraint for credit expansion. This reserve inflow is defined as liquidity bank s gets available and is willing to use for its interbank credit expansion. The reserve inflow is assumed to have two components. According to the general bank business in- and outflows, it is planned at the level of $F^P(t)$, but it is also subject to random shocks $x(t)$ with $E[x(t)] = 0$:

$$F(t) = F^P(t) + x(t). \tag{5}$$

Therefore, bank s plans the respective reserve inflows for each point in time ($F^P(t)$) according to desired credit expansion:

$$F^P(t) = \dot{C}_s^P(t)\,. \tag{6}$$

Hence, the credit expansion schedule is expected to take the following form (for a proof, see Appendix A.5):

$$E[\dot{C}_s(t)] \leqslant B + \rho\left(C_s(t)\right)^{1-\alpha} - (1+b)C_s(t) + 0 \tag{7}$$

The bank expects the available reserve inflow to cover at least the planned outflow resulting from credit expansion. In expected values, the plan works and guides the bank in the stable adjustment path of Figure 1. However, the stochastic realized credit expansion might differ from the expected one. The reserve constraint is binding for credit expansion, whenever the random realization becomes negative ($x < 0$). This stochastic realized credit expansion can be described by Equation 8.

$$\dot{C}_s(t) \leqslant B + \rho\left(C_s(t)\right)^{1-\alpha} - (1+b)C_s(t) + x(t) \tag{8}$$

In this randomly realized situation, the available reserve inflow directly restricts the bank's credit expansion, which then falls below the bank's credit expansion schedule. The bank will have to adjust and switches to a new adjustment path due to the realized reserve constraint. We refer to this new path as "adjustment path with constraints", which is now given by:

$$\begin{aligned} \dot{C}_s(t) &= x + b\tilde{C}_{d,t+\tau} + \rho(C_s(t))^{1-\alpha} - (1+b)\,C_s(t) \\ &= B' + \rho\left(C_s(t)\right)^{1-\alpha} - (1+b)C_s(t) \end{aligned} \tag{9}$$

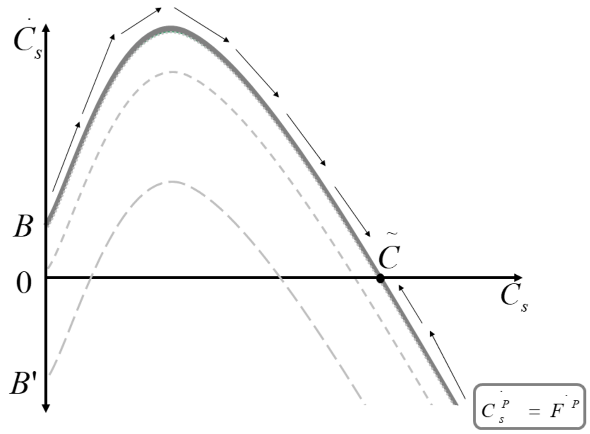

Figure 2: Planned and Realized Credit Expansion Schedule

where $B' = \left(b\tilde{C}_{d,t+\tau} + x\right)$ and $x < 0$. Figure 2 shows the adjustment paths without (solid dark grey) and with constraints (dotted and dashed lighter grey), where the solid line displays the credit expansion schedule of Equation (7). The dotted and dashed lines depict the respectively realized reserve inflows of Equation (8), which might be binding. Under normal conditions, the adjustment path without and with constraints can be identical, which would be graphically illustrated by a congruence of the solid and the dotted line, when $\dot{C}_s^P = F^P$ holds. However, under adverse conditions, the adjustment path under constraints can significantly differ from the scheduled path, which is exemplarily illustrated by the dashed lines. In this scenario, the smaller reserve inflow determines a credit expansion path along the realized reserve inflow, which is below the desired level.

As long as the random shock $x < 0$ is sufficiently small in absolute terms ($|x|$) the intersection with the vertical axis in Figure 2 remains positive, and the adjustment dynamics do generally not change. However, if in absolute terms the random shock is sufficiently large, $B' = \left(x + b\tilde{C}_{d,t+\tau}\right)$ may turn negative, and the properties of the dynamic adjustment process may change. Once we know that the intercept of the liquidity flows ($B' = \left(x + b\tilde{C}_{d,t+\tau}\right)$ with $b \in (0, 1)$, $\tilde{C}_{d,t+\tau} \geqslant 0$) is a crucial element in the analysis of the resilience of the IBM, we would like to shed light on it in further detail.

$$B' \left\{ \begin{array}{l} > 0 \text{ for } b\tilde{C}_{d,t+\tau} > -x \\ = 0 \text{ for } b\tilde{C}_{d,t+\tau} = -x \\ < 0 \text{ for } b\tilde{C}_{d,t+\tau} < -x \end{array} \right\} with \quad b \in (0,1), \quad \tilde{C}_{d,t+\tau} \geqslant 0, \quad x < 0 \tag{10}$$

3.2.1 Illustrative Example

Referring to the example in Section 3.1.1, recapture that in the **start-equilibrium-period**, bank s provides its optimal amount of credit supply, $\tilde{C}_{s,t} = 100\% = 1$ in the IBM (given the vector of equilibrium interest rates in the current period). While in the **end-equilibrium-period** $(t+\tau)$, the new equilibrium stock of credit supply is assumed to be $\tilde{C}_{s,t+\tau} = 1.1\tilde{C}_{s,t}$.

The general, sequential adjustment process can be described as in the example above, which does not assume a constrained adjustment path.

We assume bank d to repay all of its overnight credits to bank s, which receives a liquidity inflow of $1\tilde{C}_{s,t}$. Again, bank s rolls over a fraction of presumably 100% of credits of the previous day, resulting in liquidity outflows of $1\tilde{C}_{s,t}$. The further desired credit expansion of bank s requires the acquisition of new customers, where information costs allow the bank to expand its credit by no more than 2% per day. Therefore, bank s can only reach a stock of interbank credits of $C_{s,t+1} = 1.02\tilde{C}_{s,t}$ on the next day. Following the initial adjustment period, it takes 4 further days until bank s can reach its targeted credit expansion of $\tilde{C}_{s,t+\tau} = 1.1\tilde{C}_{s,t}$, where bank s can expand its credit by 2% in every period. It desires to have liquidity outflows of $1.1\tilde{C}_{s,t}$and needs liquidity inflows of $0.1C_{s,t}$, which it has to gain from bank-external sources. This takes time, with $0.02F$ manageable every day. This sequential inflow of liquidity is not automatically given, but expected.

