UNIVERSITÄT PADERBORN
Die Universität der Informationsgesellschaft

CENTER FOR INTERNATIONAL ECONOMICS

Working Paper Series

Working Paper No. 2012-10

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September 2012

CENTER FOR
INTERNATIONAL
ECONOMICS

Unsustainable Sovereign Debt – is the Euro Crisis only the Tip of the Iceberg?

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Abstract

As a direct effect of the financial crisis in 2008, public debt began to accumulate rapidly, eventually leading to the European sovereign debt crisis. However, the dramatic increase in government debt is not only happening in European countries. All major G7 countries are experiencing similar developments. What are the implications of this kind of massive deficit and debt policy for the long term stability of these economies? Are there limits in debt-ratios that qualitatively change policy options? While theory can easily illustrate these limits, where are these limits in real economies? This paper examines the relationship between sovereign debt dynamics and capital formation, and accounts for the effects of the 2008 financial crisis on debt sustainability for the four largest advanced economies. We contribute to the literature on fiscal sustainability by framing the problem in an OLG model with government debt, physical capital, endogenous interest rates, and exogenous growth. For the calibration exercise we extract data from the OECD for Germany as a stabilization anchor in Europe, the U.S., the U.K., and Japan for almost two decades before the 2008 crisis. Except for intertemporal preferences all parameters are drawn or directly derived from the OECD database, or endogenously determined within the model. The results of the calibration exercise are alarming for all four countries under consideration. We identify debt ceilings that indicate a sustainable and unsustainable regime. For 2011, all four economies are either close to-, or have already passed the ceiling. The results call for a dramatic re-adjustment in budget policies for a consolidation period and long-term fiscal rules that make it possible to sustain sufficient capital intensity so that these economies can maintain their high income levels. Current conditions are already starting to restrict policy choices. However, the results also make it very clear that none of these economies would survive a second financial crisis such as the one in 2008.

Keywords: fiscal sustainability, primary deficit, debt ceiling,
fiscal rules, OLG, calibration, advanced economies

JEL Classification: H62, H63, E62, G01

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1 Introduction

The European sovereign debt crisis is the result of the 2008 financial crisis.¹ Governments ran up large deficits in an attempt to stabilize the financial sector and the real economy. The result was a dramatic increase in net sovereign debt in the years that followed. From 2009 to 2011 net government debt-ratios in many countries increased within just three years more than they had in the three decades before. This dramatic increase in sovereign debt is happening not only in European countries. All major advanced economies (G7) have experienced more or less similar developments (Figure 1). While in the short term government deficit policies seemed to help to stabilize economic conditions, it is worth considering the implications of such a massive short term deficit and debt policy for the long term stability of these economies and for policy options in the future.² As discussed in Reinhart, Reinhart, and Rogoff (2012), due to high debt-ratios there may be future costs in terms of a capital reduction process. These long term costs need to be weighed against Keynesian short-term fiscal stimulus policies. Is there more than an intergenerational distribution effect? Are there limits where the level of debt qualitatively changes policy options because the economy is drifting off into an unstable dynamic regime? Are there limits to government intervention policies due to an unsustainable debt regime?

While these limits can be easily identified just by looking at dynamic stock flow interactions and budget constraints, the important question is: where are these limits in reality? Is a theoretically unsustainable sovereign debt scenario, defined as moving onto an unstable debt and capital trajectory that ends with the collapse of the economy, relevant for real economies?

It seems likely that the European debt crisis is a sign that Southern European economies have already crossed the line of unsustainable sovereign debt-ratios. While Southern European countries experienced strongly increasing interest spreads indicating a growing threat of default risk, Northern European countries, in particular Germany and the U.K., are regarded as a kind of safe haven. Two major questions are hence arising.

1. Are Europe's northern economies which are perceived as strong and stable, still on a sustainable debt regime, or have they already become vulnerable?
2. The dramatic increase in sovereign debt in all G7 (not Canada) countries suggests a closer look at non-European countries. Are there indicators that the current sovereign debt positions of major G7 economies are hitting sovereign debt limits, which could destabilize these economies in the longer term? Could there be a global sovereign debt crisis on the horizon?

We choose four major economies to examine whether a critical (unsustain-

¹Reinhart and Rogoff (2011) find that financial crises often precede or accompany debt crises. Causality may run in both directions. See also Gourinchas and Obstfeld (2012).

²Patillo et al. (2011), among others, show that there is a non-linear relationship between GDP growth and external debt. In particular, they find a threshold beyond which more debt affects growth negatively. Reinhart and Rogoff (2010) argue that this relationship is weak for low debt to GDP ratios.

Source: This graphic is taken from Reinhart, Reinhart and Rogoff (2012), p.72

Figure 1: Development of Sovereign Debt.

able, macroeconomically unstable) debt-ratio has already been reached. We choose Germany to link the discussion of the Euro debt crisis to a discussion of global sovereign debt problems. Germany is regarded as the major stabilizing economy in the eurozone, owing to both its size and its fiscal resilience. Germany is expected to be able to take on large shares of Southern European debt (by guarantees, sinking funds or other instruments) to reduce interest spreads as well as the interest burdens of Southern European governments, and in so doing, stabilize these economies. The U.K. is in scope because it is the largest economy in the EU outside the eurozone and, as a G7 member, one of the world's major advanced economies. The U.S. is the world's largest economy and in many cases serves as a benchmark economy. The U.S. was also the economy that triggered the financial crisis. Japan, as the second largest advanced economy, is the most puzzling but possibly the most interesting case. Japan has been running an expansionary macro policy, in both monetary and fiscal deficit terms, for almost two decades which has resulted in a tremendous sovereign debt position. So even if Japan is very difficult to understand fully we at least try to look at its debt conditions.

Even if the question of *debt sustainability* has been discussed in the literature in various ways (for a brief discussion see the next section), so far no contribution has assessed the effects of the 2008 crisis, with the massive change in debt position in the aftermath, on long term debt sustainability using a calibration exercise. Since an econometric analysis has to rely on ex post data, the sustainability of current debt-ratios cannot be analyzed on this basis, even though this question urgently requires an answer. Therefore, we choose another approach.

To assess fiscal sustainability under today's macro conditions we return to a very rudimentary, but robust dynamic macro theory. We use a simple OLG approach to theoretically assess conditions for fiscal sustainability and calibrate this model in the most straightforward way. Despite relying on an OLG framework, we are confident that the most important results are essentially driven by accounting restrictions and less by potentially controversial elements of decision models. In the calibration exercise we use OECD data to endogenously derive all required parameters and variables. Only two variables indicating time preferences are exogenous. Hence, we can assess debt dynamics and debt sustainability with the help of a very fundamental model and a consistent calibration exercise.

The paper is organized as follows. Section 2 relates the defined question to the literature. Section 3 introduces the framework of a simple OLG model. In section 4 we analyze the dynamics of the macroeconomic system, define the notion of sustainability as used in this paper, and derive the tools to assess the sustainability of fiscal regimes for calibration. Section 5 is devoted to the calibration exercise and provides policy interpretations and implications. Section 6 summarizes the main results and concludes.

2 Related Literature

A reappraisal of the economic literature reveals that the formal examination of sustainable debt is challenged by the absence of a common definition of debt sustainability. The definition of debt sustainability used in empirical studies relies, in one form or another, on a set of binding conditions concerning the government's intertemporal budget constraint. Nevertheless, the empirical strand of literature has yet to deliver conclusive and consistent results due to differences in the econometric methodology. In the spirit of Hamilton and Flavin (1986) and Threhan and Walsh (1988), the common practice is to examine if government debt or deficit follow a stationary process. In Quintos (1995) sustainability requires deficit stationarity in the differences. Blanchard et al. (1990) adopt a more relaxing measure, in which a tax-rate, which ensures that the growth in debt/GDP ratio does not exceed the interest–growth differential (the difference between the real interest rate and the growth rate), is used to identify sustainable fiscal policies in OECD countries between 1979 and 1999. The absence of a limit on the debt/GDP ratio implies that permanently increasing debt/GDP ratios are sustainable as long as the former inequality condition holds.³ Another common empirical practice is to rely on cointegration tests between government spending and revenues. In a more recent empirical study, Afonso and Rault (2010) examine the fiscal sustainability of the 15 EU member countries prior to 2001 over the period 1970-2006 using both stationarity and cointegration tests. The fulfillment of the sustainability condition requires that the present value of the future stream of primary surpluses as a share of national income matches the value of the outstanding stock of debt, which was the case for the 15 EU members as a whole. Similarly, Bajo-Rubio et al. (2010) confirm a sustainable fiscal regime for Spain for the period 1850-2000 solely using cointegration tests. A feature of the empirical literature is, as already mentioned, that the results related to sustainability solely derive from the government's intertemporal budget constraint, and thus are disadvantageous in that they cannot capture the effects which fiscal policy itself has on the wider economy, i.e., on interest rates, market expectations, and output.

The limitations of the above studies call for a different approach – one which is able to capture the interaction between the private and public sectors through its effects on macroeconomic variables. As already mentioned, the lack of a common definition of debt sustainability is one of the main reasons for the absence of an established, conventional framework. Another obstacle derives from the different schools of thought in economics. Recent followers of Keynes' view of sustainable debt issues are Aspromourgos, Rees, and White (2009), whose study establishes that permanent primary deficits are sustainable in the presence of inflation-driven strategies. In Furceri and Mourougane (2010) the trade-off between economic growth and crowding-out effects of fiscal expansionary policy are analyzed in a more complex model, where short-term interest rates are set

³This result contrasts with the pioneering results of the theoretical and older study of Sargent and Wallace (1981), where a constant increasing debt/GDP ratio is proven to be unsustainable with a limit on the demand for government debt.

by monetary decisions following the Taylor type rule. The results of a calibration exercise, that models a temporary increase in public consumption of 1% of GDP, imply that benefits from the increase in short-term activity can outweigh the increase in the debt/GDP ratio, so it is an effective short-term measure to support activity.

Following neoclassical growth theory, Diamond (1965) developed an overlapping generations model (OLG) of exogenous growth, which inspired the literature to depart from the traditional present budget balance perspective in favor of a life-cycle model. It has become the basic framework of the neoclassical approach for identifying and investigating the deeper structural determinants of fiscal sustainability and is largely split into deficit and debt sustainability issues. For example, Masson (1985) examines the sustainability of primary deficits with a two-period OLG under uncertainty to show that the dynamic inefficiency condition – which requires the long-term interest rate to be below the growth rate – does not ensure a sustainable constant primary deficit-to-output ratio. The model is calibrated with arbitrary parameter values to show a possible resulting path for interest rates. Similarly, Nielson (1992) gives conditions for a sustainable primary deficit in a rudimentary closed-version growth model but with infinite overlapping families. The dynamic inefficiency condition ensures a sustainable primary deficit only for high-growth economies and under very restrictive conditions, namely with modest-sized public sectors relative to GDP, low interest rates, a high capital's factor share in production and a low rate of time preference. The effects of fiscal expansions and contractions on the model responses as well as a calibration exercise are left open for further discussion. Chalk (2000) contributes to the literature on deficit sustainability by numerically deriving the theoretical limit to the maximal sustainable bond-financed deficit implied by a two-period OLG model. As opposed to Blanchard et al. (1990) governments cannot rely on negative interest–growth differentials to run a Ponzi-game. Instead, Chalk (2000) identifies a limit on the size of sustainable deficits which occurs at the inefficiency condition; that is, before the interest rate exceeds the growth rate. The notable contribution of Chalk (2000) is the calibration of the model to match annual facts of the U.S. economy from 1954–1980. The calibration results show deficit limits of about 5% of GDP. Azariadis and Reichlin (1996) use a two-period life-cycle model to analyze the effect of public debt on the dynamics of OLG economies with production externalities rather than focusing on the sustainability issues per se. Nevertheless, initial positive levels of public debt are shown to be responsible for low-level development traps and thus become implicitly linked to sustainability issues. De la Croix and Michel (2002) investigate the effects of the introduction of public debt on the dynamic properties of a two-period OLG model to derive conditions for ensuring debt rather than deficit sustainability. Ranking and Roffia (2003) confirm the robustness of the ‘interior maximum’ solution for a closed economy to both Cobb-Douglas and CES utility functions. Despite the fact it does not include a calibration exercise, it gives valuable insights concerning the existence of a maximum sustainable debt level. Voyvoda and Yeldan (2005) develop an OLG, small open-economy model of exogenous growth to study the growth and wel-

fare effects of Turkey's 2001 austerity program (TTSE). Similarly to the present paper, they approach the calibration exercise with a baseline scenario and expose it to exogenous growth shocks in order to illustrate possible adjustment patterns. The results of the exercise expose the TTSE program to large welfare and growth deficiencies and suggest the adaptation of other fiscal programs.

