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- Testing 200 Years of Business Cycle Dynamics -

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Abstract

We study the dynamic pattern of business cycles using US GDP data between 1790 and 2010. To address difficulties in the trend and cycle decomposition, we introduce a semiparametric estimation approach with an iterative plug-in (IPI) algorithm for endogenous bandwidth selection. This algorithm identifies continuously *Moving Growth Rate Trends* (MGT) with a time length of 24 years. Afterwards, nonlinear SETAR models are fitted parametrically. Additionally, we test the trend in a recent developed test and the estimated SETAR models against their linear alternatives. The results indicate asymmetric characteristics during booms and busts.

JEL classification numbers

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Keywords

Business Cycles, Trend and Cycle Decomposition, Semiparametric Estimation, SETAR Model

I. Introduction

“The business cycle is a fundamental yet elusive concept in macroeconomics” (Morley and Piger; 2012: 208). Hence, measuring business cycles and their properties has gained major attention over the last century as the precise measurement and understanding of business cycles allows for accurately-timed policy actions. However, the main problem of business cycle analysis lies in the interaction between trend and cycle, thus the trend and cycle decomposition plays a crucial role for analyzing further properties of the business cycle. The decomposition challenge leads to different results for business cycle properties which are sometimes described as asymmetric (Sichel (1993), Peiró (2004)). To clarify the asymmetry hypothesis for the US, we argue that a nonparametric estimation of the trend results in an appropriate decomposition and eases an analysis of business cycle properties. Therefore, we use the extended iterative plug-in (IPI) algorithm to determine the optimal nonparametric trend in the mean squared error sense. Furthermore, we introduce a test which demonstrates that the data-driven nonparametric trend is superior to the linear trend. Moreover, the test can be augmented to show that the nonparametric estimation approach is able to overcome some disadvantages of the Hodrick-Prescott filter, e.g., the selection of the smoothing parameter, which is chosen arbitrarily, and other well-known filtering methods. The nonparametric estimation approach hence comprises the possibility of smooth changes in the trend, while overcoming the drawbacks of the linear trend and the Hodrick-Prescott filter. A more precise trend and cycle decomposition is therefore vital to further analyze whether business cycles can be described as a gradual increase during expansion phases that is followed by a large and sudden decrease during a downturn. Moreover, this precise trend estimation is economically meaningful and leads to the first identification of continuously *Moving Growth Rate Trends (MGT)*, which are stable around a period of 24 years, but smoothly changing over the whole observation period. These stable trend segments will be called

continuously *MGT* with an endogenously determined time length. Hereby we suggest an alternative to the standard overall detrending models. Having adjusted the data by the estimated nonparametric trend, any model can be fitted parametrically to the residuals. The further analysis focuses on the nonlinear SETAR (*Self-Exciting Threshold Autoregressive*) model, which provides conclusive evidence that recession phases have different characteristics than expansions leading to asymmetric behavior over the cycle.

II. Related Literature

The crucial point in identifying business cycles is the *decomposition* of macroeconomic time series into long-term dynamics and short-term fluctuations. Therefore, we follow the definition of Morley and Piger (2012), who divide economic activity into long-run growth, the business cycle, and seasonal patterns. The “output-gap” definition of cycles defines the business cycle as “transitory fluctuations of the economy away from a long-run trend” (Morley and Piger; 2012: 209) level, which is not necessarily independent of long-run growth. Thus, trend and business cycles interact. As noted by Morley and Piger (2012), the estimated business cycle depends on the model specification, as different methods lead to different trend and cycle decompositions because there is no generalized method of separating both components. In order to introduce a general trend and cycle decomposition method, we propose a nonparametric approach. Zarnowitz and Ozyildirim (2006) summarize that a long phase of stagnating business conditions can weaken long-term trends and that a long period of maintained high growth can reduce cyclical instability. Hence, the detrending method is crucial and lays emphasis either on the trend or the cyclical component. Beveridge and Nelson (1981) define business cycles as transitory, stochastic fluctuations around a trend and use a decomposition method based on long-horizon forecasts. Nelson and Plosser (1982) test whether macroeconomic time series are better