However, when a shock hits the bank's total liquidity inflows on the next day, bank s receives less reserve inflows, depending on the magnitude of the shock. Consequently, its adjustment path is constrained. The following Table 2 illustrates the stocks of interbank credit supply recorded in bank s's balance sheet in every period, which is described in detail below.

Table 2: Exemplary Sequential Adjustment Process with Constraints

period	stock $\tilde{C}_{s,period}$			
	w/o	Case a	Case b	Case c
$t+1$	$1.02\tilde{C}_{s,t}$	$1.018\tilde{C}_{s,t}$	$1.00\tilde{C}_{s,t}$	$0.98\tilde{C}_{s,t}$
$t+2$	$1.04\tilde{C}_{s,t}$	$1.038\tilde{C}_{s,t}$	$1.02\tilde{C}_{s,t}$	$1.00\tilde{C}_{s,t}$
$t+3$	$1.06\tilde{C}_{s,t}$	$1.058\tilde{C}_{s,t}$	$1.04\tilde{C}_{s,t}$	$1.02\tilde{C}_{s,t}$
$t+4$	$1.08\tilde{C}_{s,t}$	$1.078\tilde{C}_{s,t}$	$1.06\tilde{C}_{s,t}$	$1.04\tilde{C}_{s,t}$
$t+5$	$1.10\tilde{C}_{s,t}$	$1.098\tilde{C}_{s,t}$	$1.08\tilde{C}_{s,t}$	$1.06\tilde{C}_{s,t}$

(Case a, $B' > 0$**)** minor shock: Instead of the expected inflows of $0.02F$, bank s receives only $0.018F$. Despite the fact that these lower inflows render the bank to a lower expansion path, it is still able to expand credit in the following

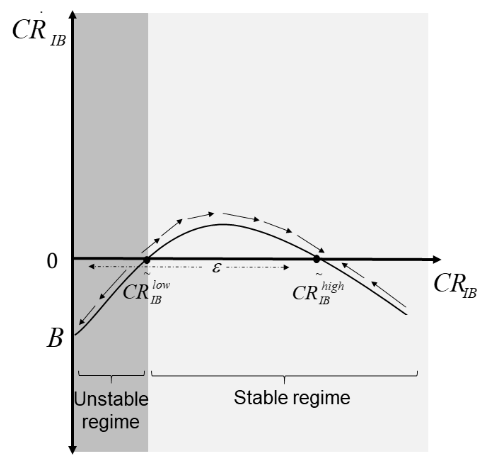

Figure 3: Adjustment Path with Constraints

period to $C_{s,t+2} = 1.018\tilde{C}_{s,t}$. On the next day, it receives again $0.02F,$ expands lending up to $C_{s,t+3} = 1.038\tilde{C}_{s,t}$and smoothly moves on track into the new, but lower end-equilibrium of $\tilde{C}_{s,t+\tau} = 1.098\tilde{C}_{s,t}$ in period $t+5$.

(Case b, $B' = 0$**)** large shock: Bank s does not receive inflows ($0F$). However, bank s can still expand credit with the help of the inflows stemming from repaid credits. All of its lending comes from a roll-over of credit. Bank s can exactly expand credit by $1\tilde{C}_{s,t}$ in period $t+1$.

(Case c, $B' < 0$**)** major shock: Bank s does no longer receive any additional inflows. The shock size prevents external liquidity inflows, with the result that bank s cannot expand yesterday's lending. Nor, and more importantly, can it roll-over all of yesterday's credit supply. The shock eats up some fraction of the repaid credits of the borrowing banks and prevents desired outflows of $0.02F$. Now, bank s has to deny a fraction of 2% of credit demand of its banking customers today, restricting bank s's credit expansion to $C_{s,t+1} = 0.98\tilde{C}_{s,t}$ only. If bank s receives a full repayment of the overnight credits on the subsequent day, and it can acquire new customers for the desired outflows, it can expand its credit provision to $C_{s,t+2} = 1.00\tilde{C}_{s,t}$.

In the worst-case scenario, a major shock cuts down bank s's interbank credit supply, Figure 3 shows this new path for a negative $B' = \left(b\tilde{C}_{d,t+\tau} + x\right)$ with $x < 0$.

3.2.2 End-Equilibrium in the IBM

Before we can discuss the implications for the dynamics, we need to formally identify the new, low equilibrium and the reactions of it with respect to changes in variables. We determine the low end-equilibrium ($\tilde{C}^{low}_{s,t+\tau}$) at time $t+\tau$ for $\left(\tilde{C}_{d,t+\tau} - C_{s,t}\right) > 0$.

$$0 = G \equiv x + b\tilde{C}_{d,t+\tau} + \rho C_s^{1-\alpha}(t) - (1+b)\, C_s(t) \tag{11}$$

Since Equation (11) cannot be solved explicitly, we need to apply the implicit function theorem. Looking at Figure 3 and using the implicit function theorem at a local point, we can state that Equation (11) implicitly defines a function C_s at potentially two equilibrium points, a low equilibrium point ($\tilde{C}^{low}_{s,t+\tau}$) and a high equilibrium point ($\tilde{C}^{high}_{s,t+\tau}$).

Proposition 1 *(Bifurcation of Equilibrium) Depending on the magnitude of the shock, the characteristics of the adjustment path can change. In an adverse scenario this results in a bifurcation of the equilibrium with two equilibrium points, which we refer to as low ($\tilde{C}^{low}_{s,t+\tau}$) and high equilibrium ($\tilde{C}^{high}_{s,t+\tau}$).*

$$\begin{aligned} \tilde{C}^{low}_{s,t+\tau} &= C_s^{low}(x,...), \qquad \textit{with} \;\; \frac{dC_s^{low}}{dx} > 0 \\ \tilde{C}^{high}_{s,t+\tau} &= C_s^{high}(x,...). \end{aligned} \tag{12}$$

For a proof, see Appendix A.6.