It must be noted that exogenous growth is not alone in the life-cycle strand of literature on debt sustainability issues. Following the seminal work of Lucas (1988), and Diamond's (1965) life-cycle model, Moraga and Vidal (2004) assess the effect of budgetary imbalances on the dynamics of the economy and the outlook for economic growth in an OLG framework with an endogenous growth model of the human capital type. The notable contribution of Moraga and Vidal (2004) is the model's calibration exercise to the three-period OLG model used to simulate the effects of Europe's demographic shock, which shows how the shock can lead to unsustainable debt levels in the absence of constant debt policy rules. Annicchiarico and Giammarioli (2004) employ a two-period OLG framework with endogenous growth of the learning-by-doing type to study the path of convergence of an economy towards well-defined fiscal targets. A simulation exercise illustrates the effect of different fiscal variables on the dynamics of the rate of growth of the economy along the adjustment process towards the long-run equilibrium. Braunigner (2005) examines the existence of permanent budget deficits in an OLG framework of endogenous growth. A limit on the sustainable overall deficit/GDP ratio ensures the existence of two steady states. Creel and Saraceno (2008) calibrate a simple non-OLG endogenous growth model to U.S., French and U.K. data to find the public spending to capital ratio which maximizes growth. The calibration results show that the three countries are below the optimal level of spending, when spending positively enters the production function. Contrary to the present paper, the variables in the calibration exercise are in capital as opposed to GDP ratios. In the form of a Ramsey-Swan endogenous growth model, Greiner (2011) analyzes the sustainability of budget balances and permanent budget deficits but not in an OLG framework. The debt management rule requires the primary surplus/GDP ratio to be a positive linear function of present and accumulated debt. Furthermore, the results of a simulation exercise show that permanent deficits are unsustainable. Despite the fact the results of the above papers derive from endogenous growth, the aim of the papers is to some extent to give a rationale for the implementation of fiscal rules to preserve sustainability. It hence supports the motivation of this paper's investigation.

The above discussion of recent literature on fiscal sustainability has identified the neoclassical OLG approach as a solid theoretical framework for long-run policy planning. The discussion also reveals how this paper can contribute to the mainstream of this strand of literature with the numerical identification of a debt/GDP limit by means of a calibration exercise. Furthermore, it should provide fiscal policy makers with an indicator for the consequences of uncontrolled debt ratios.

3 A Simple Economy

The structure of the economy takes the form of a two-period overlapping generation (OLG) model of government debt and physical capital in line with Diamond (1965). In this model we choose a specification similar to Ranking and Roffia (2003) and Farmer and Zotti (2010), where a *maximum level of sustainable debt*⁴ is shown to exist at a point of non-degeneracy capital-ratio $k \neq 0$ in both closed- and open-economy versions, respectively. Motivated by these results – where a ‘constant stock of government debt’ policy may not be sufficient for fiscal sustainability once these levels are surpassed – we develop and calibrate an OLG model of exogenous growth to a parameterization that can more plausibly be matched to actual data. Apart from the CES intertemporal utility function the notable difference in our model is the inclusion of bequest activities, which allow us to relate the discussion to the present debt and deficit values.

Before we proceed with the formal modeling a few remarks regarding the model’s specification should be made. Firstly, there is no money in this model, only real debt. Secondly, it takes place in a non-stochastic world,⁵ as it is argued that ‘the results will not be changed by introducing uncertainty’ (Chalk 2000, p. 298). Finally, as we wish to perform a calibration exercise, we present the model in discrete time. We do not intend to analyze the impact of debt itself; rather, we make use of the established tools and results of prior studies to examine the fiscal sustainability of our selection of advanced economies.

3.1 The Firm

In each period t , production occurs according to a constant returns-to-scale Cobb–Douglas production function. Perfectly competitive firms employ two factors of production, capital services K_t and labor services L_t , scaled by $A > 0$ to produce output Y_t :

$$Y_t = A (K_t)^\alpha (H_t)^{1-\alpha}, \quad (1)$$

where $H_t = h_t L_t$ denotes efficiency-weighted labor input, with $0 < h_t$ being the labor productivity per employee. As we do not analyze systematic changes in labor participation and unemployment rates, labor input moves along with population growth and can be normalized ($L_t = N_t$). Production elasticity of physical capital is denoted by $\alpha \in (0, 1)$ and of labor $1 - \alpha$, respectively. Production per labor in efficiency units is then $Y_t/H_t := \tilde{y} = A \left(\frac{K_t}{H_t} \right)^\alpha$. In order to write the model in terms of ‘income ratios’ instead of efficiency units⁶ we need to rewrite the production function as

⁴Reinhart, Reinhart, and Rogoff (2012) identify a 90% debt/ GDP threshold, above which there is a negative causal relationship between high debt and growth. Although their study takes a different approach, their long-term robust results confirm; (1) the crowding-out mechanism behind the present paper’s theoretical model, and (2) the existence of a maximum debt ratio.

⁵See for e.g., Masson (1985) for a related OLG model under uncertainty.

⁶We prefer writing intensity ratios in terms of income due to the obvious advantages linked with the calibration exercise. For identical arguments see Chalk (2000, p.304).

$$\frac{Y_t}{H_t} := \tilde{y} = A^{\frac{1}{1-\alpha}} k_t^{\frac{\alpha}{1-\alpha}}, \quad (2)$$

where $k_t = K_t/Y_t$ is defined as the *capital-ratio* (more precise capital/output-ratio). From profit maximization we obtain the net return on physical capital

$$r_t = \alpha k_t^{-1} - \delta \quad (3)$$

and the wages per labor efficiency unit w_t as

$$w_t = (1 - \alpha) A^{-1} k_t^{-\alpha}. \quad (4)$$

3.2 The Consumer

As usual, the two generations are assumed to overlap in each period t . Population grows at an exogenously given rate n . New generations of size N_t enter the economy at date t and live for two lifetime periods, working during the first and retiring and living off their savings during the second.

Households are identical within and across generations. Specifically, their preferences depend on a intertemporal CES utility function u over consumption per efficiency unit in both lifetime periods (c_t^1, c_{t+1}^2) :

$$u = \left[\beta (\tilde{c}_t^1)^{-\rho} + (1 - \beta) (\tilde{c}_{t+1}^2)^{-\rho} \right]^{\frac{-1}{\rho}}$$

where the superscript stands for each generation's lifetime period and β and ρ are the usual parameters of CES utility. During the first lifetime period the young generation allocates wage income per labor in efficiency units to consumption c_t^1 and transfers consumption between periods t and $t+1$ by saving s_t to receive $(1 + i_t)s_t$, in exchange, where i_t is the real interest rate in t . The old generation spends all savings on consumption c_{t+1}^2 in the second lifetime period. Furthermore, we assume that the old generation bequeaths the total capital K_t and bonds B_t with the share γ if total bequest does not restrict full capital transfer.⁷ Hence, this intergenerational transfer of stocks does not explicitly effect the savings (flow) decision. In addition, the government levies taxes per wage income at rate τ_t in the first period so that total income is reduced by the tax burden, given the disposable wage income $w_t(1 - \tau_t)$.⁸ A

⁷This assumption implicitly takes care of liquidity properties of the two assets. Capital will be more likely to be bequeathed and debt will be more likely to be partly consumed by the old and partly bequeathed.

⁸At this point we also differ from related modeling like Farmer and Zotti (2010). While they implement a constant tax burden on 'labor in efficiency units' we tax labor income proportionally. The implication is straightforward. In Farmer and Zotti (2010) a constant tax rule on "labor efficient units" will leave the tax payments unchanged even if wage income increases with a higher capital intensity. That is, the tax burden increases per income unit. Hence, their rule implicitly establishes a mechanism of changing taxes per wage income when capital intensity changes, which has clear implications for the reported results. In order not to rely on this specific effect we establish a fixed tax ratio per income unit. In this case higher income (for whatever reason) would be taxed with the same ratio, and the tax burden per income is fixed.

representative consumer maximizes utility of consumption from their income per efficiency unit subject to their budget constraints:

$$\begin{aligned}\tilde{w}_t(1 - \tau_t) &= \tilde{c}_t^1 + \tilde{s}_t \quad \text{and} \\ (1 + i_t)\tilde{s}_t &= \tilde{c}_{t+1}^2\end{aligned}$$

From the first order conditions we obtain $1 + i_t = \frac{\beta}{1-\beta} \left(\frac{\tilde{c}_{t+1}^2}{\tilde{c}_t^1} \right)^{\rho+1}$. Inserting the budget constraints and re-arranging gives the household's savings rate⁹ s (savings per income):

$$s_t = \frac{(1 - \alpha)(1 - \tau_t) \left(\frac{1-\beta}{\beta} \right)^\varphi}{\left[(1 + i_t)^{1-\varphi} + \left(\frac{1-\beta}{\beta} \right)^\varphi \right]} \quad (5)$$

with the elasticity of intertemporal substitution $\varphi = \frac{1}{\rho+1}$. Note that CES utility allows for a change in savings rates when interest rates change. Depending on the elasticity of substitution, the savings rate may increase ($\varphi > 1$) or decrease ($\varphi < 1$) with an increasing interest rate. For ($\varphi = 1$) we obtain a constant savings rate independent of the interest rate.

3.3 The Public Sector

Let B_t be the net value of government debt at the start of period t . Furthermore, the government levies taxes T_t , and consumes G_t . In addition, the government reimburses outstanding debt B_t with the accrued interest on government debt i_t . Interest is payable during period t . Existing debt is refinanced each period, and total government deficit is financed by issuing new bonds. Hence, rewriting total debt accumulation 'debt-ratio' $b_t := B_t/Y_t$, defines each period's debt accumulation per unit of output with the following first order difference equation in b_t :

$$\eta_t b_{t+1} = d_t + (1 + i_t)b_t = g_t - \tau_t + (1 + i_t)b_t \quad (6)$$

with $d_t = \frac{D_t}{Y_t}$, $g_t = \frac{G_t}{Y_t}$ and $\tau_t = \frac{T_t}{Y_t}$ being the primary deficit-ratio, government expenditure-ratio, and tax rate, respectively. η_t denotes the growth factor of output which in a potential steady state becomes equal to the product of the population growth factor $(1 + n)$ and the growth factor of labor productivity $(1 + \varepsilon_t)$, such that $\eta_t = (1 + \varepsilon_t)(1 + n_t)$.

3.4 Equilibrium and Stationary Conditions

1 Asset market no-arbitrage condition

We assume the two assets, public debt and productive capital, to be perfect substitutes from the investor's perspective. The no arbitrage condition equals

⁹See the difference to Ranking and Roffia (2003), p. 222.

the real return on public debt i_t and the real net return of productive capital r_t :

$$i_t = r_t = \alpha k_t^{-1} - \delta \quad (7)$$

where δ denotes the depreciation rate of physical capital.¹⁰

2 The government's fiscal rule: 'constant debt-ratio rule'

As a necessary condition for a long term sustainable government budget policy we need to establish a *fiscal rule* for the primary deficit-ratio d_t . Unless governments want a permanently increasing debt/GDP ratio¹¹ it must fix the debt/output-ratio (hereafter '*debt-ratio*') at a desired level. Hence, this chosen debt-ratio must remain constant (or at least not increase systematically) over time so that $b_{t+1} = b_t$ for all t .¹² This condition implies for society that government debt will neither automatically disappear over time as a relevant resource absorbing element ($b_{t+1} < b_t$), nor will it become an increasing and eventually overwhelming burden on future generations. Implementing this *fiscal rule* implies for (6) that the primary surplus $-d_t$ must exactly cover the interest payments on outstanding debt. Hence, it holds that

$$-d_t = (1 + i_t - \eta) b_t. \quad (8)$$

This 'constant debt ratio rule' implies that in case g_t is chosen by the government the lump-sum tax rate τ_t becomes endogenously determined to satisfy $\tau_t = g_t + (1 + i_t - \eta)b_t$.¹³ Recall that negative interest-growth differentials ($i_t < \eta - 1$) allows governments to run up primary deficits ($g > t$) while maintaining a constant b . Notwithstanding the validity of the latter in the short-term, the theoretical literature discussed above has identified dynamic inefficient-driven policies as unsustainable in the long-run.

