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Abstract: Though violent conflicts bear dramatic uncertainties, there is no closed theory addressing large uncertainties in conflict decisions. If a violent conflict is an investment in social, political, or economic change for rebels, how do large uncertainties like threatening waves of persecution affect their decision to turn violent? In order to capture such discontinuous uncertain large stochastic shocks we apply a real option model and introduce a more general stochastic process than the often used geometric Brownian motion, namely an Ito-Lévy Jump Diffusion processes. A major result is that large uncertain shocks have opposite effects on the decision to launch a conflict than small variations, indicated by normal volatility. While an increase in volatility of e.g. general economic conditions, indicated by (marginal) volatility, would give hope and postpone a potential attack, more uncertain large threats, indicated by discontinuous negative shock, will lead to an earlier outbreak of conflict. Hence, in latent conflicts, either general disaster shocks, or even deliberately announced oppressive threats by a government, often meant to awe the oppressed group, may provoke a violent outbreak, if the threat is strong enough.

JEL classifications: D74, D81, C61

Keywords: theory of social conflict, decision under uncertainty, non-marginal stochastic shocks

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1 Introduction

Almost no decision problem bears more dramatic and large uncertainties than the decision to launch a violent conflict. While empirical studies on the causes and consequences of social conflicts are manifold, consistent closed formal theories addressing uncertainties in the decision to launch a conflict remain rather limited,[1] on both the macro and the micro level.[2] Hence, in this paper we ask how uncertainty affects the decision to launch a conflict. Using option theory we can distinguish two types of uncertainty. For a rebel, an increase in marginal uncertainty which can be connected to typical fluctuations in the economy may give hope and lead them to postpone violent action. In contrast, an increase in massive threats and major uncertain events, such as extensive oppressive government actions, will have the opposite effect and encourage an earlier outbreak of conflict. The implications are straightforward: conditions, whether exogenous or deliberately provoked, that generate large uncertainties affect conflict decisions and the duration of a non-violent period that could be used to find a long-term solution to the conflict.

In the literature it is well recognized that turning to violence can be a rational choice for, e.g. an oppressed group[3]. In particular, discrimination, repressive government action, waves of political persecution, arbitrary detention, torture of group members, and bad living conditions may cause frustration and lead to violent action of all kinds. Launching a violent conflict may become an instrument for improving conditions for the rebel or their social group (Tornell, 1998; Gould and Klor, 2010), or to maintain or acquire power (Besley and Persson, 2011). As Blomberg et al. (2004) and Caplan (2006) claim, such conflicts may

[1] See the survey by Sandler and Enders (2004) and Blattman and Miguel (2010).

[2] For the case of conflicts with strategic interacting groups such as well-organized rebel group fighting to overthrow the government, game theory approaches show that conflicts can have economic roots and may be the result of strategic interactions between conflict partners. In particular, conflicts between two competing agents occur due to a lack of property rights (Skaperdas, 1992), incomplete and insufficient information about the other party's relative military power (Fearon, 1995; Yared, 2010), or due to the agent's inability to estimate the opponent's ability to win (Powell, 2002, 2006). These scenarios seem suitable for explaining certain kinds of social conflict, yet they shed light only on explicit bilateral interaction with mutual strategic behavior of clearly defined and strictly controlled conflict parties, such as a well-organized, homogeneous group of rebels engaged in a game with the government. In this case, both parties know the possible set of actions before the opponent turns to violence, and take them into account.

[3] Empirical investigations assess that attacking in the context of terrorism is a rational choice. For instance, see Anderton and Carter (2005), Finkel and Muller (1998) and Weede and Muller (1998).

even be a rational outcome if there is no other way to bring about drastic institutional change. Even if the decision to attack is based on a comparison of the perceived benefits and costs of rebellion (Collier and Höffler, 1998), and even if the goal of maximizing income or welfare prevails (Grossman, 1991), it is highly uncertain whether keeping the peace for a while longer may also improve conditions. Similarly, it is highly uncertain whether turning to violence will improve conditions for the attacking group in the aftermath. Uncertainty can trigger violence, just as it can encourage the group to wait for an ad-hoc improvement in conditions.

First thoughts about uncertainty in theoretical modelling in this context come from Morrow (1985), who claims that war decisions are based on the actors' utility of uncertain outcomes. This approach is extended by Collier and Höffler (1998) and Besley and Persson (2011). However, the role of uncertainty in formal conflict theory is rather rudimentary, even if nothing is more likely to trigger a conflict than an unforeseeable major event, and nothing is more uncertain than the outcome of a conflict once it is triggered.[4] Empirical approaches, however, have started to assess how large - not only government-induced - uncertainties, such as disasters, affect e.g. terrorist activities. Major shocks cause additional uncertainty and may strongly affect or even help to overthrow existing conditions. They impact the motivation to become violent and can lead to an outbreak or the escalation of social conflict. For instance, Brancati (2007) shows that earthquakes may be a driving force for intrastate conflicts in that they increase competition for scarce basic resources. He also concludes that the effect is greater for higher-magnitude earthquakes that strike more densely populated areas of countries with lower gross domestic products and pre-existing conflicts. Berrebi and Ostwald (2011) extend this approach by taking into account both earthquakes and hurricanes and show that terrorism may be a consequence of natural disasters. Besides the effect of natural disasters, economic disasters such as the financial crisis may encourage terrorist activities due to their destabilizing effects (Gries and Meierrieks (2013)).

Hence, in this theoretical contribution we analyse how uncertainty created, e.g., by gov-

[4] For instance, in the aftermath of a conflict rebels may become heroes, or they are just as likely to be killed or their families exposed to even more severe repression.

ernments, natural disasters, or economic crises affect the assessment of conflict benefits and hence the outbreak of a conflict. In line with Blomberg et al. (2004) we illustrate our discussion using the example of uncertain government repressions that may lead to frustration and finally to the outbreak of conflict. In particular, the government may increase the intensity and severity of discriminating and repressing activities against a particular social group. Here, uncertainty created by the government is taken as given by the rebel, and we analyse its effect on the rebel's reaction. Is higher uncertainty a driving force for conflict escalation, or does uncertainty encourage rebels to remain peaceful? Does, e.g., a higher frequency of oppressive policies or more severe government action deter the rebel group, or does it encourage them to take action and change conditions through violent means?

To analyse the effect of uncertainty on the decision to turn to violence we choose a real option approach because it considers the value of (peacefully) waiting until (violent) action is taken. Since uncertainty is evaluated in the course of time we are able to find an optimal timing decision. We can distinguish two types of uncertainty, namely marginal risk that indicates the variability of general living conditions (which are sometimes better, sometimes worse), and non-marginal stochastic shocks for sudden major threats (more repressive actions).[5] More importantly, we show that these two types of uncertainty lead to opposite effects on the outbreak of conflict.

Why is this important to know? Because we can now evaluate how government actions, such as increasing threats, will stabilize or destabilize a latent conflict. While conventional option modelling[6] suggests that higher marginal risk, with the interpretation of more economic variations, encourages the rebel to remain peaceful for longer, stochastic large threats produced by the government bring the attack forward. Hence, the decision of whether and when to attack strongly depends on the frequency and size of uncertain events. More threats will abbreviate the expected peace period and reduce the chance of a peaceful long-term solution to the conflict.