characterized as stationary fluctuations around a deterministic trend or alternatively as non-stationary fluctuations without the tendency to return to a mean or trend level. They find no evidence against the hypothesis that these time series are stochastic processes that have no tendency to return to a trend and argue that GNP is the sum of a stochastic trend that shifts every period (unit-root component) and a transitory term. In contrast, Clark (1987) concludes that it is questionable to treat economic activity as a random walk with drift and argues that economic activity comprises two independent components, a flexible trend and a stationary cycle. Furthermore, Cochrane (1988) shows that real GNP per capita has a deterministic trend, which has no permanent or random walk component and whose fluctuations are temporary. Mayoral (2006) summarizes these properties for real GNP data. Perron and Wada (2009) as well as Luo and Startz (2014) conclude that the trend function is piecewise linear and, except for a change in the slope in 1973, that the trend of real US GDP is deterministic. Besides the linear trend often used in macroeconomic analysis, Hodrick and Prescott (1997) introduce a filter where the observed time series is defined as the sum of cyclical and growth components, wherein the growth component varies smoothly over time. Although the Hodrick-Prescott filter is often used in business cycle analysis, according to Ravn and Uhlig (2002) some drawbacks exist as its cyclical estimator behaves poorly at the boundary points and its smoothing parameter λ is chosen arbitrarily. We take up this criticism and choose a method with an appropriate criterion for determining smoothness. Thus, we use a nonparametric trend estimation which is locally optimal. Moreover, the nonparametric method allows for a determination of the bandwidth for MGT.¹ The problem of different decompositions using different methods is addressed in Morley, Nelson and Zivot (2003) as well as in Perron and Wada (2005), who argue that these puzzling features are due to changes in the slope (structural breaks), rendering a linear trend unsuitable.

¹ For detailed information the reader is referred to Beran and Feng (2002) and Feng et al. (2015).

Zarnowitz and Ozyildirim (2006) require that a good trend “should be influenced by the cyclical movements in the data but it should also be smooth” (Zarnowitz and Ozyildirim; 2006: 1732). Morley and Piger (2008) present an extension of the Beveridge-Nelson decomposition for integrated time series that allows for shifts in the average growth rate. To allow for changes in the average growth rates and fulfill the requirements of Zarnowitz and Ozyildirim (2006) as well as to overcome the poor behavior of the Hodrick-Prescott filter at the boundary points, we introduce a smooth trend into the data and propose estimating this trend using a data-driven local linear estimation approach. Thus, we estimate the trend nonparametrically without making any assumptions concerning its functional form. Instead of choosing any smoothing parameter arbitrarily, the data-driven approach selects a bandwidth by minimizing the mean integrated squared error (MISE) criterion. A well-known procedure for selecting the bandwidth in a nonparametric regression with independent errors is the IPI algorithm proposed by Gasser et al. (1991), which is adapted by Beran and Feng (2002) to nonparametric regression with long- or short-range dependent errors. We develop a more flexible IPI algorithm for bandwidth selection in nonparametric regression for long macroeconomic time series without parametric assumption on the errors. The global optimal bandwidth for estimating the scale function $g(\tau)$ is that which minimizes the dominant part of the MISE of $g(\tau)$. Hence, the estimated trend is optimal in the mean squared error sense; also, it avoids possible misspecification as no functional form is supposed. As a result, the identification of the trend using a nonparametric method overcomes the difficulties of other trend estimation approaches as it allows for smooth changes in the dynamic of long-term growth. In line with the idea of dynamic waves as described by Kuznets (1930) or Kondratjew (1926), those MGT have a time length of 24 years. The general long-term trend over the complete 221 years observation period can be described as somewhat similar.

Thus, the appropriate estimation of the trend component enables us to filter a general cyclical component.

The notion of *business cycle asymmetry* traces back to Mitchell (1927) and Keynes (1936) describing cycle asymmetries in the sense that recessions are more volatile and have a higher amplitude than expansions which last for a longer period. Burns and Mitchell's (1946) concept suggest that the duration and slope of expansions are not symmetric with contractions. In order to analyze business cycle asymmetries using statistical methods Neftçi (1984) studies the nonlinearity of the US unemployment rate using finite-state Markov processes. He suggests that the probabilistic structure of macroeconomic time series may be different during upswings and downswings. The distinction between different types of asymmetry is proposed by Sichel (1993) and Sichel (1994) who differentiates steepness- and deepness-type asymmetries which can exist either separately or simultaneously. Steepness occurs when contractions are steeper than expansions, meaning that expansions are more gradual. Therefore, sharp decreases should be larger, but less frequent than increases. Deepness occurs when the troughs are deeper than the peaks are tall, hence fewer observations are below its trend than above. Ramsey and Rothman (1996) as well as Jovanovic (2006) extend these definitions by the time irreversibility of steep asymmetry, where the growth rates are negatively skewed and the time reversibility of deep asymmetry, where the frequency distribution of the time series is negatively skewed. Peiró (2004) generalizes these two types of asymmetries into asymmetries in the unconditional density function and asymmetries in the conditional density function. Thus, unconditional asymmetry is a formalization of the concept of transversal asymmetry, while conditional asymmetry formalizes the concept of longitudinal asymmetry. The consensus across all definitions is that the dynamics imply that expansions are not symmetric with recessions. To explain irregular