3 Asset market equilibrium and real capital accumulation

Asset markets clear in period t whenever the sum of the supply of resources by the first generation of households s_t , the exogenous international capital inflow rate¹⁴ f , and the total bequest $(k_t + \gamma b_t)$ equals the next generation's total

¹⁰Chalk (2000) includes a risk premium to examine possible bias. He concludes that the inclusion of a risk premium would not change qualitative results, so we will not consider this aspect further.

¹¹The unsustainability of such a scenario was first discussed by Sargent and Wallace (1981).

¹²For identical arguments see e.g., Diamond (1965 p. 1137), De la Croix and Michel (2002, pp. 216-226), and Farmer and Zotti (2010, p. 294).

¹³Note that this could have been written in terms of a government spending rule for a given tax revenue.

¹⁴In this simplifying model we only include the flow component of international assets. Including international stocks would provide more interesting information about the international component of total assets and the steady state shares. However, this would not change the fact that stationary flows reveal whether international resources add to the current budget constraint, or whether a resource outflow reduces domestically available resources.

demand for both public debt B_{t+1}/Y_t and private capital K_{t+1}/Y_t . Formally, in GDP ratios we have:¹⁵

$$\eta_t k_{t+1} + \eta_t b_{t+1} = s_t + f_t + k_t + \gamma b_t. \quad (9)$$

4 Qualitative Dynamic Analysis

4.1 Stationary Economy under Constant Debt-Ratio Rule

Stationarity in this setting contains two elements: (i) the *constant debt-ratio rule* (as *fiscla rule*) so that debt-ratios do not systematically change over and $b_{t+1} = b_t$, and (ii) for the real economy *stationary capital-ratio* as well as the *growth rate* remain constant over time so that $(k_{t+1} = k_t)$ and $(\eta_{t+1} = \eta)$. That is, we derive a curve for all potential simultaneous stationary states in this economy with respect to b_t and k_t , and therefore derive a locus of stationary debt-ratios which can be supported by stationary capital-ratios.

The general dynamics of capital-ratio, k_t :

Under the ‘*constant debt-ratio rule*’ capital-ratio dynamics are restricted to this fiscal rule (8) and hence¹⁶ can be described as

$$\eta_t k_{t+1} = \frac{(1 - \alpha)(1 - \tau_t) \left(\frac{1 - \beta}{\beta}\right)^\varphi}{\left[(1 + i_t)^{1 - \varphi} + \left(\frac{1 - \beta}{\beta}\right)^\varphi\right]} + f_t + k_t + (\gamma - \eta_t) b_t. \quad (10)$$

Stationary capital-ratio under the ‘constant debt-ratio rule’: for the implemented ‘constant debt ratio rule’ we can determine the stationary debt-ratio b_t^s for each stationary capital-ratio k_t and discuss the notion of sustainable debt.

Proposition 1 *If the government implements a “constant debt-ratio rule”, there exists a stationary sovereign debt-ratio b_t^s for each stationary capital-ratio $k_t > 0$.*

$$b_t^s := \frac{(1 - \alpha)(1 - \tau_t)}{\left[(1 + \alpha k_t^{-1} - \delta)^{1 - \varphi} \left(\frac{1 - \beta}{\beta}\right)^{-\varphi} + 1\right] (\eta_t - \gamma)} + \frac{f_t}{(\eta_t - \gamma)} + \frac{(1 - \eta)}{(\eta_t - \gamma)} k_t \quad (11)$$

The stationary debt curve, ‘ b_t^s -curve’, has a maximum $b_t^{\max}$ which denotes the maximum debt-ratio the economy can maintain. Non stationary dynamics is described by $k_{t+1} \geq k_t$ if $b_t \leq b_t^s$ and $k_{t+1} \leq k_t$ if $b_t \geq b_t^s$.¹⁷

¹⁵To keep it as simple as possible, for the formal model we do not consider potential reevaluation of assets. In recent years we have observed that real assets, in particular real estate prices developed differently from the GDP deflator. This relative price change in real assets may under certain conditions affect the budget constraint. However, for the calibration we include the change in relative prices and provide a formal modelling in appendix 1.

¹⁶Using (6) and (9).

¹⁷For a proof see appendix 1.

Figure 2: Debt Dynamics

In figure 2 the stationary debt curve, ' b_t^s -curve', describes the debt-ratio an economy can afford at each stationary capital-ratio for given parameter values of φ , like (a) $\varphi < 1$, (b) $\varphi = 1$ and (c) $\varphi > 1$.

The stability of the economy can be discussed by looking at the four areas of adjustment: A, B, C, D . The arrows show how the economy adjusts if debt is higher or lower than the stationary value. If at the current capital-ratio k_t the government chooses a debt-ratio below the b_t^s -curve (area A), the economy will adjust towards a higher capital intensity and move steadily to the right until the b_t^s -curve is reached. In the case where the government chooses a debt-ratio above the b_t^s -curve for a given k_t , like in areas B, C and D , the economy, embarking on its own law of motion, will reduce capital intensity and move to the left. The above adjustment process will lead to two qualitatively different stability regimes. The existence of a maximum on the stationary debt curve and the adjustment pattern have important implications for the ability of the economy to maintain a desired debt ratio.

Definition 2 (Sustainable Debt-Ratio) : A debt-ratio $\bar{b}_t$ is 'sustainable' if a policy of a 'constant debt-ratio rule' for $\bar{b}_t$ leads towards a stable stationary, strictly positive capital-ratio, $0 < k | \bar{b}_t$

Proposition 3 According to definition (1) areas A and B in Figure 2 describe conditions for a sustainable debt while areas C and D describe conditions of unsustainable debt.

The policy implications of a sustainable or unsustainable regime are dramatic. (i) In a sustainable debt regime fiscal policy-makers have the freedom to choose, for whatever current policy purpose, the desired ratio of debt within

the limits of the sustainable regime. (ii) Policy-makers must be aware of their limited choices. If they do not want to push the economy onto an unstable trajectory, which may lead to a degenerating capital process and an eventual collapse, they are not completely free to implement fiscal expansionary measures. (iii) Once the sustainable debt limit b^{max} has been surpassed, the freedom of choice turns into a forced policy of long term stabilization. Even if current short term conditions suggest expansionary fiscal policies the unstable trajectory needs to be maintained. The longer the economy moves on this unstable path, the stronger will be the pressure to return to a sustainable regime. The freedom of action turns into a pressure to react even under unfavorable conditions.

5 Calibrating the model

The calibration of overlapping generations has been gradually taken up by a series of papers and is well established in the literature. Yet to the authors' best knowledge none of these exercises have been parameterized to consider (1) the effect of the latest financial crisis with a massive deficit policy and the most recent debt accumulation, and (2) a potential sustainability problem of the world's largest advanced economies in its aftermath. While the debt crisis is so far seen as a problem of Europe's periphery economies, this exercise should shed more light on the debt position of the largest advanced economies, and hence reveal global threats generated by reaching upper debt thresholds.

At this point we wish to reiterate these calibration exercises since they have a number of shortcomings and should hence be seen as an instrument of broad analysis and a source of orientation rather than precise predictions. Furthermore, the resulting scenarios may be sensitive to changes in assumptions and parameter values. Despite the latter, we strongly believe that the results obtained through this tool are valuable and of immediate importance to policy-makers in light of the unprecedented speed of the recent debt accumulation which makes this issue one of extreme urgency. As this discussion seems not to have escaped recent publications, our aim is to identify the recent problem of unsustainable debt using a broad but also quick calibration exercise.

1) As usual,¹⁸ we start with a baseline scenario that covers two major ideas as far as possible: (i) for the time before the 2008 crisis we want to identify the characteristics of the real economy suggested by the model for the four largest advanced economies, namely Germany, the U.S., Japan, and the U.K.; (ii) we select a period that can be considered stationary for the real economy and simultaneously stationary (closest to a following a constant debt-ratio policy) with respect to the fiscal regime.¹⁹ In other words, we use an episode where a

¹⁸For similar OLG calibration exercises see e.g., Chalk (2000), Moraga and Vidal (2004), and Voyvoda and Yeldan (2005).

¹⁹Chalk (2000) selects a period for the baseline scenario which avoids periods of high deficits in order to calibrate the model for a balanced budget.

simultaneously stationary private economy can maintain a constant debt rule, and hence a fiscal regime with a non accelerating debt-ratio that can support a stationary private economy.

2) With the help of the baseline scenario we discuss whether the four economies have drifted on unstable debt and capital trajectories in the aftermath of the financial crisis

3) If an economy faces the conditions of unsustainable debt, what is required to return to a stable and hence sustainable debt trajectory?

5.1 Baseline Scenario

5.1.1 Baseline Period

Concept of stationarity: The purpose of the baseline scenario is to bring the economies to a base-period which reflects the latest potential ‘long-term stationarity period’ of the respective country. Guided by the concept of the model we look at (i) stationarity in the real economy and (ii) stationarity in debt behavior. Stationarity in the real economy is indicated by a rather stationary capital-ratio (k), and a stationary real growth factor (η). A fiscal regime, according to the model, is considered as stationary if the constant debt rule (8) holds.

Database: For consistency and comparability we exclusively rely on the OECD data base (for details see below). Even if this is the most comprehensive international data base available with respect to the relevant macro data we are aware that we are restricted to some extent. However, we will compute all parameters that are not available in the OECD database by endogenously using the equations of the theoretical model. Only intertemporal preferences are exogenous. The first data for Germany is available from 1991 onwards, after reunification. Therefore, we tried to cover the period 1991 to 2007. However, for the U.K. and Germany capital data (tangible fixed asset)²⁰ are not available before 1995, and for Japan not before 2001.

²⁰Since data availability and the identification of a baseline period depends on the capital concept, this interaction needs to be briefly discussed. The longest time period available for indicating real capital, as a productive capital in the production process of the model, for a capital concept that the OECD economic outlook calls ‘productive capital’. Hence, this variable is our first choice for ‘capital’ to identify a stationary capital output ratio for 1991 to 2007. However, we know that ‘productive capital’ is only a fraction of total capital input in the aggregate production process. In particular, housing capital is an important additional element of fixed capital and capital accumulation. Therefore, if housing is excluded from the calibration data we do not have consistent budget constraints, and might run into the problem of generating biased results in terms of a larger gross return on capital. E.g., for the U.K. the ‘tangible fixed capital ratio’ including dwellings is more than 2.5 times higher than a capital measure excluding dwellings. Hence, using a measure of capital that excludes dwelling will lead to an inconsistent budget constraint and overestimate the gross return on capital. As the variables are directly computed from the model, we are forced to address this problem by using the capital concept ‘tangible fixed assets’ from the national accounts of the OECD. Finding stationarity for ‘productive capital’ for 1991 to 2007 (for the U.K. 1988-2007) and being restricted to the period 1995 to 2007 for ‘tangible fixed assets’ we regard this sub-period, too, as stationary, even if we cannot apply respective tests and provide statistical support due to shortage of data.