[5] More precisely, as we use an Ito-Levy jump diffusion process that extends the Brownian motion by a jump component, we can analyse (i) marginal risk that refers to the variance in standard modelling of the Brownian motion, and (ii) the large uncertaint shocks caused by a frequency of jumps of unknown size.

[6] Here "conventional" means modelling using a Brownian motion.

2 Model

2.1 Model Idea

Many attacks are individual or small-group violent actions by agents. Irrespective of group dynamics or psychological, ethnic, or sociological reasons we focus on the idea that starting a conflict can be regarded as an investment in a better but uncertain future.[7] An attack is not something that unexpectedly enters the mind of a decision-making individual. Rather, it is the result of a dynamic process in which the current path of development of the economic and social situation (status quo) is evaluated and compared to the expected path of development after a potential attack, including conflict costs and all potential threats and opportunities (Grossman, 1991). If current conditions are dissatisfying, a latent conflict exists and violence may be considered. One of the potential outcomes of such a situation is an attack. For instance, a rebel who plans a terrorist attack will only carry out their plan if they expect to have an overall positive effect. If the assassination is successful, repression may stop and reforms could lead to more political participation or economic improvements. As rebels never know when and how strong repression will be, they take the uncertain environment created by the government as given and decide whether and if so when it is optimal for them to attack. However, even if an immediate attack may have some benefits, it is possible that a non-violent strategy comprising a potential later attack is the better option. Rebels act rationally (Caplan, 2006); they decide whether to invest immediately and pay the price by launching the conflict (attack), or to maintain the status quo, at least for a while. Since conditions are highly uncertain and may change even without an attack, sometimes a simple waiting period may be more beneficial. Hence, the hope of non-violent change or the expectation that a later attack (by someone else) is more beneficial may postpone the attack. As time goes on, rebels repeatedly consider their living conditions, which in bad times become worse, and they repeatedly decide whether to arrest this process of deterioration by violent means. As each moment's conditions determine this decision, it is a

[7] Besides the economic analysis of social conflicts there is an extensive discussion on psychological and sociological reasons for e.g. terrorist attacks. For instance, see Muller and Opp (1986) and Victoroff (2005).

sequential decision in time. The decision is also irreversible – once the attack is carried out, there is no turning back. All consequences have to be accepted, and the freedom and flexibility to choose more moderate strategies to solve the conflict are no longer present. Hence, with sequential decisions, high uncertainty, and irreversibility as major components of the decision problem, real option theory is an appropriate methodology. In order to capture large uncertain shocks we extend the conventional (Brownian motion) model by using an Ito-Lévy Jump Diffusion process.[8] Unlike the Brownian motion, which is the 'workhorse process' in real option theory, this more general process allows for evaluating parameters characterizing the frequency and the scale of major events as well as the regular risk measure which is the volatility of the geometric Brownian motion. With an Ito-Lévy Jump Diffusion process we are therefore able to analyse substantially different risk components of a stochastic process and we can show that they have opposite effects on the decision to turn violent. In this model the rebel maximizes their present discounted net value of the benefit of an attack (including the value of flexibility) by deriving a conflict threshold that determines the benefit level required to trigger the violent outbreak. Randomly reaching this threshold represents the straw that breaks the camel's back. The decision is determined by a sequential comparison of the net present value of the benefits of a potential attack with the value of postponing an attack to a later date. Having established this triggering threshold, we can also determine the 'first passage time,' that is, the expected time of attack. However, even if our sequential process identifies an expected time of conflict outbreak, the model is also able to suggest that a sudden random change in conditions may also lead to an unexpected attack at any moment.

2.2 Benefits of Conflict

As the attack is expected to change the social, economic, and political conditions created by the oppressing government, there are potentially two periods in conflict evaluation with two

[8] The importance of jump diffusions was first recognized in financial economics. For instance, Merton (1976) derived an option value of an European option similar to the Black Scholes formula. In the course of time some extensions of the Merton approach followed.

sets of conditions: first, the current period with a set of current conditions associated with a path of non-violent but dissatisfying development; and second, a new set of conditions in the period after the attack that are expected to generate a better path of development. Both elements determine the evaluation of total benefits of the conflict and eventually, the decision to launch it in order to generate a structural break in living conditions of the rebel or their group.

Current Conditions and Path of Development: In this model current conditions lead to a time path that is not satisfying for a certain social group. A harsh set of conditions for this group provokes resistance and a start of a latent conflict between an oppressive government and rebels. Increasing repression, worsening economic restrictions or discrimination, growing inequality of opportunities, and an increasing threat of persecution may lead to greater frustration in the face of deteriorating opportunities, and will eventually increase the propensity to turn violent. Further, with each additional moment of waiting and not attacking, the worsening welfare of the rebels may generate an increasing current benefit of conflict by a rate of $\delta > 0$. In other words, as the attack is a potential action, deteriorating living conditions increase the benefits of an attack. The expectation of a deteriorating current time path of welfare produces a sufficiently bleak outlook as to make an attack increasingly beneficial. However, there are marginal fluctuations due to small variations in the economic and political situation that are described by the volatility $\sigma_1 > 0$. In addition, government's repressive actions are highly uncertain to the rebel and may lead to dramatic and non-marginal change in conditions from one moment to the next. These non-marginal stochastic shocks have an intensity $\lambda_1 > 0$ and increase the benefits of conflict by a random amount $u \in U_1 \subset \mathbb{R}_+$. In particular, major disastrous events have negative effects on rebel's welfare and prospects, so that the current benefit of conflict may suddenly increase significantly. In order to consider these fundamental threats or great opportunities, we describe the development of current benefits of a potential attack during the period before the conflict is launched as an Ito-Lévy Jump Diffusion process.[9]

Definition 1 *Let $U_1 \subset \mathbb{R}_+$ be a Borel set whose closure does not contain 0. Further let W_1 be a standard Wiener process, N_1 a Poisson process with intensity λ_1 and constants $\delta, \sigma_1 \in \mathbb{R}_+$. Then the Ito-Lévy Jump Diffusion process $\tilde{Y}$, indicating the benefits of a potential attack at time T during the period before the conflict, is defined by means of the following differential equation*

$$\begin{aligned} d\tilde{Y} &= \delta\tilde{Y}dt + \sigma_1\tilde{Y}dW_1 + \tilde{Y}\int_{U_1} uN_1(t,du) \qquad (1)\\ \text{for} \quad 0 &< t < T \quad \text{and} \quad \tilde{Y}(0) = \tilde{Y}_0. \end{aligned}$$

This stochastic process includes a continuous and a discontinuous part through a combination of a geometric Brownian motion and a compound Poisson process, which models exceptional stochastic events through the integral $\int_{U_1} uN_1(t,dz) > 0$. The integrand u denotes the step height of jumps which is uncertain but limited by U_1.[10] Note that during calm periods where the government does not undertake any large repressive actions the rebel's welfare slowly becomes worse due to structural conditions which will lead to a continuous increase in the benefits of an attack. However, should the government threaten the rebel or their group significantly, the benefits of conflict will increase by a random and non-marginal amount.