We use this new path for a qualitative dynamic analysis as illustrated by Figure 3. While Figure 1 described a global overall stable process with a unique equilibrium, Figure 3 shows two equilibria. The high equilibrium in Figure 3 is comparable to the unique equilibrium in Figure 1. We see a locally stable process as the $\dot{C}_s\,(t)$-curve has a negative slope around the high equilibrium. The high equilibrium is, thus, a locally stable point. This is different for the low equilibrium. Here, the slope is positive, which implies that the low equilibrium $\tilde{C}^{low}_{s,t+\tau}$ is locally unstable. As a result, we have two dynamic regimes and overall market resilience declines. At points larger than the low equilibrium level $\tilde{C}^{low}_{s,t+\tau}$ the credit creation process will be on a stable path and move smoothly to the high equilibrium $\tilde{C}^{high}_{s,t+\tau}$, which is also the final portfolio equilibrium bank s would like to reach. However, if we look at points below the low equilibrium level $\tilde{C}^{low}_{s,t+\tau}$ the process is unstable and bank s would keep on decreasing credit supply. In this case, the credit creation of bank s is constrained by a too low reserve inflow $F\,(t)$.

Ultimately, the credit provision may potentially reduce to zero. If this is the case, the IBM will be characterized by a market freeze, indicating no further credit supply in the IBM. This brief discussion already indicates that adjustment processes are no longer only smooth and stable. If the stochastic shock described by a randomly much lower reserve inflow in the adjustment process is sufficiently large or C_s is still rather low, such that $C_s(t) < \tilde{C}^{low}_{s,t+\tau}$, the process becomes unstable. This critical mechanism is studied in more detail below.

3.2.3 Market Resilience

We have described the new adjustment path under constraints, which includes two dynamic regimes. We now focus on the determinants of the regime, in which the IBM operates, especially on the critical adjustment path. Which kind of shocks can lead the IBM into the unstable regime? We now discuss the aggregate process in the IBM as well as some elements that potentially affect the resilience of this market. The term *resilience* in this context stands for the likelihood of the IBM to be in a stable regime and to smoothly reach the new equilibrium $(\tilde{C}^{high}_{s,t+\tau})$.

A random event determines the actually realized reserve flows x, which can be binding for credit expansion $(F(t) \geqslant \dot{C}_s(t))$ and hence, determine in Equation (13) the shape of the actual adjustment process.

$$d\dot{C}_s(t) = x + b\tilde{C}_{d,t+\tau} + \rho C_s^{1-\alpha}(t) - (1+b)\, C_s(t) \tag{13}$$

These randomly realized reserve flows have to be studied in more detail. While x is a particular realized shock during the adjustment period, we now generalize to each possible shock realization, referring to the random distribution of this shock (X).

With this knowledge we determine the market's resilience.

Definition 1 *(Market Resilience) The likelihood to reach a stable interbank market equilibrium is defined as market resilience. This gives the likelihood to remain in the stable regime at any moment during the adjustment period.*

First, as described above x are randomly realized inflows of reserves, with expectation $E[x] = 0$ and variance $Var = \sigma^2$. We specify the so-far random distribution of the shock by assuming a normal distribution of realized reserve flows

$$X \sim \mathcal{N}(0, \sigma^2) \quad \text{with density} \quad f_X(x) = (1/\sqrt{2\pi\sigma^2})\exp^{(-x^2/2\sigma^2)}. \tag{14}$$

With the help of this normal distribution, we can determine a probability of each particular shock realization x. This knowledge will help us to determine the probability of being in the stable or unstable regime of the adjustment path, with the latter of primary interest.

According to Figure 3 an adjustment process becomes unstable whenever the low equilibrium is situated to the right of the current credit stock, such that $C_s(t) \leqslant \tilde{C}^{low}_{s,t+\tau}$. Starting off from any current position of current interbank credit supply $C_s(t) = \varepsilon$, we can determine a probability to be in the unstable regime of the adjustment path $(P(\varepsilon \leqslant \tilde{C}^{low}_{s,t+\tau}))$. As analyzed above, the location of $\tilde{C}^{low}_{s,t+\tau}$ is determined by the realized reserve flow x. In graphic terms, the larger the shock realization $|x|$, the further to the right will the low equilibrium be situated. Hence, the greater the extent of the unstable regime of the adjustment process, and the larger the probability to be in the unstable regime. We now determine a probability that the position is in the unstable regime ($P(\varepsilon \leqslant \tilde{C}^{low}_{s,t+\tau})$). Equation (11) has specified a function of the low equilibrium with $\tilde{C}^{low}_{s,t+\tau} = G(x)$. From (12) we know that for a particular value x the derivative of the low equilibrium function with respect to x is $\frac{dC^{low}_s}{dx} > 0$. This allows us to use a local and linear approximation at $\tilde{C}^{low}_{s,t+\tau}$ and we can rewrite $\tilde{C}^{low}_{s,t+\tau}$ as the linear function given by

$$\tilde{C}^{low}_{s,t+\tau} = g(x) = mx - c. \tag{15}$$

With the help of this linear approximation and the random distribution (14) we now arrive at the following Proposition 2:

Proposition 2 *(Systemic Instability) For a current interbank credit supply of bank s at level $C_s(t) = \varepsilon$, the probability of randomly falling in the unstable regime of the adjustment path ($P(\varepsilon \leqslant \tilde{C}^{low}_{s,t+\tau})$) is given by*

$$\mathbb{P}(g(X) \geqslant \varepsilon) = \int_{\varepsilon}^{\infty} (1/\sqrt{2\pi\sigma^2}) \exp^{-\left(\left(\frac{1}{m}(x+c)\right)^2/2\sigma^2\right)} (1/|m|) dx. \tag{16}$$

By contrast, the complementary probability of being in the stable regime of the adjustment path ($P(\varepsilon > \tilde{C}^{low}_{s,t+\tau})$), which is defined as market resilience, is given by

$$Resilience = 1 - \mathbb{P}(g(X) \geqslant \varepsilon). \tag{17}$$

For a proof, see Appendix A.7.

4 Discussion and Policy Implications

The theoretical considerations of the model presented above add to the general understanding of the mechanisms and dynamics in the IBM. The results show that the IBM can randomly fall in an unstable equilibrium, which, in the extreme case, indicates a collapse of interbank lending. Furthermore, we can state a probability of the unstable adjustment path, depending on current interbank credit provision. Based on this simple model, there are two questions ahead. First, what determines the probability of the unstable regime? And second, returning to our initial research question: What is the impact of monetary policy shocks on IBM resilience?