Identification of stationary periods for Germany, U.S., and the U.K.: During the whole period of 1991-2007, ‘productive capital-ratio’ and GDP growth rates were statistically stationary²¹. However, even if ‘productive capital’ in the OECD definition is closest to the concept of capital in an industry production function we must take into account that productive capital is only a fraction of total capital input in an economy wide aggregate process. In particular, housing is quantitatively important so we have to extend the capital concept to the concept of tangible fixed assets. However, data availability for tangible fixed assets restricts the choice of a stationary period for the real economy 1995 to 2007. Furthermore, a constant debt policy is identified when the constant debt rule is satisfied on average for a period of at least five years. A period is generally stationary if both sets of conditions (stationary real economy and constant debt rule) are simultaneously satisfied. Hence, for the US we identify 1995 to 2003, for the U.K. we find 1995 to 2004 and for Germany we regard 1996 to 2000 as the baseline period. The corresponding characteristic variable values for the stationary real economy and a stationary fiscal regimes are found in Table 1.²²

Identification of the stationary period for Japan: While Japan’s real economy²³ seemed stationary with respect to productive capital-ratio and the growth factor during the period from 1991 to 2007, the most recent ‘constant debt-ratio’ fiscal regime was between 1988 and 1992.²⁴ We hence decided to take the most recent period available in terms of appropriate asset data which is 2001 to 2007. We therefore combine data for the real economy for the period 2001 to 2007 with the latest period of constant debt policy, which was 1988 to 1993.

Note that with these benchmark periods we choose optimistic conditions in terms of the debt regime. Since the initial levels of debt are crucial when comparing and determining sustainable deficit policies, (see Chalk 2000, p. 294), we were forced to select optimistic periods as these were the only periods during the last 20 years in which debt did not substantially grow.

5.1.2 Extracted Variables, Calculated Parameters, and Baseline Scenario

As we have already written the theoretical model in output-ratios instead of the customary ‘labor in efficiency-ratios’,²⁵ we now have the obvious advantages of using empirically available GDP ratios for the calibration exercise. The results

²¹ ADF and PP tests show stationarity for the U.S., U.K., and Germany. For the U.K. we obtain this result for 1988-2007.

²² This statement is not precise with respect to the relative prices of dwellings in the U.K. In the U.K. there was a significant and ongoing relative price increase in dwellings particular after 2000 that affected tangible assets. This is also true for the United States.

²³ According to statistical stationarity tests productive capital and η were stationary for 1991 to 2007.

²⁴ Another legitimate reason why we abstract from choosing 1988-1992 as a potential stationary real economy period, despite the fact that data for Japanese ‘tangible capital assets’ is available from 2001, is the drastic development in the structure of the Japanese economy since 1992, which would make a reflection of a stationary real economy dubious at best.

²⁵ As opposed to Creel and Saraceno (2008), but following Chalk (2000).

based on GDP-ratios endow policy-makers with easy interpretations and economically meaningful figures. With that in mind, we next describe the economic variables extracted from the OECD data set and the endogenously determined parameters which, according to the model, support the stationary period. For more details about the selected variables see the tables in Appendix 2.

Table 1, 'baseline values' for fundamental variables and parameters:

		U.S.	U.K.	Ger	Jap
<i>REAL ECONOMY</i>		1995	1995	1995	2001
<i>stationary values (OECD)</i> ²⁶		-2007	-2007	-2007	-2007
-long term gov. nom. bond yield (%)	i^{nom}	5.22	5.55	4.77	1.41
-GDP deflator, percentage rate of change (%)	$\Delta p/p$	2.22	2.31	0.79	-1.31
-real long term gov. nom. bond yield (%) ²⁷	i	3.01	3.24	3.99	2.72
-foreign financial inflow/GDP ratio (%)	f	3.83	1.79	-1.62	-3.45
-GDP growth rate (%)	$\eta - 1$	3.1	3.2	1.6	1.4
-capital/GDP ratio	k	2.7	4.17	3.12	2.81
<i>endogen. computed variable and parameters</i> ²⁸					
-gross return on capital (%)	r^{gross}	15.04	9.22	13.18	14.2
-net return on capital (%)	r	3.01	3.24	3.99	2.72
-deprec. rate of capital (%)	δ	12.03	5.97	9.20	11.5
-elast. of labor prod. ²⁹	$1 - \alpha$	0.59	0.62	0.59	0.60
-bequest-ratio for gov.bonds	γ				
	$\varphi = 1.5$	0.68	0.76	0.74	0.13
	$\varphi = 1$	0.69	0.77	0.74	0.15
	$\varphi = 0.5$	0.69	0.77	0.75	0.16
<i>exogenous choices, parameters values</i>					
-elast. of intertemp. subst.	φ	0.5 / 1 / 1.5			
-eval. of future consumption ³⁰	$1 - \beta$	0.5			
<i>FISCAL REGIME</i>		1995	1995	1995	1988
<i>stationary values (OECD)</i>		-2003	-2004	-2000	-1993
-gov. debt/GDP (%)	b	43.05	26.99	33.95	14.80
-gov. disbursem./GDP (%) ³¹	g	35.44	40.78	47.73	31.25
-gov. receipts/GDP (%)	τ	33.93	39.15	45.95	32.75
-gov. prim.surplus/GDP (%) ³²	$-d$	1.21	0.80	1.08	1.49

²⁶ Data are directly taken from the OECD or derived by using standard definitions. For details see appendix 2.

In order to determine the values that are not in the OECD statistics for the defined period we need to exogenously choose plausible values for the unobservable parameters for intertemporal consumer preferences β and φ . Since β is the weight at which a consumer evaluates today's consumption, and $1 - \beta$ is the next period's consumption weight, we suggest the most conservative scheme with equal weights ($1 - \beta = \beta = 0.5$). This scheme regards future consumption as equally important to today's consumption and hence increases savings as much as possible. Realistically we would expect $1 - \beta < \beta$; however, because we hardly observe these values and we would like to describe an optimistic savings behavior, we suggest 0.5 for these values. For φ , the elasticity of intertemporal substitution, we assume $0 \leq \varphi$. For $\varphi < 1$ substitution is very low, while for $\varphi > 1$ it is high. The theory implies that a reduction in real interest rates increases the consumers' savings rate if $1 > \varphi$ and savings react negatively to increasing interest. If $1 < \varphi$ then the savings react positively to rising interest rates. In order to cover both scenarios we calculate three cases (i) $\varphi = 0.5$, (ii) $\varphi = 1.5$ and also (iii) $\varphi = 1$, which is the Cobb Douglas utility case, leading to a constant savings rate. Further, for each of the cases we need to identify the share of government debt that is passed to the young generation γ . This parameter is determined by solving the modeled system at steady state values (11) of this parameter.

Looking at Table 1 we see that the variables computed from the model equations are realistic enough to suggest a real behavior of the economies. Although we have a very simple model we obtain broad 'rule of thumb' parameter values that are within the range of what we would expect to see. It is particularly interesting to note that Japan, which has a very different monetary regime compared to the other three economies, with very low nominal interest rates and a deflationary process, has similar parameter values in real terms. Hence, the Japanese real economy seems to work conventionally under the veil of unconventional monetary conditions.

5.2 Unsustainable Debt Trajectories

With the help of the baseline scenario we discuss whether the four economies under consideration may be moving onto unstable trajectories. In order to

²⁷Nominal rate minus p.c. rate of change of GDP deflator.

²⁸To derive these variable values, equations from the theoretical model were used to calculate the values of the respective variables using OECD data. For details see appendix 2.

²⁹See e.g. Voyoda and Yeldan (2005) and Furceri and Mourougane (2010) for values of $\alpha > 0.4$, Moraga and Vidal (2004) for $\alpha = 0.3$ and Chalk (2000) for $\alpha = 0.25$.

³⁰An assumption about β is a standard feature in OLG calibration exercises. We assume no discount of the future.

³¹Total disbursement including interest payments.

³²Derived, see appendix 2.

Figure 3: Actual Deficit Policy Versus Constant Debt-Ratio Rule for Conditions of the Baseline Scenario

identify unsustainable debt conditions we look at two indicators. First, we look partially at public sector behavior and verify whether governments are following a constant or reducing debt-ratio rule. Second, we look at the macro conditions and examine whether an economy is moving on a non degenerating trajectory towards a capital-ratio strictly larger than zero once governments applied a constant debt rule at the current debt-ratio.

5.2.1 Deviation from the Constant Debt-Ratio Rule

The first indication of an unsustainable debt development is whether the constant debt-ratio rule is met at each debt ratio. Therefore, we take the rule (8) and the values from the stationary period (in particular the real interest rates and growth rates) and compute the deficit policy required to match the constant debt-ratio rule. The results are presented in Figure 3. In this figure the solid line denotes the primary budget surplus that is required at each year's debt ratio to match condition (8), while the solid line describes the actual primary surplus in the respective year. The dotted line denotes the primary surplus as a ratio of potential GDP as published by the OECD. We can see that during the last two decades this partial criterion is met only during the episode we have identified as being closest to stationarity, and hence serves as baseline period. Without this exceptional episode all four economies could not establish a fiscal regime capable of supporting a stationary debt-ratio. This very simple indicator merely tells us that the considered economies are moving on a long term path rising debt ratio.

5.2.2 Unsustainable Sovereign Debt-Trajectories

While the conditions discussed in the previous section are partial and constitute rather simple views on constant debt-ratio budget rules, we can also take a look at the full economic environment.

As suggested in definition 1, a debt level $\bar{b}_t$ is 'sustainable' if a 'constant debt rule policy' for the given debt level steadily moves towards a strictly positive stationary capital ratio ($0 < k | \bar{b}_t$) without a change in fiscal rule. In figure 2, to maintain a constant debt to GDP ratio without a change in the fiscal rule the economy has to be below the b_t^s -curve or below the maximum of this curve. Region A and B in Figure 2 support a sustainable debt development. At primary deficit levels above the maximum of this curve, region C and D in Figure 2, we obtain an unstable permanent increase in the debt ratio or a decrease in the capital ratio.

In Figure 4 we compute the stationary curves for various values of φ and identify the sustainable regions for the economies under consideration. We see that all economies are either above or close to an unsustainable trajectory. None of the conditions in the U.S., Europe, and Japan are comparable of supporting a sustainable debt development. To illustrate the effects of the relative increase

Figure 4: Baseline Scenario for Stationary Debt- and Capital-Ratios

in housing prices³³ we also calculate the 2011 ratios for an adjusted relative housing price.³⁴ Hence, all of these economies are presently playing, or are close to playing a Ponzi scheme. Even if they have not crossed the line to an unsustainable regime the capital-ratio that eventually supported the current debt-ratio would become very low. Hence, the current debt ratios would clearly lead to a strong reduction in capital intensity of the production process in the long term. However, what is the policy implication of this finding?

First, the world's largest economies and in particular their fiscal and debt regimes are not at all as sound as their governments claim. Due to the enormity of the financial crisis the governments' ability to stabilize and manage macro conditions has reached or is close to an absolute limit, rendering governments and indeed the entire economies vulnerable. While the US, the U.K., and Japan are able to react independently this finding is particularly delicate for Germany. Germany is regarded as the stabilizing anchor in the eurozone that is able to stabilize other European economies by absorbing (or give warranties on) other countries' debt. In a later section we will see that Germany's adjustment requirements are indeed less dramatic than in the other considered economies. However, there are already substantial hidden liabilities for Germany, too, that are not accounted for in this analysis. Even if Germany seems the soundest economy in the eurozone the results indicate that Germany, too, has reached its limits. If the anchor slips onto an unsustainable trajectory the entire region may fall victim to instability.

Passing this debt ceiling changes conditions for making policy. If a government does not change its deficit policies to return to a sustainable trajectory the growth path becomes unstable and the economy will not be able to maintain its capital-ratio. Policy-makers have lost the freedom to use deficit policy tools for its desired policies, unless they take the risk that the economy will degenerate.

This kind of restrictions in the ability to choose adequate policies is already apparent. The present downturn in general macroeconomic activity may indicate a call for a more stabilizing deficit policy. However, as the sustainable debt limits are reached, the risk of long term destabilization is apparent, and policy choice is reduced. Furthermore, these findings make clear that none of the economies would survive a second shock similar to that of the 2008 financial crisis.

³³See HPI for UK: http://www.nationwide.co.uk/hpi/datadownload/data_download.htm (Last accessed on September 25th 2012); for US: <http://www.fhfa.gov/Default.aspx?Page=87> (Last accessed on September 25th 2012); for Germany: <http://www.hypoport.de/hpx-hedonic.html> (Last accessed on September 25th 2012); and Japan <http://www.reinet.or.jp/en/index.html> (Last accessed on September 25th 2012).