Conflict Benefits in the Aftermath: Since conflict is assumed to pay off somehow, an attack generates a structural break with a new set of more positive economic, social, or political conditions afterwards. The rebel's increasingly dissatisfying current situation increases the benefits of conflict so that once the attack has been carried out, the rebel's living

[9] The Ito-Lévy Jump Diffusion process is a special case of geometric Lévy processes. For further information about Lévy processes see e.g. Oksendal and Sulem (2007) or Applebaum (2009). In the Appendix we provide a brief description of the process used and how we depart from the approaches known in financial economics.

[10] For a graphical illustration of Jump Diffusions see e.g. Cont and Tankov (2004) pp. 71.

conditions are expected to improve by rate $\alpha > 0$ so that the resulting benefits of conflict can be realized in the aftermath. Returning to our example, successfully assassinating an oppressive leader may lead to political and economic reforms that, in a next step, improve the welfare of the rebel. Hence, even if high uncertainty is involved, carrying out an attack is expected to lead to a satisfactory improvement in the rebel's social environment.

However, the path of future benefits of conflict is highly uncertain. A new process of uncertain developments commences, and although living conditions may be expected to improve on average, changes due to usual economic fluctuations that are captured by a volatility and unforeseen repressing events caused by the new government with intensity $\lambda_2 > 0$ and size $z \in U_2 \subset \mathbb{R}_-$ may take place and must be considered when evaluating the benefits of an attack. For instance, the government may become even more oppressive, the rebel may be caught or even tortured, and reforms, counterattacks, or military coups may fail. Since developments of future benefits in the aftermath of an attack incorporate fundamental threats, we model them as another Ito-Lévy Jump Diffusion process.

Definition 2 *Let $U_2 \subset \mathbb{R}_-$ be a Borel set whose closure does not contain 0. Furthermore, let W_2 be a standard Wiener process, N_2 a Poisson process with intensity λ_2 and constants $\alpha, \sigma_2 \in \mathbb{R}_+$. Then the Ito-Lévy Jump Diffusion process Y, indicating the future benefits in the aftermath of an attack, is defined by means of the following differential equation*

$$\begin{aligned} dY &= \alpha Y dt + \sigma_2 Y dW_2 + Y \int_{U_2} z N_2(t, dz) \qquad (2) \\ \text{for} \quad T &< t \qquad \text{and} \qquad Y(0) = Y_0. \end{aligned}$$

While the first part of the stochastic process is again an increasing geometric Brownian motion, the second part, $\int_{U_2} z N_2(t, dz) < 0$, allows for fundamental repressing and unpredictable events in the aftermath of the conflict.

2.3 Value of Conflict and the Option Value of Peacekeeping

Net Present Value of Conflict: As conflicts may lead to an improvement in living conditions, an attack enables the realization of potential benefits of conflict. Once the attack is carried out, the dynamic development of benefits is given within the limits of a random process. The economic value of conflict consists solely of its future benefit stream. Each dynamic development of benefits generates its own value of conflict. For an investor the gross and net value of conflict V^{gross} is determined by the expected present value of the benefit stream in the aftermath:[11]

Lemma 3 *Let Y be the benefit stream after the conflict described by an Ito-Lévy-Jump-Diffusion process, υ_2 a Lévy measure and r the risk-free interest rate. Further assume*

$$r > \alpha + \int\limits_{f^{-1}(U_2)} z\upsilon_2(dz) + \int\limits_{U_2} \left[\ln(1+z) - z\right] \upsilon_2(dz),$$

where $f(z) = ln(1+z)$. Then the gross value of benefits is

$$V^{gross}(T) = \frac{Y(T)}{\left(r - \int\limits_{f^{-1}(U_2)} z\upsilon_2(dz) - \int\limits_{U_2} \left[\ln(1+z) - z\right] \upsilon_2(dz) - \alpha\right)}.$$

Hence, with conflict costs I the expected net value of conflict is

$$V(T) = V^{gross}(T) - I.$$

Proof. For a proof see Appendix 2.

Note that for simplicity the rebel has an infinite lifespan. In our example, an attack would enable long term political participation of a particular social group. Furthermore, we can see that the net present value depends on the sum of repression undertaken by the

[11] A detailed solution to the SDE and the derivation of the expected value of the Ito-Lévy Jump Diffusion process, which is used for determining the gross value of conflict is presented in Appendix 1.

government after the conflict, $\int\limits_{f^{-1}(U_2)} z\upsilon_2(dz) - \int\limits_{U_2} [\ln(1+z) - z]\, \upsilon_2(dz) < 0$. This additional component acts as an additional depreciation parameter that occurs only after accounting for large and non-marginal risk. If we had used the geometric Brownian motion to describe the benefits of conflict, we would not obtain this parameter. This result is particularly convincing for our problem. The more repressive a government remains in the aftermath of the conflict, the lower the net present value of conflict and the less beneficial an attack.

Option Value of Peacekeeping: As it has been suggested in the introductory section, not attacking and waiting has its own value, because additional opportunities that could otherwise not have been foreseen and realized may open up. This value of waiting may indicate that a later conflict could be beneficial. Not attacking also protects rebels from the irreversible costs of an attack. Having the freedom to choose between alternative policies has an extra value that is particularly obvious when talking about violent conflicts. A violent attack lifts the conflict to another level. It removes any opportunity to resolve problems with peaceful measures. With a violent or even deadly attack, such as an assassination of a state representative, there is no turning back; the attackers cannot say, "Sorry, we didn't mean it!" Once the attack is carried out, rebels cannot turn back - they are tied to the expected benefit track they have chosen. This logically corresponds to a firm's investment decision (Dixit and Pindyck (1994)), where the option value of the freedom of choice is a measure of opportunities that may open up in the future when an agent does not irreversibly embark on a particular benefit stream.

Accounting for the option value F for the Ito-Lévy Jump Diffusion Process (1), we apply dynamic programming to obtain the Hamilton-Jacobi-Bellman equation:[12]

$$rFdt = E(dF).$$

[12] For a detailed discussion of the option value see Appendix 2.

2.4 Decision to Attack

The decision to attack straight away is a sequential decision where the rebel repeatedly considers their living conditions and evaluates if a conflict at this point in time is the best strategy. In order to solve the decision problem of launching the attack, the rebel compares the benefit of immediate conflict V with the option value of a later attack F. Therefore, the problem is solved by the solution to:

$$\max\left\{V^{gross}(T) - I, F(T)\right\}. \tag{3}$$

At any time during the non-violent waiting period the rebel will compare the expected net benefit of conflict with the option value of an uncertain non-violent development with the freedom to attack later. If the net value of conflict is greater than the option value ($V^{gross}(T) - I \geq F(T)$), the rebel will carry out the attack. By contrast, if the option value of postponing the attack exists and is greater than the net benefit of attacking straight away, they will not initiate the conflict and wait. Solving this continuous sequential decision problem (3) also allows us to determine the expected time of the attack.