dynamics in business cycles, Ramsey and Rothman (1996) combine the dynamic concept of time reversibility and time irreversibility with business cycle asymmetry. Therefore, longitudinal asymmetry is captured by time irreversibility and purely transversal asymmetries are time reversible. Their results indicate that many macroeconomic time series and fluctuations are time irreversible, showing that some components of the business cycle are asymmetric. Morley and Piger (2012) address the topic of asymmetry from the perspective of nonlinear time series models, noting that to analyze business cycle asymmetry the specification of the model for the cyclical component is crucial. Linear models imply symmetric fluctuations around a trend and nonlinear, e.g., regime switching models, imply asymmetric deviations away from the trend. Nevertheless, there is no conclusive evidence on the appearance and behavior of business cycles. However, Morley and Piger (2012) argue that the model specification is not unambiguous, but that there is evidence in favor of nonlinearity. The most commonly used models that fulfill these nonlinear properties are Markov switching models (Hamilton (1989)) and TAR (Threshold Autoregressive)-type models, introduced by Tong (1978). In accordance with Tiao and Tsay (1994), linearity is rejected against threshold autoregression, hence we use the TAR model, which is able to present different phases over the business cycle, in the further analysis. Among others, Teräsvirta and Anderson (1992) as well as Potter (1995) use different versions of the TAR model to analyze business cycle asymmetry. Since Clements and Krolzig (2004), multivariate models have gained attention because they include co-movements between variables over the cycle. Koop and Potter (2006) introduce the Vector Floor and Ceiling (VFC) model as a parsimonious multivariate extension of the TAR model, combining VAR (Vector Autoregressive) and TAR models. Milas et al. (2007) give an overview of the nonlinear time series analysis of business cycles. In combination with the nonparametric trend estimation, which enables us to yield an appropriate trend and cycle decomposition, the residuals of a

univariate series are used to clarify the ambiguous results of Morley and Piger (2012) concerning business cycle asymmetry. They postulate that nonlinear regime-switching models are slightly preferred, hence the nonlinear SETAR model is fitted to the data in order to demonstrate asymmetric business cycle features. It is important to note that every model can be fitted parametrically to the residuals and that our approach allows for an extension to multivariate models.

The *contribution* and main novelty of this paper is that it applies the rather developed semiparametric estimation approach, referred to as a semi-SETAR (semiparametric SETAR) model, to macroeconomic data covering a time span of over 220 years. This data set allows for a long-term analysis and provides information on changes in the frequency of business cycles. In accordance with Proietti and Luati (2008), we use a nonparametric estimation for the long-term trend of macroeconomic data to overcome the difficulties of well-known filtering methods. Moreover, the nonparametric method does not require a predetermination of the frequency of a cycle, in addition this method is useful to determine continuously MGT. We then test this local optimal trend against a linear alternative using a recently developed test method, finding that the data-driven nonparametric trend is superior to the linear trend. The standardized data provide a model-free basis for further analysis and are used to study possible asymmetry in business cycles. Thus, we fit a SETAR model to the standardized data which demonstrates a significant gradual dependence during expansions and a sudden and deep decrease during downturns. Hence, asymmetry is present over the business cycle since there are differences in the dependence structure. To show that this nonlinear regime-switching model is preferred against a linear model within the SETAR class of models, we present Hansen's (1996, 1997) extended F-test. The results show that the nonlinear semi-SETAR model is preferred over the linear semi-

AR model. This leads to new empirical evidence with the first use of the semiparametric estimation approach, as the dependence in the high regime is stronger than in the low regime, indicating that expansions are more gradual than recessions. Thus, recessions are more stochastic and comprise a sudden and deep downturn. As a consequence, business cycles cannot be described as symmetric waves around a trend. In line with Sichel (1993), steepness and deepness type of asymmetry are present over the business cycle, which is time irreversible. These findings have important policy implications and represent convincing evidence in support of the asymmetric hypothesis that was first noted by Mitchell (1927) and Keynes (1936).

III. Data and Methodology

Data

In order to analyze whether asymmetry is a possible feature of business cycles, we use US GDP data covering the period from 1790 to 2010 extracted from Johnston and Williamson “What Was the U.S. GDP Then?”, Measuring Worth 2011.² The analysis focuses on US data, because they are identified as the most undistorted data as they are less affected by wars than those of other countries. The annual real US GDP per capita is quoted in 2005 US dollars. The natural logarithms (ln) of the data are calculated to handle the exponential growth and stabilize the variability of the series. Furthermore, this simplifies the calculation of growth rates, which we use, in accordance with Potter (1995), in the further analysis to distinguish between expansion and recession phases. For the semiparametric estimation approach, in a first step the trend is estimated and removed from the original data. In a second step the growth rates of the residuals are calculated. Finally AR and SETAR models are fitted parametrically to the standardized growth rates.