³⁴Under certain conditions relative price changes in favor of real capital (here, housing) can lead to an increase in both asset ratios, k and b , without additional savings or external resources. For the time of the relative price change this effect appears almost as a kind of temporary relaxation in the resource constraints. For the theoretic argument see Appendix 1. For a discussion of the accounting principles see SNA (2008).

Under these conditions governments come under pressure to implement the necessary anti-deficit policies to stabilize the economy or to take unconventional measures. It seems obvious that the required policies will be more dramatic the larger the difference between the current debt level and a sustainable path. In general we also observe that government disbursement-ratios have dramatically increased since the baseline period, (e.g. US: 7.04 pc.pts., U.K.: 7.73 pc.pts, Japan: 11.21 pc.pts). Hence, for the economies under consideration we identify clear differences in the necessary efforts to return to a sustainable regime. These efforts can be rather painful, requiring large cuts in expenditure or a rise in taxes. As a result unconventional measures, such as reducing real debt by inflating nominal values or introducing new legislation to reduce real debt, are becoming more likely in order to return to a sustainable debt trajectory.

5.3 Conventional and Unconventional Adjustment Policies

Conventional Adjustment Policies: Having established that all economies face conditions that are likely to be unsustainable, we can determine the adjustments required to return to a stable and hence sustainable debt trajectory. While all economies are currently on or close to an unstable trajectory, the required policy will be the more dramatic the larger the difference between the current debt level and a sustainable path and the slower the stationary growth. Hence, for the economies under consideration we identify clear differences in the required effort to return to a sustainable regime. For simplicity we calculate the deficit policy and trajectory that may allow the economies to return to a sustainable debt condition. To determine a policy scenario the government chooses a debt level that is sustainable, which is the eventually desired debt-ratio. We call this level debt target b^Ω . For a historically given debt level b_t the government must implement a sequence of primary budget policies d_t to reach the target level b^Ω after a chosen number of periods θ :

$$b^\Omega = b_{t+\theta} = \sum_{i=0}^{\theta} \frac{d_{t+i} + (1 + i_t)b_{t+i}}{\eta_{t+i}}$$

Again, as the following part can be no more than a discussion of rules of thumb we keep things as simple as possible. We assume (i) for the real economy to return to *conditions of the baseline period* as kind of stationary description of the economy (see values in Table 1). This assumption is probably critical because it seems, from the current perspective, quite optimistic. However, if the identified baseline period really reflects the long term potentials of the economy, and if this period is not just another bubble in important parts of the economy, or if the 2008 crisis has not destroyed these long term potentials, then the assumption of a return pre-crisis conditions is consistent. We (ii) take the constant debt fiscal regime of the stationary period as the point of departure for each country. That is, for this period we assume that the expenditure-ratio and the tax-ratio are at non exceptional levels (values are given in Tables 1 and 2). (iii)

For *deficit consolidation policies* we decide to freeze the expenditure rate of this period to the desired ratio and determine the change in expenditure required to return from the current ratio to the ratio of the baseline period (Table 2). When considering deficit adjustments we concentrate on expenditure differentials and not on revenue differentials because they are less dependent on uncontrolled macroeconomic conditions and more determined by active policies. Hence, we either describe an expenditure cut leading to a return to the baseline, or alternatively a rise in taxes that could cover the expenditure differential between today's expenditure and the baseline period. (iv) For *debt consolidation policies* we choose a tax policy³⁵ to bring down debt from the presently unsustainable to a sustainable level. (v) As a rule of thumb, we identify the sustainable debt level around a baseline period. (vi) Since there is an obvious trade-off between fast consolidation and the duration of the increase in the tax burden we consider two scenarios with respect to the time horizon. Starting in 2011, in the first scenarios we suggest a consolidation period of 10 years, while in a second scenario we consider a longer consolidation period of 15 years.

Table 2 a) Adjustment scenario U.S.

1	Real Economy	For all variables see baseline values in Table 1	
2	Deficit Consolidation	period	variable
	<i>direction of consolidation:</i>		
a	gov.net disbursem.-ratio	baseline (1995-2003)	32.72% of GDP
b		current (2011)	39.77% of GDP
c	required adjustment	permanent	-7.04% of GDP
d	primary budget target	permanent	1.21% of GDP
	<i>policy instrument:</i>		
e	disbursement cut	permanent	-938 bn. US\$ ³⁶
f	sales tax change	permanent	20 pc. pts
3	Debt Consolidation		target: 43% of GDP
a	gov. revenue ratio	baseline (1995-2003)	33.93% of GDP
b	aver. sales tax rate	baseline (1995-2003)	6.5% ³⁷
	<i>policy instrument:</i>		
c	sales tax change	10 years consolid.	7 pc. pts
d		15 years consolid.	3 pc. pts

(vii) In order to link to real world disputes we translate the required adjust-

³⁵In line with Barro (1979), Reinhart, Reinhart, and Rogoff (2012) confirm in their long-run study that the predominance of low interest rates during periods of high debt-ratios in the advanced economies means that the latter alone cannot be a debt stabilization instrument. Public spending and tax adjustments have to keep debt below a given threshold at all times.

³⁶This ammount equals 7.04 percent of GDP in 2011.

³⁷Estimated value, based on Pagitt (2011) and the U.S. Census Bureau 2011.

ment in tax-ratios into an increase in VAT (average sales tax in the U.S.). That is, we perform a simple linear computation of the contribution of each percentage point in VAT (sales tax) to the respective total government revenue-ratio and then calculate to what extent VAT must be adjusted in order to generate a total tax-ratio that supports the respective time path of consolidation. Table 2 gives the detailed numbers for each country.

Using the example of the U.S., Table 2a should be read in the following way: If the real economy has returned to long term stationary behavior (3.1% real growth, 3.01% real interest rate, 2.22% inflation rate, and 2.7 capital-output-ratio) the fiscal regime must also return to the baseline values. For this, a deficit consolidation and a debt consolidation are necessary. For deficit consolidation, government disbursement must be permanently cut down to return to the baseline ratio. The value of this cut is about 7 percentage points of GDP or 938 billion US\$ in absolute terms for 2011. This cut would bring down the U.S. disbursement-ratio from 39.77% in 2011 to a 32.72% in the baseline period. The primary budget target would be a deficit, namely -1.21% of GDP, like in the baseline period. Instead of an expenditure cut the U.S. government could alternatively raise taxes by means of fictitious permanent increase in average sales tax by 20 percentage points.³⁸ Further, in order to reduce the debt-ratio and implement a debt consolidation to a sustainable level of 43% of GDP the U.S. would have to increase taxes by an amount equivalent to an increase in average sales tax of 7 percentage points if the planned consolidation period is 10 years. An increase in average sales tax by 3 pc. pts is required if the planned consolidation period is 15 years.

With net debt of around 80% of GDP, the *US and U.K.* are likely to be close to an unsustainable trajectory. As can be seen from Table 2 the fiscal regimes in both economies differ from their baseline period in particular because of a dramatic increase in government disbursements. Hence, in addition to tax policies to bring down debt during a consolidation period (*debt consolidation*), a return to the baseline fiscal disbursement requires a dramatic and permanent cut in expenditure for both countries (*deficit consolidation*). However, if the current expenditure level were to become the newly desired long term ratio the tax policy would have to be further adjusted substantially to raise an equivalent amount of government revenues. Deficit and debt consolidation efforts in the US and the U.K. may be reinforced by higher GDP growth and a more favorable interest to growth differential³⁹ than in Germany or Japan if they can manage to return to growth rates of the baseline values.

³⁸We are fully aware that the US has no general federal sales tax like the VAT in other countries. However, we use sales tax to achieve comparability with the systems of other countries considered.

³⁹The U.K. has been forecasted to have a lower interest-growth differential relative to the U.S. and Japan (source: the IMF Fiscal Monitor (2011), September edition, p. 9)

Table 2 b) Adjustment scenario **U.K.**

1 Real Economy	For all variables see baseline values in Table 1	
2 Deficit Consolidation	period	variable
<i>direction of consolidation:</i>		
a gov.net disbursem.-ratio	baseline (1995-2004)	38.35% of GDP
b	current (2011)	46.08% of GDP
c required adjustment	permanent	-7.73% of GDP
d primary budget target	after consolidation	0.8% of GDP
<i>policy instrument:</i>		
e disbursement change	permanent	-117 bn GBP ⁴⁰
f VAT change	permanent	-22 pc. pts.
3 Debt Consolidation		target: 27% of GDP
a gov. revenue ratio	baseline (1996-2004)	39.15% of GDP
b VAT	baseline (1995-2004)	17.5% ⁴¹
<i>policy instrument:</i>		
c VAT change	10 years consolid.	9 pc. pts
d	15 years consolid.	6 pc. pts

Table 2 c) Adjustment scenario **Germany**

1 Real Economy	For all variables see baseline values in Table 1	
2 Deficit Consolidation	period	variable
<i>direction of consolidation:</i>		
a gov. net disbursem.-ratio	baseline (1996-2000)	44.88 %of GDP
b	current (2011)	43,72% of GDP
c required adjustment	permanent	+1.15% of GDP
d primary budget target	after consolidation	1.08% of GDP
<i>policy instrument:</i>		
e disbursement change	permanent	+ 30 bn Euro ⁴²
f VAT change	permanent	-3 p.c. pts
3 Debt Consolidation		target: 30% of GDP
a gov. revenue-ratio	baseline (1996-2000)	45.96% of GDP
b VAT	baseline (1996-2000)	15.5% ⁴³
<i>policy instrument:</i>		
c VAT change	10 years consolid.	4 p.c. pts
d	15 years consolid.	3 p.c. pts

⁴⁰This amount equals 7.73 percent of GDP in 2011

⁴¹See OECD (2011).

⁴²This amount equals 1.08 percent of GDP in 2011

With a current net debt of slightly above 50% of GDP *Germany* is also above or close to a non sustainable level even for the benchmark case of a constant savings-ratio. With respect to Germany two findings are important. 1) Despite the Maastricht deficit rule and despite never having drastically violated it, on average Germany has experienced a continuous increase in net debt. Even if the latest acceleration of debt was due to the 2008 crisis, the Maastricht rules do not seem sufficient to follow a constant net debt rule. This is apparent when looking again at Figure 4. Most of the time, no matter if the Maastricht rule was violated or not, Germany has experienced an increase in net debt and could not match the constant net debt rule. 2) However, Germany was not too far away from the Maastricht and/or constant debt rules, and implemented reforms and raised VAT between 2003 and 2006 so that elements of a budget consolidation policy are already active. Therefore, today's consolidation measures are less severe. From the perspective of a long term deficit policy Germany could even cut VAT by 3 pc. points.⁴⁴

Table 2 d) Adjustment scenario Japan

1 Real Economy		For all variables see baseline values in Table 1	
2	Deficit Consolidation	period	variable
	direction of consolidation:		
a	gov.net disburse.-ratio	baseline (1988-1992)	30.81% of GDP
b		current (2011)	42.02% of GDP
c	required adjustment	permanent	-11.21% of GDP
d	primary budget target	after consolidation	1.49% of GDP
	policy instrument:		
e	disbursement change	permanent	+ 52476 bn Euro ⁴⁵
f	VAT change	permanent	26 pc. pts
3	Debt Consolidation		target: 30% of GDP
a	gov. revenue-ratio	baseline (1988-1992)	32.97% of GDP
b	VAT	baseline (1988-1992)	3% ⁴⁶
	policy instrument:		
c	VAT change	10 years consolid.	20 pc. pts
d		15 years consolid.	13 pc. pts

However, the debt resulting from the 2008 crisis must first return to a sustainable debt trajectory and hence be reduced by an increase in VAT according

⁴³See OECD (2011).

⁴⁴Or alternatively increase spending by 30 bill Euro yearly.

⁴⁵This ammount equals 11.21 percent of GDP in 2011

⁴⁶See OECD (2011).

to the scenarios in Table 2c. Therefore, on balance, then a moderate increase in VATs would be capable of funding a return to a sustainable debt trajectory.