3 Solving for the Expected Time of Conflict

Identifying the conditions that eventually trigger the attack and also determining the expected time of attack involves two steps.

First, for each non-violent period during which deteriorating living conditions generate increasing benefits of the attack, we need to determine the benefit value of conflict in the current period (Y^* threshold) that would trigger the outbreak. This threshold is the required current benefit level that would make the attack preferable. It marks the boundary at which conditions become so bad that a conflict becomes unavoidable for the rebel. Then, the expected value of conflict exceeds the option value of peace and hence the attack becomes more profitable. Peaceful waiting, even if uncertain positive events are still possible, is no longer rational.

Second, as the threshold indicates the start of the conflict, rebels simultaneously observe the development of the current period's benefit $\tilde{Y}$. Under worsening conditions during the waiting period they compare the threshold Y^* with the corresponding current period's benefit level of conflict $\tilde{Y}$ and verify if the threshold has already been reached. Even if the hope of positive events that improve living conditions currently lets the rebel to remain peaceful, the expected end of the peaceful period can be predicted. Hence, understanding this mechanism allows for an extension of peaceful episodes in latent conflicts and may help to generate more time to look for peaceful solutions.

3.1 Conflict Threshold

In order to determine the benefit value that triggers the conflict we need to consider the standard conditions of a stochastic dynamic programming problem. In addition to the Hamilton-Jacobi-Bellman equation for the option value F and applying Ito's lemma for jump diffusions to dF, we have to use the well-known boundary conditions, namely (4), the value matching condition (5) and the smooth pasting condition (6)

$$F(0) = 0 \tag{4}$$

$$F(Y^*) = V^{gross}(Y^*) - I \tag{5}$$

$$\frac{dF(Y^*)}{dY} = \frac{d(V^{gross}(Y^*) - I)}{dY} \tag{6}$$

to solve for the threshold benefit Y^*. The setting of the decision problem implies that the net benefits of conflict must be sufficiently large to launch the attack. Reaching this threshold triggers a change in strategy from peace to conflict. Therefore, determining this threshold is the first part of a solution to the expected timing of attack.

Proposition 4 *Let I be the constant costs of conflict,* (1) *a sequence of increasing current benefit levels while remaining peaceful, and* (2) *future benefit developments after the attack.*

Further let β be an implicit function resulting from the differential equation $rFdt = E(dF)$ with solution $F = B\tilde{Y}^{\beta}$, with B being a constant. Then the threshold Y^* that would trigger a conflict is

$$Y^* = \frac{\beta}{\beta - 1}(r - \int_{f^{-1}(U_2)} z v_2(dz) - \int_{U_2} [\ln(1+z) - z]\, v_2(dz) - \alpha)I. \tag{7}$$

Proof. For a proof see Appendix 3.

The threshold is the current benefit that an attack needs to generate as a minimum if all positive values of peaceful waiting are accounted for. It is the ultimate limit to what one can bear in terms of discrimination, oppression, or persecution. As long as this threshold is not reached, the latent conflict is not triggered and the rebel will somehow tolerate the conditions in the hope of improvement. The situation remains calm. However, it is a calm before the storm. Even if the rebel does not attack straight away, their expectations about the future suggest that there will be a point in time when the attack is beneficial enough to be launched. That is, knowing the value of the threshold and the random process of living conditions during the peaceful period, the expected time of attack becomes predictable.

3.2 Expected Time of Conflict

Once the rebel knows from the threshold at which current benefit level they should attack, the question is when they can expect to obtain this benefit for the first time. This time is referred to as the first passage time.

As we consider Jump diffusion processes a potential overshooting has to be taken into account. This means that the horizontal boundary marking the potential start of conflict does not have to be hit exactly, but may be overshot. In our model we are particularly interested in the effect of repressive actions by the government that are modelled by stochastic events occurring at a random point in time with an unpredictable size. In such a context we can use the exponential distribution for which an analytical solution for the first passage time exists. Similar to Kou and Wang (2003) the exponential distribution is given by

$$h(z) = \eta e^{-\eta z},$$

where $\frac{1}{\eta}$ denotes the mean of the exponential distribution. By using the Girsanov theorem we can derive the probability density function of T^* [13] which is sometimes referred to as the Inverse Gaussian Distribution.[14]

Proposition 5 *Let $\tilde{Y}$ be an Ito-Lévy Jump Diffusion process in (1), Y^* a constant threshold in (7) and $h(z) = \eta e^{-\eta z}$ be the density function of the double exponential distribution. Then the first hitting time of Y^* is*

$$E(T^*) = \frac{1}{\bar{u}} \left[Y^* + \frac{\mu_2^* - \eta}{\eta \mu_2^*} (1 - e^{-Y^* \mu_2^*}) \right] \tag{8}$$

with $\bar{u} = \delta + \lambda_1 \frac{1}{\eta}$ denoting the overall drift. μ_2^* is defined as the unique root of $G(\mu_2^*) = 0$ with $G(x) := x\delta + \frac{1}{2}x^2\sigma_1^2 + \lambda_1 \left(\frac{\eta_1}{\eta - x} - 1 \right)$ and $0 < \eta < \mu_2^* < \infty$.

Proof. For a proof see Appendix 3.

$E(T^*)$ is the result of a sequential optimization and hence the solution to a timing problem. Furthermore, it is the time when the rebel expects to carry out the terrorist attack in order to end repression and discrimination. It is the time that introduces new and improved living conditions. However, this point in time is just an expected value. In the course of time a major event can trigger an attack at any, even an unexpected, moment.

4 Determinants of the Expected Time of Attack

In this section we examine how economic conditons or government policies affect uncertatinty and hence rebels's decisions. In general, we could look at both, conditons during

[13] An extensive discussion is offered by Karatzas and Shreve (1991, p.196) and Karlin and Taylor (1975, p.363).

[14] The term "Inverse Gaussian Distribution" stems from the inverse relationship between the cumulant generating functions of these distributions and those of the Gaussian distributions. For a detailed discussion of the inverse Gaussian distribution see Johnson, Kotz, and Balakrishnan (1995) or Dixit (1993).

the time of latent conflict which is still non-violent, and changes in expectations about the aftermath of an outbreak. However, we focus on the condions during the first, the peace period, where we can already identify a latent conflict but the rebels have not yet turned to violence, and analyse how uncertainty-describing parameters may shorten this peaceful episode.

We start with the effect of economic variations that are described by the volatility of the geometric Brownian motion that is part of the Ito-Levy jump diffusion process. How does an increase in volatility affect the rebel's decision to attack?

Proposition 6 *An increase in economic variation indicated by the variance of the Brownian motion leads to a later attack*

$$\frac{\partial E(T^*)}{\partial \sigma_1} > 0.$$

Proof. For a proof see Appendix 4.

According to Proposition 6, larger variations in the economic, social, or political conditions of the rebel increase the peaceful period. Greater volatility implies both unexpected improved conditions as well as their deterioration. Hence, with this increase in marginal uncertainty the rebel expands the time he is willing to hope and wait for the conflict to resolve itself without any violent means.