² The database is at <http://www.measuringworth.org/usgdp/>.

Local Linear Estimation of the Trend Function

Some properties of the well-known parametric time series models make them unsuitable for the application to real data in practice. The problem of macroeconomic data lies mainly in the existence of long-term dynamics over a long time-span. By not taking this trend into account, the quality of the fitted model is impaired due to biased estimators. To examine long time series, the component model is the main way to overcome this difficulty since it decomposes a time series into (unobserved) components. Thus, the component model is extended to a semiparametric time series model of the following form. Let X_t be a sequence of macroeconomic time series with time $t = 1, \dots, T$. An additive component model, in accordance with Beran and Feng (2002), is given by

$$X_t = g(\tau_t) + Y_t, \quad (1)$$

where $\tau_t = \frac{t}{T}$ denotes the rescaled time, $g(\tau)$ is some smooth function and Y_t denotes a stationary time series. If the additive trend component $g(\tau)$ is a constant, the semiparametric model reduces to any standard time series model. If $g(\tau)$ is not constant, the time series observations following equation (1) are modeled via a two-step procedure, where in the first step the long term trend is removed and the trend adjusted data (residuals) are obtained by

$$\hat{Y}_t = X_t - \hat{g}(\tau_t). \quad (2)$$

In a second step any time series model can be fitted to the residuals $\hat{Y}_t$ parametrically via (Q)MLE or OLS methods. Hence the benefits of the semiparametric estimation approach become obvious as the trend estimation is a previous step for the further analysis of macroeconomic phenomena besides economic growth. The essence of nonparametric methods is smoothing. The procedure described in the following is based on Beran and Feng (2002). The authors propose a data-driven local polynomial estimator for the trend function in time series with short- or long-

range dependence using an IPI algorithm for selecting the optimal bandwidth. We extend this approach with short-range dependence only for the application to macroeconomic time series. In accordance with their idea, the v -th derivative of $g(\tau)$, defined as $g^{(v)}(\tau)$, can be estimated by minimizing the locally weighted least squares

$$Q = \sum_{t=1}^T \{x_t - \alpha_0(\tau) - \alpha_1(\tau)(\tau_t - \tau) - \dots - \alpha_p(\tau)(\tau_t - \tau)^p\}^2 K\left(\frac{\tau_t - \tau}{b}\right), \quad (3)$$

where $K(u)$ is a kernel function and b is the relative bandwidth, respectively. K and b need to be selected in accordance with the data. The obtained estimates are $\hat{g}^{(v)}(\tau) = v! \hat{\alpha}_v$ and accordingly we have $\hat{g}(\tau) = \hat{\alpha}_0$. For our purpose, the local linear estimator with $p = 1$ and the optimal (Epanechnikov) kernel function is used. A crucial problem is to choose the optimal bandwidth b . In line with Gasser et al. (1991) and Ruppert et al. (1995), in this paper b is selected by minimizing the approximate mean integrated squared error (AMISE) given by

$$AMISE = b^4 \frac{I(g'')I^2(K)}{2} + \frac{2\pi c_f R(K)}{bT}, \quad (4)$$

where $I(g'') = \int [g''(\tau)]^2 d\tau$, $I(K) = \int u^2 K(u) du$, $R(K) = \int K^2(u) du$ and c_f denotes the value of the spectral density of Y_t at the origin. The asymptotical global optimal bandwidth b_A for estimating $g(\tau)$ is the one that minimizes the AMISE. As required by Morley and Piger (2012), our estimates of trend and cycle are optimal in the minimum mean squared error sense. For estimating $g(\tau)$ on $[0, 1]$, the formula for the asymptotical global optimal bandwidth is given by:

$$b_A = \left(\frac{2\pi c_f R(K)}{I^2(K) I(g'')} \right)^{1/5} T^{-1/5}. \quad (5)$$

These results correspond to those in equations (3.17) to (3.19) of Beran and Feng (2002) for local linear regressions with short-range dependent errors. The IPI algorithm of Beran and Feng (2002) based on (5) can be used for selecting b_A under the parametric assumption that Y_t is an

ARMA process. We propose selecting b_A without this parametric assumption, so that any suitable time series model can be fitted to the residuals, where c_f is estimated nonparametrically using the well-known lag-window method with the Bartlett-window. The modified IPI algorithm, which is not described in detail, consists of five steps which are repeated until convergence or the set maximal number of iterations is reached.