4.1 Determinants of Systemic Instability

If bank s is a representative credit provider in our system, all described mechanisms hold for the entire IBM. Therefore, it is interesting to identify elements that can increase or decrease the probability of market instability, and identify the elements that affect market resilience (see Definition (1)).

First, if current interbank credit provision ($\varepsilon = C_s(t)$) increases, the probability of being in the unstable regime decreases

$$
\begin{aligned}
\frac{d}{d\varepsilon}\mathbb{P}(f(X) &\geqslant \varepsilon) = \frac{d}{d\varepsilon}\int_{\varepsilon}^{\infty}(1/\sqrt{2\pi\sigma^2})\exp^{(-\left(\frac{1}{m}(x+c)\right)^2/2\sigma^2)}(1/|m|)dx \quad (18)\\
&= -(1/\sqrt{2\pi\sigma^2})\exp^{(-\left(\frac{1}{m}(\varepsilon+c)\right)^2/2\sigma^2)}(1/|m|) < 0. \quad (19)
\end{aligned}
$$

That is, close to the high equilibrium the probability of instability is low, the IBM is rather resilient.

Second, it is interesting to note that it is not necessarily the level of the reserve flows that determines the probability of falling into the unstable regime of the adjustment path. The probability of being in the unstable regime increases if we take the derivative of (16) with respect to the variance σ^2,

$$\frac{d}{d\sigma^2}\mathbb{P}(f(X) \geqslant \varepsilon) = (1/|m|)(1/\sqrt{2\pi\sigma^2})(\varepsilon/2\sigma^2)\exp^{(-\varepsilon^2/(2m^2\sigma^2)} > 0. \quad (20)$$

Thus, a higher volatility of reserve flows increases the probability of switching to an unstable adjustment path. This volatility of reserve flows may even be generated in other financial markets instead of directly resulting from the market for central bank money.[2] When a shock applies to the liquidity inflows of bank s, and these threaten to restrict its lending process, bank s can try to gain liquidity from other sources in order to remain on its schedule. Supposed, bank s decides to sell a fraction of the bonds recorded on the asset side of its balance sheet. However, the bond price is subject to volatility, too. When asset markets are characterized by high volatility, the in exchange received liquidity inflows will also be highly volatile. Consequently, the probability for bank s to switch to the unstable adjustment path will also be higher under higher volatility in the bond market. If it is assumed that volatility in asset markets increases, and shocks to individual banks in group s are correlated, i.e., non-diversifiable, this implies that lenders in the IBM are affected simultaneously. The whole IBM may fall into an unstable adjustment path with more volatile reserve flows. Hence, market resilience declines under more volatile conditions.

Under these volatile conditions, the higher volatility of liquidity flows changes the characteristics of the market's adjustment path, resulting in a bifurcation of the equilibrium. By assuming the shock to be normally distributed, the credit creation path depends crucially on the variance. A higher variance translates into a flatter density curve, which increases the probability of extreme values. Thus, a higher variance implies a higher likelihood of the unstable adjustment path. In our interpretation, market resilience declines in this case, i.e., the probability of switching to an instable adjustment path increases with higher volatility, which can lead to a potential market freeze.

[2] Reasons for higher volatility in other financial markets are shown, for instance, by Daniel et al. (1998). Their model shows that investor overconfidence can cause stock market bubbles and increase volatility in asset markets.

4.2 Monetary Policy Tightening and Systemic Instability

We have shown that a higher volatility of reserve flows reduces the resilience of the IBM. In order to investigate the effect of monetary policy on IBM resilience, we first examine the effect of monetary policy tightening on volatility in asset markets. Figure 4 shows the monthly relationship between the MRO (ECB, 2019b) and the VSTOXX (Deutsche Börse, 2019), i.e., a volatility index for the EURO STOXX 50, between January 2000 and December 2018.

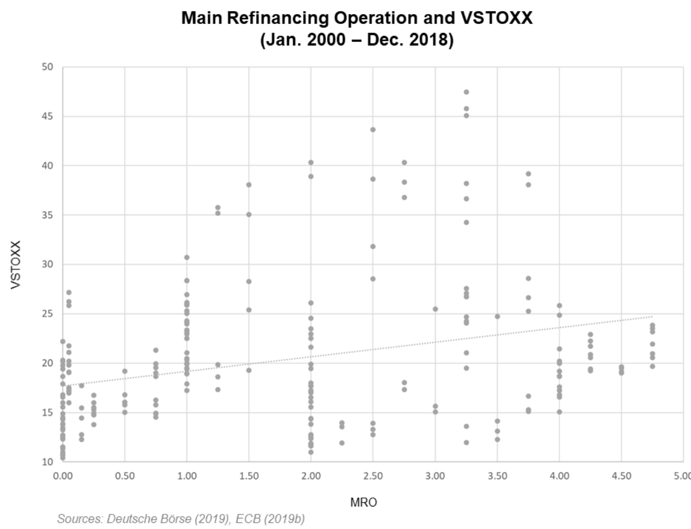

Figure 4: Key Policy Rate and Volatility

Overall, a positive correlation between the monetary policy rate and asset price volatility could potentially be suggested to hold. However, as this is no thorough empirical regression analysis, this Figure 4 has to be interpreted in a very prudent manner and requires further investigations. Although the general relationship between monetary policy and asset prices is also of high interest in recent literature, especially with respect to the assessment of consequences of a monetary policy shock on financial stability (cf. Allen and Gale, 2004; Disyatat, 2010), the theoretical idea of monetary policy tightening coming along with higher volatility in asset markets has to the best of our knowledge not been proven so far. First insights are offered by Alessi and Kerssenfischer (2016) who examine asset market responses to monetary policy shocks. The authors use a Structural Factor Model for the euro area, which yields a wide information set of various impulse response functions (IRF) following a contractionary monetary policy shock of a 50 bp rise in the short-term rate. The IRF of a volatility index of stock prices (EURO STOXX 50) is u-shaped and shows that the contractionary monetary policy shock leads volatility to go up on impact. As time passes, the effects of the shock in volatility decay, reaching zero impact after approximately 10 months. The results of Alessi and Kerssenfischer (2016) might indicate a higher volatility of the EURO STOXX 50 following monetary policy tightening in the current environment. At least, they suggest that monetary policy is an important determinant of asset prices as well as asset price movements.