However, what happens if there are non-marginal stochastic threats, e.g., if the government steps up the fight against the discriminated group and takes severe oppressive action such as the persecution of group members, waves of detention, or assassination of rebel leaders?

Proposition 7 *An increase in the frequency λ_1 of large scale repressive government actions during the latent conflict leads to an earlier attack*

$$\frac{\partial E(T^*)}{\partial \lambda_1} < 0.$$

Similarly an increase in the magnitude of large scale repressive government actions shortens the peaceful period

$$\frac{\partial E(T^*)}{\partial u} < 0.$$

Proof. For a proof see Appendix 4.

In general, large uncertainty and non-marginal stochastic shocks, indicated by λ_1 and u, affect the outbreak of conflict. Even more, these effects are different than the effects of marginal shocks, indicated by σ_1.

An increase in λ_1 implies that more fundamental events are occurring that imply non-marginal changes in the expected path of benefits associated with the attack. Deteriorations in welfare conditions like unfavourable regime changes, increasing number of oppressive government actions, or even external disasters occur more often. The political implications are straight. In a latent conflict, signals pointing to randomly deteriorating fundamental welfare conditions for rebels would escalate the situation. A sooner attack can be exxpected. In contrast, a policy that promises significant opportunities for the group represented by the rebels may not terminate the conflict but postpone the attack and give more time for a peaceful conflict resolution.

Furthermore, an increase in u means that threats by the government become larger and a greater benefit of the attack determined by deteriorating events that worsen the rebel's situation is created. The rebel group faces more severe threats when following the current welfare path. For instance, torture instead of detention, or expropriation instead of taxation during the non-violent period, will make the attack more benefical since an uprising could lead to a new regime that is more beneficial for the rebels, so we can expect the attack to be carried out sooner.

5 Summary

There is hardly any decision problem that bears more dramatic and large uncertainties than the decision to launch a violent conflict. In this model the decision to turn to violence is based on the idea that an attack is a form of investment in a change of conditions. The major focus of this paper is on the impact of large (non-marginal) uncertaint shocks - such as fundamental threats often created by an oppressive government - on the decision to launch a conflict. In order to capture these major uncertain events, we suggest a real option decision model in the context of social conflicts that predicts the expected outbreak of a conflict. To model such discontinuous large stochastic shocks we introduce a more general stochastic process than the often used geometric Brownian motion, namely an Ito-Lévy Jump Diffusion processes. With this stochastic modelling we can show that *large uncertaint shocks* have opposite effects on the decision to launch a conflict than what we usually refer to as *risk* indicated by the marginal variability, or volatility, included in the geometric Brownian motion.

In more detail, under dynamic conditions an attack is the result of an evaluation of highly uncertain developments in economic, social, and political conditions. Rebels compare the current conditions with an expected path of development after a potential attack, including conflict costs. If current conditions are expected to develop sufficiently badly turning violent can be considered and a latent conflict can be identified. During the non-violent period of this latent conflict the benefits of attack are still not sufficiently high, so rebels will remain peaceful and wait until the attack is capable of paying off. The decision to attack is a sequential decision in the course of time that is irreversible. Since we are not interested in the interaction between the government and the rebel but rather in the rebel's reaction on a given but uncertain set of conditions, we suggest that real option theory is an appropriate methodology in this context to evaluate the timing of and predict the outbreak of a conflict. In order to include discontinuities and large uncertain events in our model we need to extend current methods in real option theory by introducing an Ito-Lévy Jump

Diffusion process. For this discontinuous modelling we analytically derive the threshold that triggers the attack, and we determine the time this is expected to happen. Because we propose an "option to attack decision" for formal modelling in conflict theory, we can show that large uncertain shocks have opposite effects than the so far considered risk as pure volatility. While marginal variability, often modelled by the volatility of a geometric Brownian motion and associated with usual economic or social variation, prolongs the peaceful period, major uncertain shocks, modelled by a jump process and associated with major oppressive government actions, make an early attack more likely. Hence, these large uncertain shocks should be considered more carefully when policy is designed.

The political implication is straight. Even if latent conflicts are not immediately solved, prospects of significant improvements are able to extend the peaceful period and provide more time to find a solution to the conflict. In contrast, more uncertain threats, often meant to awe the oppressed group may provoke a soon attack if the threat is strong enough.

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6 Appendix

Appendix 1: Solution to and Expected Value of the SDE

Solution to the SDE for Y

In this section we determine the solution of the SDE for Y and derive the expected value. A more general formulation of this process can be found in Oksendal and Sulem (2007). They describe under which conditions a solution to these SDE exists and discuss some characteristics. For our purpose we assume that the existence conditions are fulfilled. A further discussion of Lévy processes and their characteristics can be found in e.g. Applebaum (2009) and in Cont and Tankov (2004). However, in order to obtain the respective results for $\tilde{Y}$ replace Y by $\tilde{Y}$.

Lemma 8 *The Ito-Lévy Jump Diffusion process described by the SDE*

$$dY = \alpha Y dt + \sigma_2 Y dW_2 + Y \int_{U_2} z N_2(t, dz) \quad \textit{for} \quad T < t,$$

has the solution

$$Y(t) = Y(0) \exp \left[\begin{array}{c} \left(\alpha - \frac{1}{2}\sigma_2^2\right) t + \sigma_2 W_2(t) + \int_0^t \int_{U_2} \left[\ln(1+z) - z\right] \upsilon_2(dz) ds \\ + \int_0^t \int_{U_2} \ln(1+z) N_2(dt, dz) \end{array} \right].$$

Proof. Similar to Oksendal and Sulem (2007) we define $X(t) = \ln Y(t)$ and use Ito's Lemma for jump processes to obtain the solution to the SDE. Note that according to $ln(1+z)$ the function $Y(t)$ is only defined for $z > -1$.

Lemma 9 *Let Y be an Ito-Lévy Jump Diffusion process. The expected value of Y is*

$$EY(t) = Y(0) \exp \left[t \left(\alpha + \int_{f^{-1}(U_2)} z \upsilon_2(dz) + \int_{U_2} \left[\ln(1+z) - z\right] \upsilon_2(dz) \right) \right].$$

Proof. Assume that the Wiener and the compound Poisson process, are independent. Then the expected value of Y can be decomposed into

$$EY(t) = \underbrace{EY(0) \cdot Ee^{\left(\alpha - \frac{1}{2}\sigma_2^2\right)t + \sigma_2 W_2(t)}}_{(*)} \cdot \underbrace{E\left[\exp\left(\int_0^t \int_{U_2} \ln(1+z) N_2(ds, dz)\right)\right]}_{(**)}$$

$$\cdot \underbrace{E\left[\exp\left(\int_0^t \int_{U_2} \left[\ln(1+z) - z\right] \upsilon_2(dz) ds\right)\right]}_{(***)}.$$

In this case compute the respective values for all three components.