Following this algorithm, the global optimal (absolute) bandwidth for estimating the trend function with respect to the observation time points is $\hat{h}_{0,US} = [\hat{b}_A \cdot T + 0.5] = 12$, for the US GDP data, where $[\bullet]$ denotes the integer part. Using this optimal bandwidth, the long-term trend is estimated and the residuals are calculated following equation (2). Figure 1 shows the $\ln$ transformed US GDP data (labeled in red) together with the estimated local linear trend (labeled in black) and the linear trend (blue dashed line). The nonparametric estimation approach appears to fit the data of long term dynamics well, resulting in a stable trend. That is, the estimated bandwidth fits the data well. Whereas a bigger bandwidth would smooth out some characteristic structures, a smaller bandwidth would give more weight to those structures, resulting in an increasing variance. In accordance with Zarnowitz and Ozyildirim (2006), the estimated trend is upward sloping and smoothly changing over time, making a linear model for trend estimation and a direct fitting of standard time series models unsuitable. An interesting point is that the bandwidth of 12 years in one direction results in a bandwidth of 24 years around one observation. This indicates different phases of economic growth and different economic conditions, which tend to last about 24 years, defined as continuously MGT. Hence, the growth process is stable within a middle term period of 24 years, but somewhat variable over the long-term. Although this interesting fact is in line with the idea of dynamic waves (Kondratjew, Kuznets), the dynamic is obviously nonlinear. In addition, Figure 1 suggests that the estimated trend displays asymmetric behavior. Thus, a deterministic asymmetric effect is present in the

analyzed data. That means that on average, more positive deviations or effects that are above any mean value exist, resulting in an increasing trend function. The typical increasing trend indicates Sichel's (1993) deepness type of asymmetry. In addition, the findings support those of Baudry and Koop (1993), who state that the effects of negative shocks are temporary and the effects of positive shocks are persistent. Moreover, changes in the slope of the estimated trend indicate the presence of structural breaks. Nonetheless, the identification of structural breaks is far beyond the scope of this analysis, which is in line with Morley and Piger (2012) who argue that permission for structural breaks does not resolve the sensitivity of business cycle model specification.

Figure 1: Linear trend and data-driven local linear trend for the LN-US GDP.

Important to mention is that the standardized data are obviously stationary as the ACF decays rapidly to zero as the lag increases, hence any well-known time series model can be fitted parametrically to the standardized residuals and their growth rates. In line with Morley and Piger (2012), business cycle asymmetry is defined as stochastic asymmetry in the residuals. The

standardized data in Figure 2 show that the deviations from the estimated trend change over the observation period. It is obvious that the deviations in the period from 1925 to 1950 are the largest. Furthermore, the amplitude seems to be larger in the prewar than in the postwar period. Thus, the mean value of a peak is smaller and the mean of a trough is higher in absolute values in the postwar period, resulting in a smaller amplitude. A business cycle tends to increase gradually, followed by a rapid decrease. A first hint of asymmetry for the US GDP residuals is displayed in Figure 2, where Sichel's (1993) steepness type of asymmetry is indicated, occurring when contractions are steeper than expansions, meaning that expansions are more gradual and sharp decreases should be larger, but less frequent than increases. In addition, the average difference between zero and a peak is 0.02306 and the average difference between zero and a trough is 0.02622 in absolute values for the postwar period. Thus, deepness type of asymmetry is demonstrated in the data while on average troughs are deeper than peaks are tall, hence fewer observations are below its trend than above. This definition is commonly used to identify business cycle dates, which can be detected in Figure 2 a). A business cycle for the US GDP series tends to last about seven years, which is obvious especially for the residuals of the postwar period. This is in line with the NBER Business Cycle Dating Committee that defines average cycles of about six years and lies within the range of maximum cycle duration (42 quarters) of Everts (2006). Nevertheless, the differences between the NBER cycles and our cycles are due to the differing frequency and definitions. Whereas the NBER follows the "alternating phases" definition (Morley and Piger (2012)), our definition is that a peak is a local maximum that belongs to the 15% largest positive deviations and no other peak of higher magnitude is present in the vicinity of two years. Moreover, our findings are in line with recent results of Jordà et al. (2011), who argue that long time series display a complex picture of cycles due to different pre- and postwar dynamics.

Figure 2: Residuals of the local linear regression and growth rates of the residuals.