If this assumption holds true, monetary policy tightening would go hand in hand with higher asset price volatility and hence a higher volatility of reserve flows in banks' balance sheet management, which would reduce the resilience of the IBM.

A positive correlation between monetary policy tightening and volatility in asset markets is also suggested when the trading volume in asset markets is taken into consideration. Figure 5 shows the monthly relationship between the MRO (ECB, 2019b) and the trading volume of the EURO STOXX 50 (Investing, 2019) between February 2006 and March 2019. Figure 6 shows the relationship between the trading volume of the EURO STOXX 50 (Investing, 2019) and the VSTOXX (Deutsche Börse, 2019) over the same period of time. Due to the fact that both Figures show a positive relationship, a positive correlation between the monetary policy rate and volatility in asset prices could be suggested to hold. However, these Figures have to be interpreted with due caution and have to be examined in detail in future research.

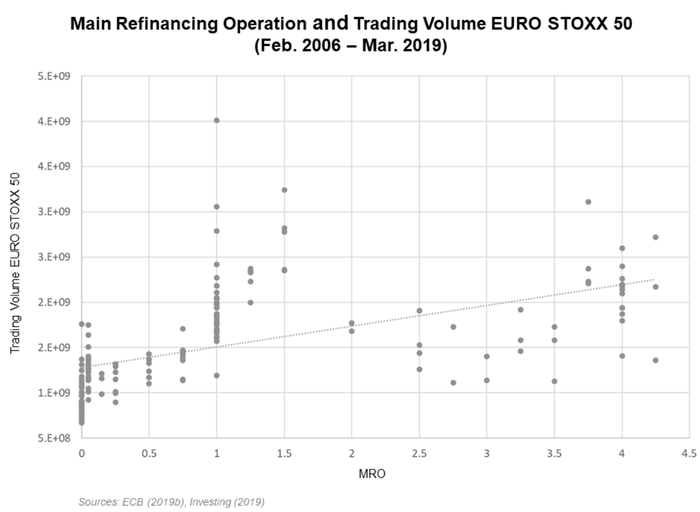

Figure 5: Key Policy Rate and Asset Trading Volume

Given the assumption that volatility is positively related to monetary policy tightening holds, monetary policy tightening would be likely to reduce the resilience of the IBM. By contrast, expansionary monetary policy would decrease asset market volatility and hence add to the resilience of the IBM. This

Figure 6: Asset Trading Volume and Volatility

asymmetric reaction scheme to monetary policy shocks needs further investigation. Given it holds, it should be taken into consideration when entering a contractionary monetary policy phase. When switching the regime, policymakers should be aware of a potential reduction in interbank credit market resilience and the consequences for financial stability.

4.3 Policy Implications

Financial stability, i.e. liquidity in the IBM, can be safeguarded ex-ante with the help of preventive measures as well as emergency operations in case the adverse scenario has already materialized.

Preventive measures should enable banks to absorb or reduce the effects of a shock on reserves by providing them with a sufficiently high liquidity buffer. In the context of our model, they should also safeguard the ability of a bank to provide interbank credit at all times, even if a limited cash inflow constrains its credit provision. Furthermore, risk indicators of volatility triggers and amplifiers as well as early warning systems are important so that countermeasures can be taken at an early stage. In this vein, macroprudential policies have been developed to ensure the stability of the financial system as a whole. However, the discussion on the effectiveness of these tools is still underway.

Macroprudential policy tools, such as the LCR or the Net Stable Funding Ratio (NSFR) as introduced by the Basel Committee on Banking Supervision (BCBS, 2013) are regarded as a reasonable first step. The LCR requires banks to hold a stock of high-quality liquid assets (HQLA) to cover expected net cash outflows over the next 30 days. However, in the most extreme case, banks will still have to rely on the central bank to step in as a "lender of last resort". Furthermore, research on the interaction of the LCR with monetary policy implementation is rather scarce. Theoretical contributions include, for instance, Bindseil and Lamoot (2011), Schmitz (2013) or Bech and Keister (2017). Schmitz (2013) finds that the implementation of the LCR leads to a shift from operations within the unsecured IBM to the central bank. In addition, his analysis reveals that feedback and network dynamics can lead to a more volatile structural liquidity deficit of the banking sector and, ultimately, to a collapse of the arbitrage relationship between borrowing in the unsecured IBM and the central bank money market. By contrast, Aaron et al. (2015) suppose that lending in the IBM remains stable under the LCR, but shifts towards shorter maturities. The detailed analysis of Bech and Keister (2017) examines the effect of the LCR under monetary policy normalization. Bech and Keister (2017) show that under a binding LCR requirement, banks' funding costs are rather determined by the quantity of available HQLA than by central bank reserves, which can hinder monetary policy implementation. Based on the results of Bech and Keister (2017), the implementation of the LCR shifts attention away from central bank reserves, which, according to our model, can contribute to a less resilient financial system.

In addition, in an environment of market liquidity tensions, the tool of forward guidance can become an essential element to mitigate the shock and allow for anticipation on a timely basis.

In our view, macroprudential policies should give further attention to volatility shocks and resulting effects on banks' portfolio and liquidity management. For instance, the LCR could be refined regarding a binding fraction of central bank money holdings. In addition to the minimum reserve requirement on commercial bank deposits, which also insures against a too high withdrawal of customer deposits, a reserve requirement on highly volatile assets would potentially ex ante reduce the probability of reaching the unstable equilibrium, contributing to financial system resilience.

However, once a sufficiently large shock has materialized and banks can no longer borrow in the IBM, the central bank may be forced to meet excess demand for liquidity by providing emergency operations on a temporary basis. This approach was also adopted in 2008, when the ECB provided unprecedented "Enhanced Credit Support", which included unlimited credit supply to the banking sector at a fixed rate (fixed-rate full allotment) and relaxed collateral requirements for its refinancing operations (ECB, 2011).