- The expectation value (*) is the same as for the geometric Brownian motion, which can be found in Dixit and Pindyck (1994)

$$EY(0)e^{\alpha t}.$$

- In order to compute the expected value (**) we use Theorem 2.3.7 (i) in Applebaum (2009). As $\int_0^t \int_{U_2} \ln(1+z) N_2(ds, dz)$ is compound Poisson distributed with the characteristic function

$$E\left[\exp\left(iu \int_0^t \int_{U_2} \ln(1+z) N_2(ds, dz)\right)\right] = \exp\left(t \int_{U_2} \left(e^{iuz} - 1\right) \upsilon_{2f}(dz)\right)$$

  it follows from $\upsilon_{2f} = \upsilon_2 \circ f^{-1}$, $f = \ln(1+z)$ and $u = -i$

$$E\left[\exp\left(\int_0^t \int_{U_2} \ln(1+z) N_2(ds, dz)\right)\right] = \exp\left(t \int_{f^{-1}(U_2)} z \upsilon_2(dz)\right).$$

  This result only holds for $\int_{U_2} e^{uz} \upsilon_2(dz) < \infty$. In contrast to Theorem 2.3.7 (i) in Applebaum (2009) where $u \in \mathbb{R}$, we assume u to be complex. This is possible since the according integral exists anyway.

- As the expected value (***) is given by

$$E\left[\exp\left(\int_0^t\int_{U_2}[\ln(1+z)-z]\,\upsilon_2(dz)ds\right)\right]=\exp\left(t\int_{U_2}[\ln(1+z)-z]\,\upsilon_2(dz)\right).$$

Accordingly, the resulting expected value for Y is

$$EY(t)=Y(0)\exp\left[t\left(\alpha+\int_{f^{-1}(U_2)}z\upsilon_2(dz)+\int_{U_2}[\ln(1+z)-z]\,\upsilon_2(dz)\right)\right].$$

$EY(t)$ is an increasing function for $\alpha+\int_{f^{-1}(U_2)}z\upsilon_2(dz)+\int_{U_2}[\ln(1+z)-z]\,\upsilon_2(dz)>0$, otherwise it decreases with t.

Appendix 2: Value of Conflict and Option Value of Peacekeeping

In order to determine the optimal investment in rebellion the value of conflict and the option value of waiting to attack are optimized for each period.

Proof of Lemma 3. For the value of conflict all benefits per period are summarized through an integral. Similar to Dixit and Pindyck (1994) we obtain

$$V^{gross}=E\int_T^\infty Ye^{-r(t-T)}Y\,dt$$

Under the assumption $r>\alpha+\int_{f^{-1}(U_2)}z\upsilon_2(dz)+\int_{U_2}[\ln(1+z)-z]\,\upsilon_2(dz)$ we obtain the result.

Corollary 10 *Let $\tilde{Y}$ be an Ito-Lévy Jump Diffusion process. Then the Hamilton-Jacobi-Bellman equation defined in Dixit and Pindyck (1994)*

$$rF=\frac{1}{dt}E(dF)$$

has the solution $F = B\tilde{Y}^{\beta}$ *where* β *is defined implicitly.*

Proof. From Ito's Lemma we know:

$$dF = (\frac{\partial F}{\partial t} + \delta\tilde{Y}\frac{\partial F}{\partial\tilde{Y}} + \frac{1}{2}\sigma_1^2\tilde{Y}^2\frac{\partial^2 F}{\partial\tilde{Y}^2})dt + \sigma_1\tilde{Y}\frac{\partial F}{\partial\tilde{Y}}dW_1$$
$$+ \int\limits_{U_1}\left[F(\tilde{Y}(t^-) + u\tilde{Y}(t^-)) - F(\tilde{Y}(t^-)) - \frac{\partial F}{\partial\tilde{Y}}u\tilde{Y}(t^-)\right]\upsilon_1(du)dt$$
$$+ \int\limits_{U_1}\left[F(\tilde{Y}(t^-) + u\tilde{Y}(t^-)) - F(\tilde{Y}(t^-))\right]N_1(dt, du).$$

In order to determine $E(dF)$, we use Theorem 2.3.7 (ii) in Applebaum (2009). For the expectation value of

$$\int\limits_{U_1}\left[F(\tilde{Y}(t^-) + u\tilde{Y}(t^-)) - F(\tilde{Y}(t^-))\right]N_1(dt, du)$$

we obtain

$$E\int\limits_{U_1}\left[F(\tilde{Y}(t^-) + u\tilde{Y}(t^-)) - F(\tilde{Y}(t^-))\right]N_1(dt, du)$$
$$= \; t\int\limits_{U_1}\left[F(\tilde{Y}(t^-) + u\tilde{Y}(t^-)) - F(\tilde{Y}(t^-))\right]\upsilon_1(du)$$

and with $E(dW_1) = 0$, this leads us to

$$\Rightarrow \quad E(dF) = (\frac{\partial F}{\partial t} + \delta\tilde{Y}\frac{\partial F}{\partial\tilde{Y}} + \frac{1}{2}\sigma_1^2\tilde{Y}^2\frac{\partial^2 F}{\partial\tilde{Y}^2})dt$$
$$+2\int\limits_{U_1}\left[F(\tilde{Y}(t^-) + u\tilde{Y}(t^-)) - F(\tilde{Y}(t^-)) - \frac{1}{2}\frac{\partial F}{\partial\tilde{Y}}u\tilde{Y}(t^-)\right]\upsilon_1(du)dt.$$

From the Bellman and the last equation we obtain the following differential equation:

$$\delta\tilde{Y}\frac{\partial F}{\partial\tilde{Y}} + \frac{1}{2}\sigma_1^2\tilde{Y}^2\frac{\partial F}{\partial\tilde{Y}^2} + 2\int\limits_{U_1}\left[\begin{array}{c} F(\tilde{Y}(t^-) + u\tilde{Y}(t^-)) - F(\tilde{Y}(t^-)) \\ -\frac{1}{2}\frac{\partial F}{\partial\tilde{Y}}u\tilde{Y}(t^-) \end{array}\right]\upsilon_1(du) - rF = 0.$$

This is a second-order homogenous ordinary differential equation with a free boundary. A general solution to this differential equation is of the form

$$F = B\tilde{Y}^{\beta}.$$

Hence,

$$\delta\beta + \frac{1}{2}\sigma_1^2\beta(\beta-1) + 2\int_{U_1}\left[(1+u)^{\beta} - (1+\frac{1}{2}\beta u)\right]\upsilon_1(du) - r = 0.$$

If we define

$$g(\beta) := \delta\beta + \frac{1}{2}\sigma_1^2\beta(\beta-1) + 2\int_{U_1}\left[(1+u)^{\beta} - (1+\frac{1}{2}\beta u)\right]\upsilon_1(du) - r$$

with h being the distribution of the jump sizes, then it follows

$$\begin{aligned} g(1) &= \delta + \int_{U_1} u\upsilon_1(du) - r, \\ \lim_{\beta\to\infty} g(\beta) &= \infty. \end{aligned}$$

Accordingly, we can assume that $r > \delta + \int_{U_1} u\upsilon_1(du)$ leading to $g(1) < 0$. With the intermediate value theorem we find $\beta_1 \in (1, \frac{r}{\delta + \int_{U_1} u\upsilon_1(du)})$ such that $g(\beta_1) = 0$. It follows immediately that β_1 is a function of r, δ and $\int_{U_1} u\upsilon_1(du)$ determined as the implicit function of $g(\beta_1) = 0$, and $\beta_1 > 1$.