Test of the Linear Trend against the Local Linear Growth Trend

To test the linear trend against the local linear trend, the idea is to test the null hypothesis H_0 : the trend is linear with a constant slope β against the alternative hypothesis H_1 : the trend is local linear with a time-varying slope β_t . The test is carried out using two methods. Method A is based on the differences between the two estimated trends and their confidence intervals and Method B is based on the differences of the estimated slopes and their confidence intervals. Let $\hat{g}_L(\tau)$ denote the simple linear regression estimate of g . H_0 will be rejected if clearly more than 5% of the confidence intervals for $\hat{g}(\tau) - \hat{g}_L(\tau)$ (or $\hat{\beta}_\tau - \hat{\beta}$) do not intersect with zero. Figure 3 clearly shows that more than 5% of the confidence intervals do not intersect with zero for the US GDP. These observations indicate that economic activity and economic conditions have changed over the centuries; the results confirm that the trend is stable over 24 years. In addition, the differences between the local linear slopes and the constant slope are tested. Thus, the

differences between the first derivatives are significantly different if more than 5% of the confidence intervals do not intersect with zero. It is obvious from Figure 3 c) that more than 5% of the intervals do not intersect, therefore the differences between the local linear slopes and the constant slope are significant in the sense that the slope is not a constant. Hence, the hypothesis that the trend is linear with a constant slope is rejected in favor of the nonparametric trend with time-varying slopes. It is important to mention that the differences between the local linear slopes and the constant slope obviously change over time. After 2000 the differences decrease slowly and seem to become negative, which indicates an unprecedented change. Again these findings show that economic conditions for the US change significantly over the observation period. However, the nonparametric estimation fits the data better, taking the complete observation period into account. Furthermore, the findings are in line with Perron (1989) as well as Perron and Wada (2009) since they indicate a reduction in mean growth rates. To summarize the results, it is shown that the trend estimation method is crucial as it has a large impact on the further analysis. Moreover, the test results show that the trend behaves differently during different periods due to changes in economic activity, which are significant over a period of 24 years. Therefore, there is no single and stable long-term trend for the complete observation period. This is in line with the general idea of dynamic waves. Thus, the relevant period for decision-making seems to be within this timespan. In addition, the local linear approach and the Hodrick-Prescott trend yield quite similar results. However, the data-driven local linear regression improves the estimation results at the boundary and chooses the bandwidth in a statistical appropriate way. Hence, the data-driven local linear estimation approach is used to estimate the long-term trend of the macroeconomic time series as it is optimal in the mean squared error-sense.

Figure 3: Test of the linear trend against the local linear trend for the LN-US GDP.

Fitting and Testing semi-SETAR(k,p,d) Models

Next, semi-SETAR(k,p,d) models are fitted to the growth rates of the standardized US GDP series. In this section the residuals are simply denoted Y_t . The rather improved tsDyn package of di Narzo (2015) is used. This package was developed in order to improve the estimation of nonlinear as well as nonparametric time series models. The piecewise linear TAR(k,p,d) model, introduced by Tong (1978), allows for k different regimes. Hence, the model is a nonlinear

extension of the AR(p) model and allows for different regimes during expansion and downturn phases. Furthermore, it is able to present asymmetric adjustment over the business cycle. Teräsvirta and Anderson (1992) use a smooth version of the TAR model for characterizing nonlinearities in business cycles. Potter (1995) argues that the nonlinear SETAR model outperforms standard linear models by investigating US GNP data. In accordance with Tong (1983), a TAR model is referred to as a *Self-Exciting* TAR (SETAR) model if it uses its own lagged value Y_{t-d} as a threshold variable. It generalizes to the TAR model by allowing any constant value c_t for the threshold variable. A time series Y_t follows a k -regime SETAR(k,p,d) model if it satisfies

$$Y_t = \phi_0^{(j)} + \phi_1^{(j)}Y_{t-1} - \dots - \phi_p^{(j)}Y_{t-p} + a_t^{(j)}, \quad \text{if } \gamma_{j-1} \leq Y_{t-d} < \gamma_j, \quad (6)$$

where $k > 0$ and $d > 0$, $j = 1, \dots, k$, Y_{t-d} is the threshold variable and γ_i are real numbers such that $-\infty = \gamma_0 < \gamma_1 < \dots < \gamma_{k-1} < \gamma_k = \infty$. The superscript (j) is used to signify the regime and $a_t^{(j)}$ are i.i.d. random noise sequences with zero mean and variance σ_j^2 , which are mutually independent for different regimes j . The parameter d is the delay parameter and γ_j denotes the thresholds. The threshold parameter for the application to real macroeconomic data is determined in accordance with Potter (1995) and Hansen (1997). Potter (1995) uses zero as a threshold. Hansen (1997) estimates a threshold near zero, allowing for changes in the autoregressive structure in expansions relative to contractions. Therefore the threshold variable is set to $Y_{t-d} = 0$, resulting in $k = 2$ different regimes. These regimes are labeled low regime (LR), if $Y_{t-1} \leq 0$, indicating a non-positive growth rate, and high regime (HR), if $Y_{t-1} > 0$, demonstrating a positive growth rate. The choice of the order of the lag is not obvious due to the unclear dynamic dependence structure. Therefore $p = 1, 2, 3$ are tested under the assumption that the AR order is

equal for different regimes and the results compared. The estimated coefficients and the corresponding standard errors are displayed in Table 1.