5 Concluding Remarks

This study pays special attention to dynamics in the IBM and financial system resilience. Based on a stable unique equilibrium in financial markets, we have introduced a stochastic model of the interbank credit market, focusing on the flow process between lending and borrowing institutions. In our model credit supply is restricted by the availability of stochastic liquidity inflows to lending institutions. Following a monetary policy shock, a sequential flow adjustment process sets in and the IBM moves towards a new equilibrium.

The characteristics of this adjustment path are found to depend on surrounding conditions. In "normal times" the flow dynamics remain smooth within a stable adjustment regime and the planned credit expansion schedule can be realized. However, under more volatile conditions, the lending process can change, resulting in a bifurcation of the equilibrium. Further, our model yields a probability of randomly falling in the unstable adjustment regime, depending on current interbank credit provision. Defining interbank "market resilience" as the probability of remaining in the stable regime, we show that a change in the volatility of reserve flows may threaten the resilience of the IBM and increase the probability of the market to fall into a regime of unstable dynamics. Given that monetary policy tightening increases the volatility of asset prices and as a potential consequence the volatility of reserve flows in banks' balance sheet management, a contractionary monetary policy would reduce the resilience of the IBM. The likelihood of the market to fall into a regime of unstable dynamics would be increased. Thus, we stress that tightening monetary policy could incidentally reduce IBM resilience up to a potential market collapse, even in the absence of contagion phenomena.

To prevent this adverse scenario, we emphasize the importance of system-wide, macroprudential policies that should pay special attention to the risk of asset price volatility and resulting effects on banks' portfolio and liquidity management to support and ensure the resilience of the financial system. A first step to safeguard liquidity in the IBM are preventive macroprudential policy tools, such as the LCR implemented under Basel III. However, once a sufficiently large shock has materialized, the central bank can only try to revitalize the IBM, for instance, with the help of emergency liquidity injections. Furthermore, the monetary policy tool of forward guidance becomes essential in avoiding the adverse scenario in the IBM. Therefore, policymakers should be aware of asymmetric effects of different monetary policy regimes on the resilience of the financial system.

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Appendix

A.1 Asset Markets

This supplementary section discusses the three markets in further detail.

Central Bank Money Market Overall, the aggregate liquidity of the banking sector is managed by the central bank, which provides and absorbs liquidity in the geographical headquarter location of the respective bank. The banking sector can hold reserves in the form of deposits at the central bank account or currency. The banking sector's need for reserves results from minimum reserve requirements and autonomous liquidity factors. To manage the euro area banking sector's liquidity, the ECB refers to its standard instruments of the open market operations and two standing facilities (see ECB (2011) for a detailed description of its instruments). Open market operations include the MRO, longer-term refinancing operations, fine-tuning operations, and structural operations. We briefly present the corridor system, which is formed by the three key interest rates of the main refinancing operation (i.e., MRO) and the two rates for the standing facilities.

The MRO is a weekly credit operation of one-week loans, which traditionally satisfies approximately 74% of the liquidity needs of the euro area's banking sector (ECB, 2002). Borrowers have to pay the main refinancing rate (i_R) and pledge adequate collateral. In "normal times" (between June 2000 and October 2008), the ECB used variable rate tenders, under which banks bid both, the amount of credit as well as the rate they are willing to pay. The estimated allotment volume reflected the liquidity needs of the banking sector during the maturity of the MRO. In these "normal times", bids in the MRO were rationed if aggregate bids exceeded the estimated allotment, serving the highest interest rate bids first. Since October 2008, the ECB has used fixed rate tenders with full allotment to mitigate adverse effects of the dysfunctional money market (ECB, 2011). Our analysis will refer to the operations in "normal times" in order to emphasize the presence of a vulnerability in the financial system even under normal conditions.

In addition, the ECB sets short-term interest rates on its standing facilities. Under normal conditions, the usage of standing facilities is unfavorable when compared with market rates. The rate on the marginal lending facility (i_{LF}) provides overnight liquidity from the central bank at higher costs than those of the one-week MRO, while banks also have to pledge adequate collateral for this short-term liquidity. By contrast, the deposit facility absorbs liquidity from the banking sector, where the central bank pays an interest rate (i_{DF}) on overnight deposits held at central bank accounts. Usually, the interest rates on these facilities determine a symmetric corridor around the MRO-rate (cf. Figure 7), with the deposit facility acting as a floor and the lending facility as a ceiling.

Interbank Money Market In addition to the central bank money market, liquidity is traded in the IBM for reserves. Banks reallocate the reserves provided by the central bank mainly for two reasons. First, usually, the one-week MRO is the shortest maturity available for liquidity from the central bank. While the banking sector's aggregated need of liquidity may remain broadly constant within one-week, individual bank's liquidity needs usually vary daily (Hauck and Neyer, 2014). Second, only a fraction of banks covers its liquidity need directly from the central bank. There are also banks, which prefer to cover their needs of reserves exclusively in the IBM.[3]

In accordance with Hauck and Neyer (2014), the market is characterized by two types $i = d, s$ of banks, according to the bank's position in the IBM. In this context, we define liquidity as excess reserve holdings of single banks. Each type of banks consists of a continuum of banks with unit mass. Banks of type s supply excess reserves in the IBM, whereas banks of type d demand liquidity in the IBM. Each single bank is randomly assigned to its type, while Brousseau and Manzanares (2009) claim that its main business activities are a key indicator for the bank's liquidity situation.