Japan is far from any sustainable trajectory. With a net debt-ratio of 125% of GDP and an extraordinary increase in disbursement-ratio compared to the baseline period, Japan would have to permanently cut government expenditure by 50,416 bn Yen to consolidate its budget. If Japan wanted to maintain expenditure at the current ratio the government would have to permanently raise VAT by very considerably high percentage points compared to the baseline period. For *debt consolidation* Japan would have to raise VAT in line with the numbers in Table 2d to run a *debt consolidation* program.

The results show that a considerable adjustment is required to ensure sustainability. In fact the figures for the U.K. and the US are very close to the IMF's forecasted primary balance adjustment (see IMF Fiscal Monitor, Table A3.3 in the Appendix).⁴⁷

In particular, for Japan but also for the U.S. and the U.K., the required actions seem so dramatic that we doubt that conventional austerity will be politically enforceable. Hence, unconventional measures will become ever more likely the longer governments hesitate to take drastic conventional action.

Unconventional Adjustment Policies: By unconventional measures we mean policies such as cutting debt by means of new legislation or inflation. However, when governments consider these unconventional measures, they should be aware of the drastic effects of non-anticipated inflation on distortions in allocation and wealth distribution. These policies are an expropriation of nominal wealth holders. Nominal wealth holders are mostly regular less wealthy people. While nominal wealth holders will experience a loss of their real wealth, debtors and real asset holders - which are often wealthier people - gain massively from a real devaluation of debt (private and public) without a depreciation of real values of these assets. In democratic societies where governments should act in the interest of the majority we doubt that these kinds of unconventional measures are an appropriate instrument even if they enable the government to eliminate its debt and claim to stabilize the economy.

There is another unconventional measure worth discussing. As the point of departure for the recent rapid acceleration in sovereign debt was a drastic market failure in the global financial system, we should have a closer look at this dysfunction. The 2008 crisis revealed a massive allocative inefficiency that was probably generated or at least favored by principal-agent, asymmetric information and moral hazard problems within the financial system. Financial instruments in these markets should improve allocative efficiency and not increase information asymmetries and moral hazard. As long as we do not find

⁴⁷For another illustrative analysis see e.g. the IMF's fiscal adjustment strategy in the 2011 Fiscal Monitor edition. It gives the adjustment required to bring back debt to the *target ratios* of the IMF by 2030.

Figure 5: Deficit and Debt Consolidation Scenarios

a design for the financial markets and financial firms can address these kinds of distortions the economies will be at risk of experiencing another, then unaffordable, crisis. Therefore, there should be no taboo in thinking about even unconventional new designs of the financial system.

Ideologies and beliefs: Government budget policies are very often discussed with a certain preconception in mind. The choice of one model family or model variation over another can have very different implications. Therefore, we would like to clearly identify the elements that are most controversial and we also evaluate briefly how we believe these elements affect our major findings.

1) We model and discuss long term developments and perspectives. We agree that macroeconomic stabilization policies may be helpful in the short term. However, there is a long term effect which we focus on, and which cannot be neglected forever.

2) The choice of an OLG model with exogenous growth is a choice for neo-classical growth mechanics. The key element responsible for most resulting mechanics are the properties of an aggregate neoclassical production function and resource constraints. We follow the notion of stationary capital-ratios, long term growth rates at a given stationary level, and returns on capital that move inversely to capital-ratios. The statistics suggest that these notions do not obviously contradict the stylized facts on long term developments in the considered economies. Factor payments according to marginal productivity are regarded as a simplification (not a contradiction) of a more complex real-life payment scheme.

3) A clear undervaluation of productive government activities is connected to the implicit assumption that the government does not generate a productive public input good, let alone a public stock of infrastructure. The government simply absorbs resources for whatever purpose. In the model a public infrastructure would clearly improve aggregate productivity. However, in the calibration exercise we implicitly account for this idea by assuming that the expenditure rate of the baseline period is the desired rate that may account for these kind of effects. During baseline periods governments earned sufficient revenues but did not spend more, signaling a kind of optimal spending.

4) The neoclassical savings decision is a way to incorporate intertemporal resource decisions into a macro model. Since debt issues clearly affect intertemporal allocation and distribution we choose to model this link. However, the qualitative results are not much affected by this issue. Even if we assume that savings rates are independent of interest rates or constant, the main results still hold.

5) In our understanding of the model mechanics the major results in terms of the dynamic effects of government deficits are generated by pure accounting identities and resource restrictions. Additional resources cannot easily be mobilized, neither within a period nor intertemporally.

6) Even if a calibration is a rule of thumb exercise, as long as we do not see other more sophisticated calculations and discussions, our results ring alarm bells. If the world's major economies are apparently forced to increase their

debt-ratios in just three years to a greater extent than in the entire three-decade period before that, something is clearly not working properly.

6 Summary and Conclusion

As a response to the financial and economic crisis in 2008 huge government deficits in most high income economies led to a rapid accumulation of public debt. The extremely fast increase in sovereign debt may raise doubts as to the fiscal sustainability of some advanced economies. This paper examines the relationship between debt dynamics, capital formation, and economic development in the four largest advanced economies, and is the first contribution to account for the effects of the 2008 financial and economic crisis on debt sustainability .

We contribute to the literature on fiscal sustainability by framing the problem in an OLG model with government debt, physical capital, endogenous interest rates, and exogenous growth. We demonstrate the existence of a maximum debt ratio and the described dynamics of adjustments. Debt is unsustainable if the economy is on an unstable trajectory where even a ‘constant debt ratio rule’ for the current debt ratio would not stabilize the economy at any positive capital ratio. The policy implications are serious. (i) In a sustainable debt regime decision makers are free to choose a desired permanent debt ratio within the limits of the sustainable regime. Policy makers must be aware of their limited choices. (ii) Once the economy has crossed over to an unsustainable regime, freedom of action turns into pressure to react even under unfavorable conditions. Governments should establish debt ceiling rules in order to avert this kind of threat, which is the result of short term opportunistic policies.

Yet to the authors’ best knowledge none of the calibration exercises performed so far have been parameterized to consider (1) the effect of the 2008 financial crisis with a massive deficit policy and the most recent debt accumulation, and (2) a potential sustainability problem of the world’s largest advanced economies in its aftermath. While the debt crisis has so far essentially been a problem for some smaller Southern European economies, this exercise sheds more light on the debt position of the largest advanced economies. We are aware that calibration exercises are only good for broad analysis, and have a number of shortcomings and produce no more than rules of thumb, so they are more a source of orientation than of precise predictions, yet we believe that our results are more than alarming.

For the calibration exercise we extract data from the OECD for a period of almost two decades prior to the 2008 crisis. Except for parameters describing intertemporal preferences all parameters are drawn or directly endogenously derived from the OECD database.

1) In our discussion we start with a baseline scenario that covers, as much as possible, two major ideas: (i) to describe the real economic conditions (real growth, real interest rate, capital ratio, productivity) of the four largest advanced economies, namely the Germany (as the European stability anchor), the

U.S., the U.K., Japan, and that have been broadly observable in recent years (before the 2008 crisis), and (ii) to combine the characteristics of each real economy with the fiscal regime that is closest to a constant debt policy during that same period.

2) With the help of the baseline scenario we discuss if these economies may be moving on unstable trajectories because they have drifted into an unsustainable sovereign debt regime. Our results suggest (i) that without an exceptional period none of the four economies under review would be able to establish a fiscal regime capable of supporting a stationary debt ratio during the last two decades; (ii) that all four are likely to have crossed or are about to cross the line to an unsustainable regime. The overall results of this simple calibration exercise are therefore alarming. They call for a strong readjustment in longterm fiscal activity of at least three of the four major high-income economies, namely the US, the UK, and, most urgently, Japan.

3) What changes and adjustments in deficit policies are required to return to a stable and hence sustainable debt trajectory? The required policies will be the more dramatic the larger the difference between the current debt ratio and a sustainable path and the less pronounced the stationary growth. These measures are discussed and translated into simple consolidation instruments and actions. Reaching debt ceilings requires the rethinking of deficit and debt rules, of which the Maastricht deficit rules are an example. In particular, for Japan but also for the US and the UK the required actions seem so painful that we have doubts that conventional austerity programs like expenditure cuts and higher taxes will be politically enforceable. As a result unconventional measures, such as new legislation to cut debt or raise inflation, are more and more likely the longer the government waits with implementing drastic conventional actions.

However, even if a calibration is a rule of thumb exercise, as long as we do not see other more sophisticated calculations and discussions, our results are ringing the alarm bells. If the world's major economies are seemingly forced to increase their debt ratios in just four years to a greater extent than in the entire four-decade period before that, something is clearly not working properly.

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Appendix

This is an extended appendix for the working paper version:

Notation

- α = share of physical capital
- $1 - \beta$ = the future discount factor
- γ = bequest share of sovereign debt
- A = exogenous scale parameter reflecting technological level
- n = population growth rate
- ε_t = labor efficiency growth
- η = total growth factor
- c_t^1 = individual's consumption per capita of the young generation at time t
- c_{t+i}^2 = individual's consumption per capita of the old generation in period $t + i$ of life
- B_t = stock of government net financial liabilities at the start of period t
- D_t = principal of debt issued at the start of period t
- b_t = government net financial liabilities as a share of GDP at the start of period t
- k_t = capital efficiency-labour ratio at the start of period t
- g_t = government expenditure as a share of GDP at time t
- τ_t = tax revenues as share of wage income at the time t
- d_t = primary balance as share of GDP at time t
- i_t = real interest rate on public debt
- r_t = net real physical rate of return on physical capital at time t
- δ_t = depreciation rate for real capital stock

6.1 Appendix 1: Solving the Model

6.1.1 Production per labor in efficiency units

The output of a firm is defined by the Cobb-Douglas production function

$$Y_t = A (K_t)^\alpha (H_t)^{1-\alpha}.$$

In order to determine the production in efficiency units we have to compute the ratio each efficiency-weighted labour input provides. Hence, we obtain

$$\begin{aligned} \tilde{y} & : = \frac{Y_t}{H_t} = A \left(\frac{K_t}{H_t} \right)^\alpha \\ & = A \left(\frac{K_t}{Y_t} \frac{Y_t}{H_t} \right)^\alpha \\ & \Rightarrow \tilde{y} = A^{\frac{1}{1-\alpha}} k_t^{\frac{\alpha}{1-\alpha}} \end{aligned} \tag{12}$$

with $k_t := K_t/Y_t$.