TABLE 1

Estimated model parameters for the SETAR(k,p,d) model

<i>US GDP</i> <i>Coeff.</i>	<i>SETAR(2,1,0)</i>		<i>SETAR(2,2,0)</i>		<i>SETAR(2,3,0)</i>	
Regime	LR	HR	LR	HR	LR	HR
	48.62%	51.32%	48.62%	51.38%	48.85%	51.15%
$\hat{\phi}_0$	-0.0048 (0.0053)	-0.0006 (0.0056)	-0.0047 (0.0053)	-0.0003 (0.0056)	-0.0041 (0.0053)	-0.0003 (0.0056)
$\hat{\phi}_1$	0.1288 (0.1194)	0.3072** (0.1461)	0.1626 (0.1227)	0.3180 ** (0.1466)	0.1618 (0.1219)	0.3046** (0.1473)
$\hat{\phi}_2$			-0.1196 (0.1003)	-0.0777 (0.0931)	-0.0723 (0.1027)	-0.0602 (0.0961)
$\hat{\phi}_3$					-0.1892* (0.0980)	-0.0661 (0.0953)
$\hat{\phi}_1^1 \hat{\phi}_1^2$	0.0396					
$\hat{\sigma}^2$	0.0016		0.0016		0.0015	
AIC	-1409		-1407		-1408	

Notes: Model parameters, standard errors in parentheses and p-values, *p(z)<0.1, **p(z)<0.05, ***p(z)<0.01.

It is obvious that the threshold variable $Y_{t-d} = 0$ separates the US GDP data into two equally spaced regimes as around 49% of the observations are in the low and 51% of the observations in the high regime. The results show that the low and the high regime are different and not symmetric. The differences in the low and the high regime of the fitted models for the US GDP

may indicate asymmetry. Thus, i.e. in the SETAR(2,1,0), the values for the low regime do not depend significantly on each other, whereas the value of Y_t in the high regime depends significantly on Y_{t-1} . If Y_{t-1} is negative, it needs multiple time indexes to increase to a positive value. The estimated coefficient for $Y_{t-1} > 0$ is $\hat{\phi}_1 = 0.3071$, hence Y_t tends to remain positive for a few periods before it changes to negative. This verifies a deterministic, slow, and gradual increase during expansions, which corresponds to the high regime. There is no significant dependence in the low regime, indicating stochastic behavior during downturns. Consequently, the US GDP series is not time reversible and show Sichel's (1993) steepness and deepness types of asymmetry. In summary, the dynamic dependence in the underlying data appears to be stronger when the value is in the high regime. Another indicator in favor of asymmetry is the change in the sign of the coefficients when a second and third lagged variable are introduced, but this is less obvious due to non-significant estimates for higher lags, except $\hat{\phi}_3$ in the low regime. Nonetheless, the differences of SETAR models with different orders and the general additional power of nonlinear SETAR models for explaining business cycle asymmetry are not unambiguous. After linear AR models, which are displayed in the appendix, and nonlinear SETAR models are fitted to the growth rates of the standardized US GDP data the generalized F-test of Hansen (1996) is applied in order to test the improvement of nonlinear alternatives.³ Testing for linearity within the SETAR class of models is a test of the null hypothesis of a SETAR(1) model against the alternative hypothesis of a SETAR(k) model. The test statistic and the corresponding p-values for the standardized US GDP series displayed in Table 2,

³ Furthermore, AR models are fitted to the data set of Morley and Piger (2012) using our nonparametric regression and compared to their estimated AR models to further check model robustness. The results indicate that we are able to improve Morley and Piger's (2012) model quality due to the local linear trend estimation.

demonstrate that the different models are in general not significantly different at the 5% critical value. Consequently, the parsimonious AR(1) model may be preferred, indicating that a nonlinear SETAR model with 2 or 3 regimes (allowing for recovery phases) provides no additional explaining power. Nevertheless, relatively small p-values indicate that there may be a difference in the models.