Corollary 11 *The derivatives of β_1 according to σ_1, λ_1 and u are*

$$\frac{\partial\beta_1}{\partial\sigma_1} = -\frac{\sigma_1\beta(\beta-1)}{\delta - \frac{1}{2}\sigma_1^2 + 2\lambda_1\int_{U_1}\left[\ln(1+u)(1+u)^{\beta_1} - \frac{1}{2}u\right]\upsilon_1(du)} < 0,$$

$$\frac{\partial\beta_1}{\partial\lambda_1} = \frac{2\int_{U_1}\left[-(1+u)^{\beta_1} + (1+\frac{1}{2}\beta_1 u)\right]h(du)}{\delta - \frac{1}{2}\sigma_1^2 + 2\lambda_1\int_{U_1}\left[\ln(1+u)(1+u)^{\beta_1} - \frac{1}{2}u\right]h(du)}$$

and

$$\frac{\partial \beta_1}{\partial u} = -\frac{2\beta \int\limits_{U_1} \left[(1+u)^{\beta-1} - \frac{1}{2} \right] \upsilon_1(du)}{\delta - \frac{1}{2}\sigma_1^2 + 2\lambda_1 \int\limits_{U_1} \left[\ln(1+u)(1+u)^{\beta_1} - \frac{1}{2}u \right] \upsilon_1(du)}.$$

Proof. Apply the rules

$$\frac{\partial \beta_1}{\partial \sigma_1} = -\frac{\frac{\partial g}{\partial \sigma_1}}{\frac{\partial g}{\partial \beta_1}}, \frac{\partial \beta_1}{\partial \lambda_1} = -\frac{\frac{\partial g}{\partial \lambda}}{\frac{\partial g}{\partial \beta_1}} \text{ and } \frac{\partial \beta_1}{\partial u} = -\frac{\frac{\partial g}{\partial u}}{\frac{\partial g}{\partial \beta_1}}$$

in order to obtain the derivatives. However, their sign is obtained by discussing the jump integral. The numerator of $\frac{\partial \beta_1}{\partial \lambda_1}$ contains a measure integral $\int\limits_{U} \left[-(1+u)^{\beta_1} + (1 + \frac{1}{2}\beta_1 u) \right] h(du)$ with a measurable function f and a measure h. This integral is defined as

$$\int\limits_{U_1} f h(du) = \int\limits_{U_1} f^+ h(du) - \int\limits_{U_1} f^- h(du) \tag{9}$$

and is a measure integral where, depending on U_1, f^+ is the positive and f^- the negative part of f. The difference between the two integrals in (9) depends on the size of each integral leading to an ambiguous sign of (9). Hence, if the negative jumps outweigh the positive jumps the sign of the integral will be negative and otherwise positive. In our case where we have repressive actions that lead to a positive jump in the benefits the integral is positive as well.

In the denominator we have

$$\delta - \frac{1}{2}\sigma_1^2 + 2\lambda_1 \int\limits_{U_1} \ln(1+u)(1+u)h(du) - \lambda_1 \int\limits_{U_1} u h(du),$$

consisting of

$$\delta - \frac{1}{2}\sigma_1^2 > 0$$

and a jump component

$$2\lambda_1 \int\limits_{U_1} \ln(1+u)(1+u)h(du) - \lambda_1 \int\limits_{U_1} u h(du)$$

where the logarithm is only defined for $u > -1$. Again, in our case positive jumps lead to a positive integral. Assuming the denominator and numerator are positive, the derivative according to λ_1 becomes negative.

With the same argumentation the derivative according to z becomes positive.

Appendix 3: Investment Threshold and Expected Time of Conflict

Proof of Proposition 4. Apply the boundary conditions

$$
\begin{aligned}
F(0) &= 0 \\
F(Y^*) &= V^{gross}(Y^*) - I \\
\frac{dF(Y^*)}{dY} &= \frac{d(V^{gross}(Y^*) - I)}{dY}
\end{aligned}
$$

and solve the equation system for Y^*.

Proof of Proposition 5. For the jump diffusion case the first passage problem can be solved analytically if we assume an explicit distribution of the jump sizes. According to Kou and Wang (2003) the moment-generating function for $\tilde{Y}(t)$ with $\theta \in (0, \eta_1)$ is

$$\phi(\theta, t) := E(e^{\theta \tilde{Y}(t)}) = \exp(G(\theta)t)$$

where the function G is defined as

$$G(x) := x\delta + \frac{1}{2}x^2\sigma_1^2 + \lambda_1 \left(\frac{\eta}{\eta - x} - 1 \right).$$

For jump diffusion processes the study of first passage times has to consider the exact hit of a constant boundary as well as an overshoot. Accordingly, two cases have to be distinguished. The Laplace transformation of the first hitting time, when $\tilde{Y}(t)$ hits the boundary Y^* exactly[15] is :

$$E(e^{-\varepsilon\tilde{T}_i}1_{\{\tilde{Y}(\tilde{T}_i)=Y^*\}}) = \frac{\eta - \beta_{1,\varepsilon}}{\beta_{2,\varepsilon} - \beta_{1,\varepsilon}} e^{-Y^*\beta_{1,\varepsilon}} + \frac{\beta_{2,\varepsilon} - \eta}{\beta_{2,\varepsilon} - \beta_{1,\varepsilon}} e^{-Y^*\beta_{2,\varepsilon}}$$

with $\mu_{1,\varepsilon}$ and $\mu_{2,\varepsilon}$ being the only positive roots of $G(\beta) = \varepsilon$ and $0 < \beta_{1,\varepsilon} < \eta < \beta_{2,\varepsilon} < \infty$. For every overshoot $\tilde{Y}(\tilde{T}) - Y^*$ the Laplace transformation is

$$E(e^{-\varepsilon\tilde{T}}1_{\{\tilde{Y}(\tilde{T}_i)-Y^*>y\}}) = e^{-\eta y}\frac{(\eta - \mu_{1,\varepsilon})(\mu_{2,\varepsilon} - \eta)}{\eta_1(\mu_{2,\varepsilon} - \mu_{1,\varepsilon})}\left(e^{-Y^*\mu_{1,\varepsilon}} - e^{-Y^*\mu_{2,\varepsilon}}\right) \quad \text{for all } y \geq 0.$$

The expectation of the first passage time is finite, i.e., $E(T^*) < \infty$, if and only if the overall drift of the jump diffusion process is positive. Hence,

$$E(T^*) < \infty \Leftrightarrow \bar{u} = \delta + \lambda_1 \frac{1}{\eta_1} > 0.$$

Now for $\bar{u} > 0$ we determine the first passage time as

$$E(T^*) = \frac{1}{\bar{u}}\left[Y^* + \frac{\mu_2^* - \eta}{\eta\mu_2^*}(1 - e^{-Y^*\mu_2^*})\right]$$

where μ_2^* is defined as the unique root of $G(\mu_2^*) = 0$ with $0 < \eta < \mu_2^* < \infty$.