The unsecured interbank credit market is characterized by an OTC market, where contracts are negotiated bilaterally (e.g., Vollmer and Wiese, 2014; Bianchi and Bigio, 2017). In the absence of public information, the empirical analysis of these contracts remains a major challenge. Although individual interest rates are negotiated in every OTC contract, we assume banks to trade at one common rate (i_C). Furthermore, we refer to the unsecured market.[4] Generally, the IBM is regarded as a very short-term market, where mainly overnight credits are traded among banks of groups s and d (cf. Green et al., 2016). Empirical studies confirm the majority of interbank trades to be overnight, although varying between 59% in the Dutch (Heijmans et al., 2010), 67% in the American (Kuo et al., 2013), and more than 90% in the Italian IBM (Arciero et al., 2016). This result is also confirmed by, e.g., Cocco et al. (2009) (Portugal), de Bonis et al. (2013) (Italy) or Raddant (2014) (Italy). The empirical reference rate is given by the European Overnight Index Average (EONIA), which is a market index calculated by the ECB as a weighted average of the interest rates on unsecured overnight lending transactions denominated in Euro, undertaken by a representative panel of 28 contributing banks.[5]

How do the central bank money market and the IBM relate to each other? Figure 7 depicts the key interest rates charged by the ECB (ECB, 2019b) as well as the market interest rate in the unsecured European overnight interbank market (EMMI, 2019) between July 2000 and October 2008. We see that a bank with excess liquidity could hold an overnight deposit at the central bank, which yields a return of the deposit facility rate (grey dashed) or supply liquidity in

[3] At the end of January 2011 the euro area recorded 6334 credit institutions. Only 2267 credit institutions were eligible for participation in the open market operations by fulfilling the ECB's criteria, 2395 credit institutions had access to the marginal lending facility and 2789 to the deposit facility (ECB, 2011, p. 97).

[4] In the euro area, the secured IBM (repo market) gained prominence following the GFC (see ECB, 2011), although data availability is restricted (cf. ECB, 2019a). For the analysis under "normal conditions", however, we stick to a traditional focus on unsecured lending in the IBM.

[5] For further information on this reference rate see EMMI (2019).

the IBM, which yields the higher EONIA rate (dashed black). Consequently, arbitrage opportunities would suggest that a surplus bank rather invests in the IBM and lends to other banks than stores its liquidity at the central bank. Second, if the bank has to cover a liquidity need, it has three possibilities. First, it can take a loan from the central bank for one-week, which would charge the MRO-interest rate (solid thick grey). This is the cheapest way of covering its liquidity need, though the bank has to pledge adequate collateral and the liquidity can only be accessed at a one-week frequency. Second, the bank can take a short-term credit from the central bank, which is given at the marginal lending facility rate (solid thin grey) for one day (overnight). However, the bank can also receive short-term liquidity from other banks, borrowing in the IBM is OTC, but, on average, the EONIA rate is charged (dashed black). Hence, if the bank needs liquidity on short-term notice, arbitrage conditions would suggest to rather take an IBM credit (dashed black), which is cheaper than borrowing from the central bank (solid thin grey).

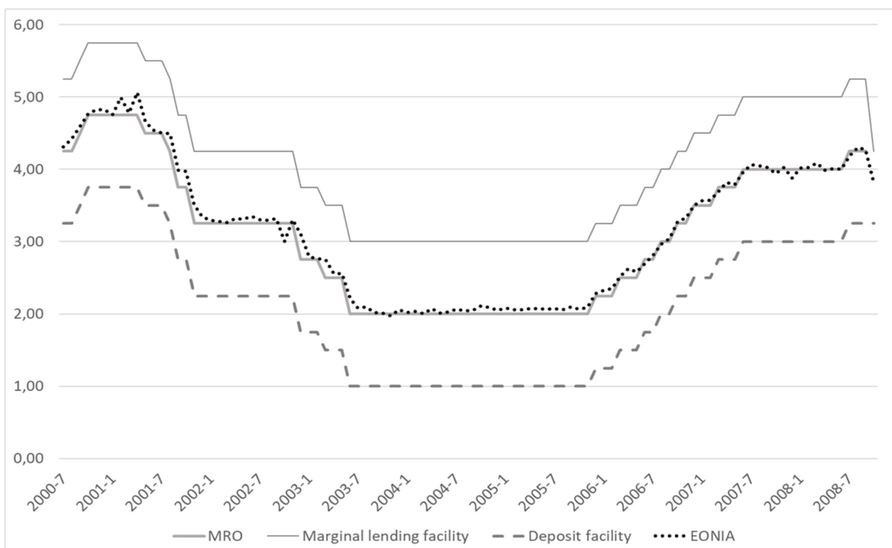

Figure 7: Selected Interest Rates of the ECB and the IBM (EONIA)

Moreover, Figure 7 reveals the interest rate relations applying in the markets for reserves under normal conditions with $i_{LF} > i_R > i_{DF}$. First, in the overnight market, the interest rate on the deposit facility (i_{DF}) acts as a floor for the market rate, while the interest rate on the marginal lending facility (i_{LF}) acts as a ceiling. The facilities thus determine a corridor, within which the overnight interest rate in the IBM can fluctuate, usually settling of close to the middle of the corridor. Furthermore, in normal times, the overnight IB rate for liquidity has generally remained close to the one-week central bank rate, but mainly averaging slightly above the interest rate charged by the central bank

($i_{IB} \geqslant i_R$). This positive spread between the overnight IB rate and the one-week central bank rate is of highest interest, indicating a non-perfect adjustment of IB market rates to monetary policy rates.

Asset Market Besides the markets for reserves, several investment opportunities for each bank exist in the asset market, regardless of its bank type. We simplify to a representative asset, such as a zero-coupon bond, which provides an investment opportunity for banks of both groups. It matures after one period and is traded at the bond's price (in %) of $P_B = 100/(1 + i_B)$ where i_B is a fixed interest payment (in %). This bond is assumed to carry various types of risks, such as interest rate risk or default risk.

A.2 Balance Sheet

The following Table 3 shows a simplified balance sheet of investment bank s, which supplies liquidity in the IBM. We explicitly abstract from money creation via the private sector, i.e. the balance sheet does not record credits to the private sector, neither private deposits.

Table 3: Simplified Balance Sheet of a Lending Bank

Bank s

Assets		Liabilities	
Reserves	R_s	Central bank credit	M_s
Interbank lending	C_s		
Bonds	$P_B B_s$	Equity	E_s
Balance sheet total	$\mathbf{T}_s$	**Balance sheet total**	$\mathbf{T}_s$

The asset side shows the use of funds, where we distinguish between three groups of assets, i.e., reserve holdings (R_s), overnight-lending of bank s in the interbank market (C_s) as well as the value of bank s's investment in the asset market as represented by bonds ($P_B B_s^D$). On the liability side, we simplify to one-week credits received from the central bank within the MRO (M_s) and bank equity (E_s).

Bank s is assumed to be risk averse and profit maximizing. It takes an investment decision based on the asset's characteristics, focusing on its return and portfolio risk combination.