TABLE 2

Test results for different SETAR models

<i>US GDP</i>		
<i>Number of regimes</i>	<i>Test statistic</i>	<i>p-value</i>
1 vs 2	11.0649	0.1650
1 vs 3	23.0039	0.1480
2 vs 3	13.3365	0.2020

In addition to the test results, the AIC values for the nonlinear SETAR models fitted to the US GDP residual growth rates are smaller than those of the fitted AR models. The fitted SETAR models are hence preferred over the AR models. This is evidence of the identified hints for asymmetric cycles in the US GDP, which can be described by a nonlinear model. This is in line with the results of Morley and Piger (2012) as well as Lopes and Zsurkis (2015), who argue that linear and nonlinear models are able to produce business cycle features. However, it is worth mentioning that the permission for nonlinearities in TAR types of models avoids a possible misspecification of linear models. Hence nonlinear models are more suitable to describe business

cycle asymmetries. The results argue in favor of asymmetric business cycles, hence expansions are more gradual and followed by a stochastic as well as a deep decrease. Connecting these results with macroeconomic policy, the position on the business cycle has strong implications for policy decisions. Our findings may also have direct implications for growth and business cycle models with respect to the literature on financial crisis. Furthermore, they are important for investors as they could align their decisions with economic conditions.

IV. Conclusion

This paper extends the trend and cycle decomposition by a nonparametric estimation approach of the trend function. We introduce an extended IPI algorithm for selecting the optimal bandwidth for the continuously MGT and apply it to low-frequency macroeconomic data covering an extensive time span of over 220 years. The estimated trend indicates that economic conditions change smoothly, as periods with a stable trend over 24 years exist within the variable long-term trend which covers the whole observation period. Thus, we argue that a time series does not have the one trend. Hence, a macroeconomic time series may be disassembled into continuously MGT of endogenous time length. This time length is estimated to be 24 years. In order to show that this local optimal trend is superior to the linear trend, we test both trends in a newly developed test. Consequently, the nonparametric estimation allows for a more precise separation of trend and cycle and yields an appropriate basis for further analysis. In a next step, the standardized US GDP data are obtained by subtracting the estimated trend from the original data. After the growth rates of these residuals are calculated, linear AR and nonlinear SETAR models are fitted parametrically to the trend adjusted growth rates. The robust results show that a nonlinear semi-SETAR model is preferred for the US GDP data as it allows for asymmetric adjustment during booms and busts. Thus, business cycles display asymmetric behavior in the sense that deterministic expansion phases are more gradual than sudden stochastic decreases in recession

phases. In line with the definition of Sichel (1993), steepness and deepness types of asymmetry are identified in the US GDP residual growth rates. Consequently, the US economy behaves differently during different phases over the business cycle, which according to Ramsey and Rothman (1996) is time irreversible and asymmetric. Thus, it is shown that a deterministic structure in the residuals exists, especially in the high regime and that the business cycle can be described as asymmetric. Moreover, the results of Morley and Piger (2012) are improved by the more appropriate decomposition which enables us to clarify that nonlinear models are preferred over their linear alternatives. Directions for future research include the extension to a multivariate context as well as the refinement of the IPI algorithm and the linearity test in order to compare the nonparametric trend with other possible trend specifications.

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Appendix A

TABLE 3

Estimated AR(p) models for the standardized US GDP growth rates

<i>Series</i> <i>Coeff.</i>	<i>US GDP</i>				
<i>Model</i>	<i>AR(1)</i>	<i>AR(2)</i>	<i>AR(3)</i>	<i>AR(4)</i>	<i>AR(5)</i>
$\hat{\phi}_0$	0.0002 (0.0036)	0.0002 (0.0032)	0.0002 (0.0028)	0.0001 (0.0023)	0.0001 (0.0020)
$\hat{\phi}_1$	0.2427*** (0.0652)	0.2668*** (0.0670)	0.2541*** (0.0667)	0.2271*** (0.0659)	0.2000*** (0.0667)
$\hat{\phi}_2$		-0.0985 (0.0672)	-0.0644 (0.0690)	-0.0783 (0.0677)	-0.0887 (0.0673)
$\hat{\phi}_3$			-0.1284* (0.0669)	-0.0774 (0.0675)	-0.0880 (0.0671)
$\hat{\phi}_4$				-0.2059*** (0.0663)	-0.1771*** (0.0674)
$\hat{\phi}_5$					-0.1307* (0.0671)
$\widehat{\sigma^2}$	0.0016	0.0016	0.0016	0.0015	0.0015
Log-likelihood	395.36	396.43	398.25	402.96	404.84
AIC	-784.60	-784.67	-786.23	-793.53	-795.15
BIC	-774.53	-771.28	-769.54	-773.56	-771.93

Notes: Model parameters, standard errors in parentheses and p-values, *p(z)<0.1, **p(z)<0.05, ***p(z)<0.01.

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