6.1.2 Profit Maximization of Firms

In order to determine the input factor prices we consider the profit function

$$\Pi = A(K_t)^\alpha (H_t)^{1-\alpha} - (r_t + \delta)(K_t) - \tilde{w}_t(H_t).$$

Maximizing subject to L_t leads to the first order conditions

$$\begin{aligned} \frac{\partial \Pi}{\partial H_t} &= (1 - \alpha) A(K_t)^\alpha (H_t)^{-\alpha} - \tilde{w}_t \stackrel{!}{=} 0 \\ \Rightarrow \tilde{w}_t &= (1 - \alpha) A \left(\frac{K_t}{H_t} \right)^\alpha. \end{aligned}$$

After rearranging and using (12) we obtain

$$\begin{aligned} \tilde{w}_t &= (1 - \alpha) A (k_t \tilde{y})^\alpha \\ &= (1 - \alpha) A^{\frac{\alpha}{1-\alpha}} k_t^{\frac{\alpha}{1-\alpha}} \end{aligned}$$

Similarly, after maximizing subject to K_t we obtain

$$\begin{aligned} \frac{\partial \Pi}{\partial K_t} &= \alpha A(K_t)^{\alpha-1} (H_t)^{1-\alpha} - (r_t + \delta) \stackrel{!}{=} 0 \\ \Rightarrow r_t &= \alpha A(K_t)^{\alpha-1} (H_t)^{1-\alpha} - \delta. \end{aligned}$$

After rearranging and using (12) we obtain

$$\begin{aligned} r_t &= \alpha A \left(\frac{K_t}{H_t} \right)^{-(1-\alpha)} - \delta \\ &= \alpha k_t^{-1} - \delta. \end{aligned}$$

6.1.3 Lifetime Consumption Maximization

Using the Langrangeian function for the consumer's maximization problem

$$L = \left(\beta (\tilde{c}_t^1)^{-\rho} + (1 - \beta) (\tilde{c}_{t+1}^2)^{-\rho} \right)^{\frac{-1}{\rho}} + \lambda_1 \left(\tilde{w}_t (1 - \tau_t) - \left(\tilde{c}_t^1 + \frac{1}{1 + i_t} \tilde{c}_{t+1}^2 \right) \right)$$

we obtain for a first order conditions

$$\begin{aligned} \frac{\partial L}{\partial \tilde{c}_t^1} &= \beta (\tilde{c}_t^1)^{-\rho-1} \left(\beta (\tilde{c}_t^1)^{-\rho} + (1 - \beta) (\tilde{c}_{t+1}^2)^{-\rho} \right)^{\frac{-1}{\rho}-1} - \lambda_1 = 0 \\ \Rightarrow \beta (\tilde{c}_t^1)^{-\rho-1} \left(\beta (\tilde{c}_t^1)^{-\rho} + (1 - \beta) (\tilde{c}_{t+1}^2)^{-\rho} \right)^{\frac{-1}{\rho}-1} &= \lambda_1 \end{aligned}$$

and

$$\begin{aligned} \frac{\partial L}{\partial \tilde{c}_{t+1}^2} &= (1 - \beta) (\tilde{c}_{t+1}^2)^{-\rho-1} \left(\beta (\tilde{c}_t^1)^{-\rho} + (1 - \beta) (\tilde{c}_{t+1}^2)^{-\rho} \right)^{\frac{-1}{\rho}-1} - \lambda_1 \frac{1}{1 + i_t} = 0 \\ \Rightarrow -(1 + i_t) (1 - \beta) (\tilde{c}_{t+1}^2)^{-\rho-1} \left(\beta (\tilde{c}_t^1)^{-\rho} + (1 - \beta) (\tilde{c}_{t+1}^2)^{-\rho} \right)^{\frac{-1}{\rho}-1} &= \lambda_1. \end{aligned}$$

Combining the two equations results at the consumer's optimum

$$\begin{aligned}
& \beta (\tilde{c}_t^1)^{-\rho-1} \left(\beta (\tilde{c}_t^1)^{-\rho} + (1-\beta) (\tilde{c}_{t+1}^2)^{-\rho} \right)^{\frac{-1}{\rho}-1} \\
&= - (1+i_t) (1-\beta) (\tilde{c}_{t+1}^2)^{-\rho-1} \left(\beta (\tilde{c}_t^1)^{-\rho} + (1-\beta) (\tilde{c}_{t+1}^2)^{-\rho} \right)^{\frac{-1}{\rho}-1} \\
&\Leftrightarrow 1+i_t = \frac{\beta}{1-\beta} \left(\frac{\tilde{c}_{t+1}^2}{\tilde{c}_t^1} \right)^{\rho+1}. \tag{13}
\end{aligned}$$

For the derivative with respect to λ_1 we obtain

$$\begin{aligned}
\frac{\partial L}{\partial \lambda_1} &= \tilde{w}_t (1-\tau_t) - \left(\tilde{c}_t^1 + \frac{1}{1+i_t} \tilde{c}_{t+1}^2 \right) = 0 \\
&\Rightarrow \tilde{w}_t (1-\tau_t) = \tilde{c}_t^1 + \frac{1}{1+i_t} \tilde{c}_{t+1}^2.
\end{aligned}$$

Plug in $(1+i_t)\tilde{s}_t = \tilde{c}_{t+1}^2$ and use (13) in order to obtain the household's savings rule in efficiency units:

$$\begin{aligned}
\tilde{w}_t (1-\tau_t) &= \left[(1+i_t) \frac{1-\beta}{\beta} \right]^{-\frac{1}{\rho+1}} \tilde{c}_{t+1}^2 + \frac{1}{1+i_t} \tilde{c}_{t+1}^2 \\
&= \left[\left[(1+i_t) \frac{1-\beta}{\beta} \right]^{-\frac{1}{\rho+1}} + (1+i_t)^{-1} \right] (1+i_t) \tilde{s}_t \\
&\Leftrightarrow \tilde{s}_t = \frac{\tilde{w}_t (1-\tau_t) \left(\frac{1-\beta}{\beta} \right)^\varphi}{\left[\left(\frac{1-\beta}{\beta} \right)^\varphi + (1+i_t)^{1-\varphi} \right]}
\end{aligned}$$

with $\varphi = \frac{1}{\rho+1}$.

Hence, for the savings rule in terms of the savings rate (savings per income) it holds

$$\begin{aligned}
s_t &: = \frac{\tilde{s}H}{Y} = \frac{\tilde{s}}{\tilde{y}} \\
&= \frac{(1-\alpha)(1-\tau_t) \left(\frac{1-\beta}{\beta} \right)^\varphi}{\left[(1+i_t)^{1-\varphi} + \left(\frac{1-\beta}{\beta} \right)^\varphi \right]}.
\end{aligned}$$

We can show that the derivative of s_t according to i_t is increasing

$$\frac{\partial s_t}{\partial i_t} = - \frac{(1-\varphi)(1-\alpha)(1-\tau_t) \left(\frac{1-\beta}{\beta} \right)^\varphi}{\left[(1+i_t) + (1+i_t)^\varphi \left(\frac{1-\beta}{\beta} \right)^\varphi \right]^2} > 0 \quad \text{for } \varphi > 1$$

6.1.4 The Public Sector

Each period debt changes by

$$\begin{aligned} B_{t+1} - B_t &= D_t + i_t B_t = G_t - T_t + i_t B_t. \\ \Leftrightarrow \frac{B_{t+1}\eta_t}{Y_t\eta_t} &= \frac{D_t}{Y_t} + (1 + i_t) \frac{B_t}{Y_t} = \frac{G_t}{Y_t} - \frac{T_t}{Y_t} + (1 + i_t) \frac{B_t}{Y_t}. \end{aligned}$$

In order to obtain the total debt accumulation in ‘debt ratio’ $b_t =: B_t/Y_t$ substitute $d_t = \frac{D_t}{Y_t}$, $g_t = \frac{G_t}{Y_t}$ and $\tau_t = \frac{T_t}{Y_t}$. Hence, we obtain

$$\eta_t b_{t+1} = d_t + (1 + i_t) b_t = g_t - \tau_t + (1 + i_t) b_t$$

with η_t denoting the growth factor of output which in a potential steady state becomes equal to the product of the population growth factor $(1 + n_t)$ and the growth factor of labor productivity $(1 + \varepsilon_t)$, such that $\eta_t = (1 + \varepsilon_t)(1 + n_t)$.

6.1.5 Equilibrium and Stationarity Conditions

Pure transmission of capital and bonds from generation to generation:

We assume that the old generation bequeaths the total capital K_t and bonds B_t with the share γ if the restriction is fulfilled that total bequest does not restrict full capital transfer. Therefore the income and expenditure equation becomes:

$$wH + K_t + \gamma B_t - T - C^y + K_t r_t + B_t r_t + (1 - \gamma) B_t = C^o + K_{t+1} + B_{t+1} - F$$

$$\begin{aligned} \Rightarrow \overbrace{\frac{wH}{Y_t} - \frac{T}{Y_t} - \frac{C_t^y}{Y_t}}^{s_t} + \frac{(K_t + \gamma B_t)}{Y_t} + \frac{F_t}{Y_t} &= \frac{K_{t+1}}{Y_t} + \frac{B_{t+1}}{Y_t} \\ \Leftrightarrow s_t + f_t + k_t + \gamma b_t &= \eta_t k_{t+1} + \eta_t b_{t+1} \end{aligned}$$

General stationarity curve:

For the general stationarity curve we use that $b_{t+1} = b_t$ and $k_{t+1} = k_t$:

$$s_t + f_t + k_t + \gamma b_t = \eta_t k_{t+1} + \eta_t b_{t+1}$$

$$\Rightarrow b_t = \frac{s_t}{(\eta_t - \gamma)} + \frac{f_t}{(\eta_t - \gamma)} + \frac{(1 - \eta_t)}{(\eta_t - \gamma)} k_t$$

$$b_t^s =: b_t = \frac{(1 - \alpha)(1 - \tau_t)}{\left[(1 + \alpha k_t^{-1} - \delta)^{1-\varphi} \left(\frac{1-\beta}{\beta} \right)^{-\varphi} + 1 \right] (\eta_t - \gamma)} + \frac{f_t}{(\eta_t - \gamma)} + \frac{(1 - \eta_t)}{(\eta_t - \gamma)} k_t$$

Properties of the general stationarity curve:

Case1: $\varphi > 1$

$$\lim_{k \rightarrow 0} b_t^s(k_t) = \frac{(1 - \alpha)(1 - \tau_t)}{(\eta_t - \gamma)} + \frac{f_t}{(\eta_t - \gamma)}.$$

Case 2: $\varphi = 1$

$$\lim_{k \rightarrow 0} b_t^s(k_t) = \frac{(1 - \alpha)(1 - \tau_t)}{\left[\left(\frac{1 - \beta}{\beta} \right)^{-1} + 1 \right] (\eta_t - \gamma)} + \frac{f_t}{(\eta_t - \gamma)},$$

Case 3: $\varphi < 1$

$$\lim_{k \rightarrow 0} b_t^s(k_t) = \frac{f_t}{(\eta_t - \gamma)}.$$

For $k \rightarrow \infty$ we obtain

$$\lim_{k \rightarrow \infty} b_t^s(k_t) = -\infty.$$

For the derivative we obtain

$$\begin{aligned} \frac{\partial b_t^s(k_t)}{\partial k_t} &= \frac{(1 - \alpha)(1 - \tau_t)}{(\eta_t - \gamma)} \left[\frac{\alpha \left(\frac{1 - \beta}{\beta} \right)^{-\varphi}}{k_t^2 (1 + \alpha k_t^{-1} - \delta)^\varphi \left[(1 + \alpha k_t^{-1} - \delta)^{1 - \varphi} \left(\frac{1 - \beta}{\beta} \right)^{-\varphi} + 1 \right]^2} \right] \\ &\quad + \frac{(1 - \eta)}{(\eta_t - \gamma)} \end{aligned}$$

$$\lim_{k \rightarrow 0} \frac{\partial b_t^s(k_t)}{\partial k_t} = \infty.$$

$$\lim_{k \rightarrow \infty} \frac{\partial b_t^s(k_t)}{\partial k_t} = \frac{(1 - \eta)}{(\eta_t - \gamma)} < 0.$$

Since we have parts where the derivative of the b_t^s -curve is positive and parts where it is negative we can state that with the intermediate value theorem there is at least one point where the derivative is 0, hence a maximum.

Non-stationary dynamics: From (10) and (11) it is apparent that below the b_t^s - curve capital intensity will increase and above the b_t^s - curve capital intensity will decrease.

$$k_{t+1} \geq k_t \quad \text{if} \quad b_t \leq b_t^s \quad \text{and} \quad k_{t+1} \leq k_t \quad \text{if} \quad b_t \geq b_t^s$$

6.1.6 Relative Price Changes of Real Assets

In order to include the relative price change of real assets into the budget constraint we assume the price change to be

$$p_{t+1}^K = \underbrace{\left(\frac{(p_{t+1}^K - p_t^K)}{p_t^K} + 1 \right)}_{:=\pi_t^K} p_t^K = \pi_t^K p_t^K.$$

Then the income and expenditure equation becomes

$$wH + p_t K_t + \gamma B_t - T - C^y + K_t r_t + B_t r_t + (1 - \gamma) B_t = C^o + p_{t+1} K_{t+1} + B_{t+1} - F$$

$$\begin{aligned} \Rightarrow \overbrace{\frac{wH}{Y_t} - \frac{T}{Y_t} - \frac{C_t^y}{Y_t}}^{s_t} + \frac{(p_t K_t + \gamma B_t)}{Y_t} + \frac{F_t}{Y_t} &= \frac{p_{t+1} K_{t+1}}{Y_t} + \frac{B_{t+1}}{Y_t} \\ &\Leftrightarrow s_t + f_t + p_t k_t + \gamma b_t = \eta_t \pi_t^K p_t^K k_{t+1} + \eta_t b_{t+1} \end{aligned}$$

With the same procedure we obtain for the general stationarity curve:

$$b_t^s := b_t = \frac{(1 - \alpha)(1 - \tau_t)}{\left[(1 + \alpha p_t k_t^{-1} - \delta)^{1-\varphi} \left(\frac{1-\beta}{\beta} \right)^{-\varphi} + 1 \right] (\eta_t - \gamma)} + \frac{f_t}{(\eta_t - \gamma)} + \frac{(1 - \eta \pi_t^K p_t^K)}{(\eta_t - \gamma)} k_t$$

Note that the price change could also be implemented before deriving the savings rule. However, since the total capital stock is bequeathed an implementation would not change the interpretation.