Appendix 4: Determinants of the Expected Time of Conflict

Proof of Proposition 6. In order to determine the derivative of $E(T^*)$ we first have to find out the sign of $\frac{\partial Y^*}{\partial \sigma_1}$ and $\frac{\partial \mu_2^*}{\partial \sigma_1}$.

$$\begin{aligned}
\frac{\partial Y^*}{\partial \sigma_1} &= \frac{\frac{\partial \beta}{\partial \sigma_1}(\beta - 1) - \frac{\partial \beta}{\partial \sigma_1}\beta}{(\beta - 1)^2}\left(r - \int_{f^{-1}(U_2)} z\upsilon_2(dz) - \int_{U_2}[\ln(1+z) - z]\,\upsilon_2(dz) - \alpha\right) I \\
&= -\frac{\overset{(-)}{\frac{\partial \beta}{\partial \sigma_1}}}{(\beta - 1)^2}\left(r - \int_{f^{-1}(U_2)} z\upsilon_2(dz) - \int_{U_2}[\ln(1+z) - z]\,\upsilon_2(dz) - \alpha\right) I > 0
\end{aligned}$$

[15] See Kou and Wang (2003) Theorem 3.1.

Now define $G := x\delta + \frac{1}{2}x^2\sigma_1^2 + \lambda\left(\frac{p\eta}{\eta - x} - 1\right)$ and determine

$$\begin{aligned}\frac{\partial \mu_2^*}{\partial \sigma_1} &= \frac{\partial x}{\partial \sigma_1} = -\frac{\frac{\partial G}{\partial \sigma_1}}{\frac{\partial G}{\partial x}} \\ &= -\frac{x^2\sigma_1}{\delta + x\sigma_1^2 + \lambda\frac{\eta}{(\eta-x)^2}} < 0.\end{aligned}$$

With these results we can now determine

$$\begin{aligned}\frac{\partial E(T^*)}{\partial \sigma_1} &= \frac{1}{\bar{u}}\frac{\partial Y^*}{\partial \sigma_1} + \frac{1}{\bar{u}}\left(\frac{\partial Y^*}{\partial \sigma_1}\mu_2^* + Y^*\frac{\partial \mu_2^*}{\partial \sigma_1}\right)\frac{\mu_2^* - \eta_1}{\eta_1\mu_2^*}e^{-Y^*\mu_2^*} + \frac{\frac{\partial \mu_2^*}{\partial \sigma}\eta_1\mu_2^* - \frac{\partial \mu_2^*}{\partial \sigma}\eta_1\mu_2^*}{(\eta_1\mu_2^*)^2}(1 - e^{-Y^*\mu_2^*}) \\ &= \frac{1}{\bar{u}}\underbrace{\frac{\partial Y^*}{\partial \sigma_1}}_{(+)} + \frac{1}{\bar{u}}\left(\underbrace{\frac{\partial Y^*}{\partial \sigma_1}}_{(+)}\mu_2^* + Y^*\underbrace{\frac{\partial \mu_2^*}{\partial \sigma_1}}_{(-)}\right)\frac{\mu_2^* - \eta_1}{\eta_1\mu_2^*}e^{-Y^*\mu_2^*} > 0\end{aligned}$$

Proof of Proposition 7.

$$\begin{aligned}\frac{\partial E(T^*)}{\partial \lambda_1} &= -\underbrace{\frac{\frac{1}{\eta}}{\left[\delta + \lambda_1\frac{1}{\eta}\right]^2}}_{(\#)}\underbrace{\left[Y^* + \frac{\mu_2^* - \eta}{\eta\mu_2^*}(1 - e^{-Y^*\mu_2^*})\right]}_{(\#\#)} \\ &\quad + \underbrace{\frac{1}{\delta + \lambda_1\frac{1}{\eta}}}_{(\#\#\#)}\underbrace{\left[\begin{array}{c} -\frac{\frac{\partial \beta}{\partial \lambda_1}}{(\beta-1)^2}(r - \int\limits_{f^{-1}(U_2)} z\upsilon(dz) \\ -\int\limits_{U_2}\left[\ln(1+z) - z\right]\upsilon_2(dz) - \alpha)I \end{array}\right]}_{(\#\#\#\#)}\underbrace{(1 - \frac{\mu_2^* - \eta}{\eta}e^{-Y^*\mu_2^*})}_{(\#\#\#\#\#)}\end{aligned}$$

For the first term (#) and term (###) we obtain

$$\frac{\frac{1}{\eta}}{\left[\delta + \lambda_1\frac{1}{\eta}\right]^2} > 0$$

and

$$\frac{1}{\delta + \lambda_1\frac{1}{\eta}} > 0.$$

For the second term (##) it holds that

$$\underbrace{Y^*}_{>0} + \underbrace{\frac{\mu_2^* - \eta}{\eta\mu_2^*}}_{>0}\underbrace{(1 - e^{-Y^*\mu_2^*})}_{\geq 0} > 0.$$

The sign of the fourth term (####) depends on whether $\frac{\partial\beta}{\partial\lambda_1}$ is positive or negative. Assuming $\frac{\partial\beta}{\partial\lambda_1} < 0$ then term (####) becomes

$$-\frac{\frac{\partial\beta}{\partial\lambda_1}}{(\beta-1)^2}\underbrace{(r - \int\limits_{f^{-1}(U_2)} z\upsilon(dz) - \int\limits_{U_2} [\ln(1+z) - z]\,\upsilon_2(dz) - \alpha)}_{>0}\underbrace{I}_{>0} > 0$$

The last term (#####)

$$1 - \frac{\mu_2^* - \eta}{\eta}e^{-Y^*\mu_2^*}$$

is negative if

$$1 < \frac{\mu_2^* - \eta}{\eta}e^{-Y^*\mu_2^*}.$$

Summarizing all conditions leads to $\frac{\partial E(T^*)}{\partial\lambda_1} < 0$.

$$\begin{aligned}\frac{\partial E(T^*)}{\partial u} &= \underbrace{\frac{1}{\delta + \lambda\frac{1}{\eta}}}_{(x)}\underbrace{\left[\begin{array}{c} -\frac{\frac{\partial\beta}{\partial u}}{(\beta-1)^2}(r - \int\limits_{f^{-1}(U_2)} z\upsilon(dz) \\ -\int\limits_{U_2} [\ln(1+z) - z]\,\upsilon_2(dz) - \alpha)I(T^*) \end{array}\right]}_{(xx)} \\ &\quad \cdot\underbrace{(1 - \frac{\mu_2^* - \eta}{\eta}e^{-Y^*\mu_2^*})}_{(xxx)}.\end{aligned}$$

Similar to the last proof, the term (x) is positive. Accordingly, the last component (xxx) is negative for $1 < \frac{\mu_2^*-\eta}{\eta}e^{-Y^*\mu_2^*}$. The sign of (xx) depends on whether $\frac{\partial\beta}{\partial u} \gtrless 0$. Assuming that $\frac{\partial\beta}{\partial u} < 0$ it follows that $\frac{\partial E(\tilde{T})}{\partial u} < 0$.

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