In a portfolio choice model bank s would maximize its profits as recorded in a value function, referring to the asset side of its balance sheet. Central bank money holdings are assumed to yield the overnight interest rate of the deposit facility (i_{DF}) without contributing to portfolio risk ($\frac{d\sigma_s}{dR_s} = 0$). Interbank lending yields the overnight interest rate (i_{IB}), while increasing portfolio risk due to unsecured lending in the IBM ($\frac{d\sigma_s}{dCR_{IB_s}} > 0$). The value of government bonds in bank s's portfolio equals $P_B B_s^D$, where bonds are regarded as a volatile investment contributing to portfolio risk ($\frac{d\sigma_s}{dB_s^D} > 0$). Bank s decides which share of its balance sheet total to invest in the respective asset, expecting a yield of the pre-determined return-risk combination, while this investment decision is constrained by the balance sheet total (T_s).

A.3 Credit Expansion Schedule

Proof. The unconstrained credit expansion process is characterized by the following properties of the non-linear, non-homogeneous differential equation in C_s.

$$\frac{d\dot{C}_s}{dC_s} = \rho\left(1-\alpha\right)C_s^{-\alpha} - \left(1+b\right)\begin{cases} >0 \text{ for } C_s^{\alpha} < \rho\left(1-\alpha\right)\left(1+b\right)^{-1} \\ =0 \text{ for } C_s^{\alpha} = \rho\left(1-\alpha\right)\left(1+b\right)^{-1} \\ <0 \text{ for } C_s^{\alpha} > \rho\left(1-\alpha\right)\left(1+b\right)^{-1} \end{cases}$$

Hence, the first derivate of the credit expansion process with respect to the stock of interbank credits recorded in bank s's balance sheet can be positive or negative, depending on which of the gross forces described above dominates the process. By contrast, the second derivative is negative, such that:

$$\frac{d\dot{C}_s^2}{d\left(C_s\right)^2} = \rho\left(-\alpha+\alpha^2\right)C_s^{-\alpha-1} < 0,$$

which shows the concavity of the flow function. That is, higher interbank credit flows imply a higher stock of interbank credits, but at a decreasing rate. ■

A.4 Equilibrium Credit Supply

Proof. Derivation of the equilibrium credit supply as denoted by $\tilde{C}_s$.

$$\begin{aligned} \dot{C}_s\left(t\right) &= b\tilde{C}_{d,t+\tau} + \rho(i_R)C_s^{1-\alpha} - \left(1+b\right)C_s = 0 \\ &\Longleftrightarrow 0 = \rho(i_R)C_s^{1-\alpha} - C_s \\ &\Longleftrightarrow 1 = \rho(i_R)C_s^{-\alpha} \\ &\Longleftrightarrow \tilde{C}_s = \rho(i_R)^{1/\alpha} \end{aligned}$$

■

A.5 Expected Credit Expansion Schedule

Proof. Derivation of the expected credit expansion schedule:

The reserve inflow requirement is given by

$$\begin{aligned}
\dot{C}_s(t) &\leqslant F(t) \\
\dot{C}_s(t) &\leqslant F^P(t) + x(t) \\
\dot{C}_s(t) &\leqslant \dot{C}_s^P(t) + x(t) \\
\dot{C}_s(t) &\leqslant B + \rho\left(C_s(t)\right)^{1-\alpha} - (1+b)C_s(t) + x(t)
\end{aligned}$$

with $\dot{C}_s^P(t) = F^P(t)$ and $\dot{C}_s^P(t) = B + \rho\left(C_s(t)\right)^{1-\alpha} - (1+b)C_s(t)$.

Given $E[x(t)] = 0$, this yields the expected credit expansion schedule given by $E\left[\dot{C}_s(t)\right]$:

$$E\left[\dot{C}_s(t)\right] \leqslant B + \rho\left(C_s(t)\right)^{1-\alpha} - (1+b)C_s(t)$$

■

A.6 Proof of Proposition 1

Proof. Bifurcation of equilibrium, derived using implicit function theorem:

The stochastic realized credit expansion schedule as given by Equation (11)

$$0 = G = x + b\tilde{C}_{d,t+\tau} + \rho C_s^{1-\alpha} - (1+b)C_s$$

implicitly defines a function $C_s^{low} = C_s^{low}(x, ...)$ if $\frac{dG}{dC_s} \neq 0$.

Thus, taking the derivative of G with respect to C_s gives:

$$\begin{aligned}
\frac{\partial G}{\partial C_s} &= (1-\alpha)\rho C_s^{-\alpha} - (1+b) > 0 \qquad \text{for} \quad C_s < \left(\frac{\rho(1-\alpha)}{1+b}\right)^{\frac{1}{\alpha}} \\
\frac{\partial G}{\partial C_s} &= (1-\alpha)\rho C_s^{-\alpha} - (1+b) < 0 \qquad \text{for} \quad C_s > \left(\frac{\rho(1-\alpha)}{1+b}\right)^{\frac{1}{\alpha}}.
\end{aligned}$$

Hence, $\tilde{C}_s^{low} = C_s < \left(\frac{\rho(1-\alpha)}{1+b}\right)^{\frac{1}{\alpha}}$ is the low credit level stationary equilibrium. The derivative of this implicit function is $\frac{\partial G}{\partial x} = 1$.

$$\frac{dC_s}{dx} = -\frac{\frac{\partial G}{\partial x}}{\frac{\partial G}{\partial C_s}} = -\frac{1}{\frac{\partial G}{\partial C_s}} < 0$$

■

A.7 Proof of Proposition 2

Proof. Given density $f_X(x) = (1/\sqrt{2\pi\sigma^2}) \exp^{(-x^2/2\sigma^2)}$, we apply the theorem on the transformation of integral equations.

$$
\begin{aligned}
\mathbb{P}(g(X) \geqslant \varepsilon) &= \int_{x \geqslant \varepsilon} f_X(g^{-1}(x)) \frac{1}{|g'(g^{-1}(x))|} dx \\
&= \int_{x \geqslant \varepsilon} f_X\left(\frac{1}{m}(x+c)\right) \frac{1}{|m|} dx \\
&= \int_{\varepsilon}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{\left(\frac{1}{m}(x+c)\right)^2}{2\sigma^2}} \frac{1}{|m|} dx.
\end{aligned}
$$

■