6.2 Appendix 2: A Calibration Exercise

6.2.1 Definition of extracted input-data:

All extracted variables are taken as simple averages for the respective country and base line period

Variable as termed in this paper	Notation	Variables as officially termed by the OECD & other sources	OECD Denotation	Definition	Source/ Ref. Parameter
Government gross debt/GDP ratio		General government gross financial liabilities, as a percentage of GDP	GGFLQ	OECD: Gross financial liabilities refer to the debt and other liabilities (short and long-term) of all the institutions in the general government sector, defined by ESA95/ISNA93. Here it is presented as a percentage of nominal GDP at market prices.	1)
Government debt/GDP ratio	b	General government net financial liabilities, as a percentage of GDP	GNFLQ	OECD: Net financial liabilities measure the gross financial liabilities of the general government sector less the financial assets of the general government sector. Here it is presented as a percentage of nominal GDP at market prices.	1)
Government disbursement/GDP ratio	g	Total disbursements general government, as a percentage of GDP	YPGTQ	OECD: Total disbursements general government represents: Final consumption expenditure + social security benefits paid by general government + property income (payable) + interest (payable) + subsidies & other current payments + gross fixed capital formation + capital transfers & other capital payments + consumption of fixed capital. Here it is presented as a percentage of nominal GDP at market prices.	1)
Government revenues/GDP ratio	r	Total receipts general government, as a percentage of GDP	YRGTD	OECD: Total receipts general government represents: Taxes on production and imports (receivable) + current taxes on income, wealth, etc. (receivable) + property income received by the government + interest (receivable) + social contributions (receivable) + other current receipts of general government + capital transfers (receivable). Here it is presented as a percentage of nominal GDP at market prices.	1)
Government primary deficit/GDP ratio	d	Government primary balance, as a percentage of GDP	NLGXQ	Own calculation (correctness confirmed by OECD): Government primary balance is defined as total disbursements general government - total receipts general government - net government interest payments. Here it is presented as a percentage of nominal GDP at market prices.	YPGTQ, YRGTD, GNINTQ
GDP		Gross domestic product, value, market prices	GDP	Nominal GDP at market prices. OECD: Gross domestic product at market prices is the sum of the gross values added of all resident producers at market prices, plus taxes less subsidies on imports.	1)
Real GDP		Gross domestic product, volume, market prices	GDPV	Real GDP at market prices. OECD: Gross domestic product at market prices is the sum of the gross values added of all resident producers at market prices, plus taxes less subsidies on imports.	1)
Capital/GDP ratio	k	Capital output ratio on tangible fixed assets		Own calculation: Capital/GDP ratio = Tangible fixed assets / Gross domestic product, value, market prices	NI11, GDP

Detailed Definition of Variables

Variable as termed in this paper	Notation	Variable as officially termed by the OECD & other sources	OECD Denotation	Definition	Source/ Ref. Parameter
GDP deflator	p	Gross domestic product, deflator, market prices	PGDP	OECD: The GDP deflator (implicit price deflator for GDP) is calculated as GDP at current prices divided by GDP at "constant prices" (chained volume estimates or fixed-base volume estimates, depending on countries). Presented at market prices here.	1)
Producer price growth	p	Percentage rate of change GDP-Deflator		Own calculation: Annualised growth rate of GDP deflator.	PGDP
GDP growth factor	$\square$	Gross domestic product, volume, growth factor, annualised rate	GDPV_ANNPCT	Annualised volume based GDP growth rate at market prices	1)
Long term government bond yield	i_{long}	Long term interest rate on government bonds		OECD: 'Long term' rates are secondary market yields of long term (usually 10 year) bonds. (http://stats.oecd.org/mei/default.asp?lang=en&subject=1.5)	1)
Real long term government bond yield	i		IRL	Own calculation: Real long term government bond yield = Long term government bond yield - percentage rate of change GDP-Deflator	IRL, PGDP
Elasticity of labor production	$1-\alpha$			Own calculation: (Total employment * Compensation rate, total economy) / Gross domestic product, value, market prices.	ET, WEST, GDP
Foreign financial inflow/GDP ratio	f	Current account balance, as a percentage of GDP	CBGDPR	OECD: The current account includes all the transactions (other than those in financial items) that involve economic values and occur between resident and non-resident entities. Also covered are offsets to current economic values provided or acquired without a quid pro quo. Specifically, the major classifications are: goods and services; income; current transfers. Here it is presented as a percentage of nominal GDP at market prices.	1)
Required primary surplus for current constant debt	$-d$			Own calculation: Required primary surplus for current constant debt = General government net financial liabilities, as a percentage of GDP * (1 + average (base line period) long-term interest rate on government bonds / 100 - average (base line period) percentage rate of change GDP-Deflator / 100 - (1 + average (base line period) gross domestic product growth factor / 100))	GNFLQ, IRL, PGDP, GDPV_ANNPCT

Detailed Definition of Variables

Implicitly used variables (not termed in this paper)	OECD Denotation	Definition	Source/ Ref. Parameter
Net government interest payments, as a percentage of GDP	GNINTQ	Own definition: Net government interest payments = Gross government interest payments - interest income of the government. Here it is presented as a percentage of nominal GDP at market prices.	1)
Government net lending, as a percentage of GDP	NLGQ	OECD: Net lending is the net amount a unit or a sector has available to finance, directly or indirectly, other units or other sectors. It is the balancing item in the capital account and is defined as: (Net saving plus capital transfers receivable minus capital transfers payable) minus (the value of acquisitions less disposals of non-financial assets, less consumption of fixed capital). Negative net lending may also be described as "net borrowing". Here it is presented as a percentage of nominal GDP at market prices.	1)
Total employment	ET	OECD: Employment - Persons in civilian employment include all those employed above a specified age who during a specified brief period, either one week or one day, were in the following categories: i) paid employment; ii) employers and self-employed; iii) unpaid family workers; unpaid family workers at work should be considered as being self-employed irrespective of the number of hours worked during the reference period. For operational purposes, the notion of some work may be interpreted as work for at least one hour. Total employment is defined as the sum of civilian employment and members of the armed forces. (http://www.oecd.org/employment/emp/employmentpolicyanddata/onlineoecdemploymentdatabase.htm)	1)
Compensation rate, total economy	WSST	Average remuneration of employees at current market prices. OECD: Compensation of employees is the total remuneration, in cash or in kind, payable by an enterprise to an employee in return for work done by the latter during the accounting period. Compensation of employees has two main components: (a) Wages and salaries payable in cash or in kind; (b) The value of the social contributions payable by employers: these may be actual social contributions payable by employers to Social Security schemes or to private funded social insurance schemes to secure social benefits for their employees, or imputed social contributions by employers providing unfunded social benefits.	1)
Productive capital stocks, volume	KTPV	OECD: Productive capital stock is the stock of a particular, homogeneous, asset expressed in 'efficiency' units. The importance of the productive stock derives from the fact that it offers a practical tool to estimate capital services. Typically, the latter are assumed to be proportional to the former. Here it is presented at constant prices.	1)
Tangible fixed assets	N111	Tangible fixed assets are non-financial produced assets that consist of dwellings, other buildings and structures, machinery and equipment and cultivated assets.	2)

Sources

- 1) Economic Outlook No 91 - June 2012; Available online: <http://www.oecd-ilibrary.org/statistics> (Accessed on August 27th 2012)
- 2) OECD (2010), "Detailed National Accounts: Balance sheets for non-financial assets", OECD National Accounts Statistics (database). doi: 10.1787/data-00368-en (Accessed on August 27th 2012)

6.2.2 Results stationarity tests:

Germany				
Period	kapital output ratio productive		GDP growth rate	
	mean: 2.289744	ci: [2.2730,2.3065]	mean: 1.2916	ci: [0.4703,2.1129]
	Augmented DF Test (1 lag) p-value	Phillip Perron Test (1 lag) p-value	Augmented DF Test (1 lag) p-value	Phillip Perron Test (1 lag) p-value
	1991-2007	0.0005	0.0969	0.0529
Japan				
Period	kapital output ratio productive		GDP growth rate	
	mean: 3.7986	ci: [3.4931,4.1041]	mean: 1.9267	ci: [0.9538,2.8995]
	Augmented DF Test (1 lag) p-value	Phillip Perron Test (1 lag) p-value	Augmented DF Test (1 lag) p-value	Phillip Perron Test (1 lag) p-value
	1991-2007	0.0715	0.0002	0.0073
UK				
Period	kapital output ratio productive		GDP growth rate	
	mean: 1.4782	ci:[1.4285,1.5278]	mean: 2.3202	ci: [1.5622,3.0782]
	Augmented DF Test (1 lag) p-value	Phillip Perron Test (1 lag) p-value	Augmented DF Test (1 lag) p-value	Phillip Perron Test (1 lag) p-value
	1988-2007	0.0000	0.0140	0.1082
	1991-2007	0.0165	0.0003	0.0013
US				
Period	kapital output ratio productive		GDP growth rate	
	mean: 2.0340	ci: [1.8886,2.1794]	mean: 2.8016	ci: [2.0955,3.5078]
	Augmented DF Test (1 lag) p-value	Phillip Perron Test (1 lag) p-value	Augmented DF Test (1 lag) p-value	Phillip Perron Test (1 lag) p-value
	1991-2007	0.0348	0.0188	0.0022

Stationarity tests

6.2.3 Determining Values for the Baseline Scenario from the Model:

Determining values for baseline scenario from the model:

			U.S.	U.K.	Ger	Jap
computed values of variables or param. ⁴⁸	<i>(equ.)</i>	model relation	1995 -2007			2001 -2007
- <i>elast. of labor prod.</i> ⁴⁹ ,	(1,4)	$1 - \alpha^* = \frac{w^*H}{Y}$	0.59	0.62	0.59	0.60
- <i>elast. of capital prod.</i>		$\alpha^* = -\frac{w^*H}{Y} + 1$	0.41	0.38	0.41	0.40
- <i>net return on capital (%)</i>	(7)	$r^* = i$	3.01	3.24	3.99	2.72
- <i>gross return on capital (%)</i>	(3)	$r^{gross*} = \alpha^* k_t^{-1}$	15.04	9.22	13.18	14.2
- <i>deprec. rate of capital (%)</i>	(3)	$\delta^* = r^{gross*} - r^*$	15.04	5.97	9.20	11.5
- <i>bequeath share for sovereign debt (%)</i>	(11)	<i>see above</i>				
		$\gamma, \varphi = 0.5$	0.69	0.77	0.75	0.16
		$\gamma, \varphi = 1$	0.69	0.77	0.74	0.15
		$\gamma, \varphi = 2$	0.68	0.76	0.74	0.13
exogenous choices, parameters values						
- <i>elast. of intertemp. subst.</i>		φ	0.5/1/2	0.5/1/2	0.5/1/2	0.5/1/2
- <i>evalu. of future consum.</i>		$1 - \beta$	0.5	0.5	0.5	0.5

⁴⁸To derive these variable values, OECD equations from the theoretical model were used to calculate the values of the respective variables.

⁴⁹See for example Voyoda and Yeldan (2005), and Furceri and Mourougane (2010) for values of $\alpha > 0.4$, Moraga and Vidal (2004) for $\alpha = 0.3$ and Chalk (2000) for $\alpha = 0.25$.

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