



**UNIVERSITÄT PADERBORN**  
*Die Universität der Informationsgesellschaft*

# CENTER FOR INTERNATIONAL ECONOMICS

## Working Paper Series

---

Working Paper No. 2013-07

### **Cooperative Transfer Price Negotiations under Incomplete Information**

Sonja Brangewitz, Claus-Jochen Haake

July 2013

---

# Cooperative Transfer Price Negotiations under Incomplete Information

Sonja Brangewitz and Claus-Jochen Haake\*

July 17, 2013

## Abstract

In this paper, we analyze a model in which two divisions negotiate over an intrafirm transfer price for an intermediate product. Formally, we consider bargaining problems under incomplete information, since the upstream division's (seller's) costs and downstream division's (buyer's) revenues are supposed to be private information. Assuming two possible types for buyer and seller each, we first establish that the bargaining problem is regular, regardless whether incentive and/or efficiency constraints are imposed. This allows us to apply the generalized Nash bargaining solution to determine transfer payments and transfer probabilities. Furthermore, we derive general properties of this solution for the transfer pricing problem and compare the model developed here with the existing literature for negotiated transfer pricing under incomplete information. In particular, we focus on the models presented in Wagenhofer (1994).

**Keywords:** Transfer Pricing, Negotiation, Generalized Nash Bargaining Solution, Incomplete Information

**JEL Classification Numbers:** C78, D82, M41

---

\*Department of Economics, University of Paderborn, Warburger Str. 100, 33098 Paderborn, Germany; Sonja.Brangewitz@wiwi.uni-paderborn.de and Claus-Jochen.Haake@wiwi.uni-paderborn.de. This work was partially supported by the German Research Foundation (DFG) within the Collaborative Research Centre "On-The-Fly Computing" (SFB 901). Haake acknowledges financial support from the DFG project TR 120/15-1, "Projekt (USA): Analysis of the Interplay between Markets and Cooperative Games".

# 1 Introduction

We consider a model of intrafirm trade of an indivisible intermediate product that is produced by the upstream division (division 1) and then sold to the downstream division (division 2). Division 2 is able to sell the product on an external market (or to another division). In the following, we also refer to division 1 as *the seller*, and to division 2 as *the buyer*. To focus on the bilateral trade of the two divisions, the traded product itself cannot be sold or bought on an external market. The trade of the intermediate product is supposed to be an additional profit opportunity for the two divisions. The transfer process is illustrated in Figure 1.

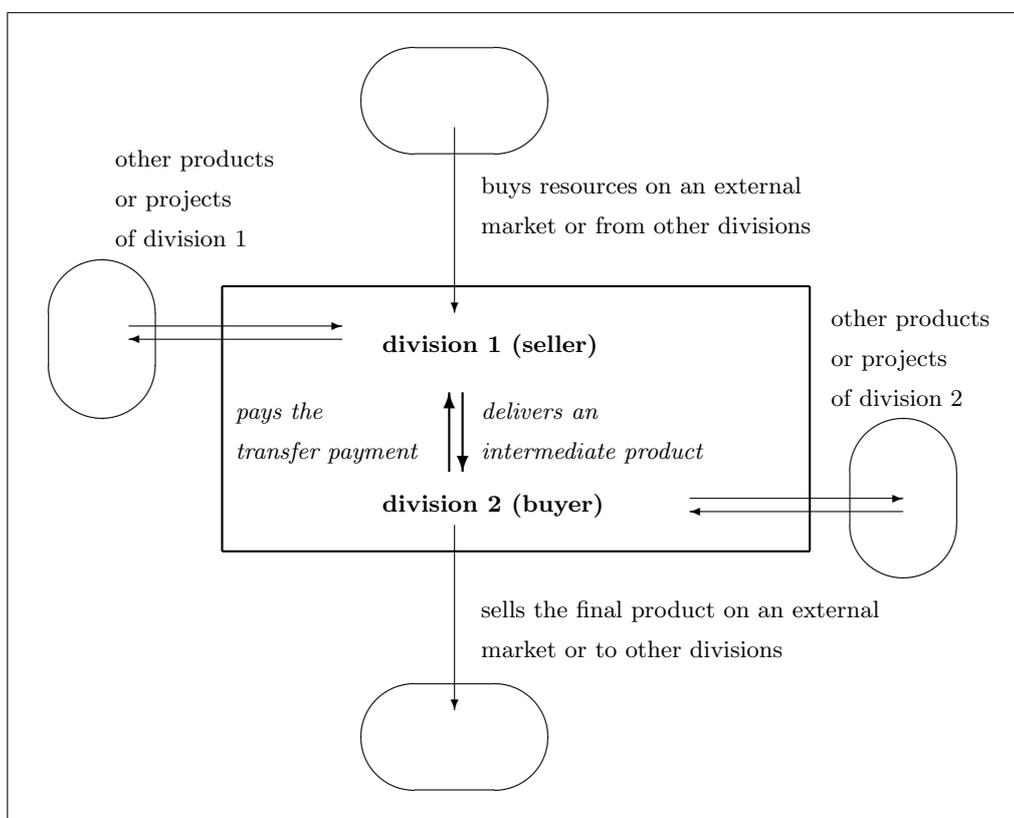


Figure 1: Transfer process.

Both divisions act as profit centers and are assumed to maximize their divisional profits, whereas headquarters (HQ) seeks to maximize the overall profit of the company.<sup>1</sup> Both divisions, or more precisely its managers, are assumed to have private information about costs and revenues that HQ cannot observe.

<sup>1</sup>The performance of the division might be used as an indicator to evaluate the abilities and the effort of the division managers. Thus, each division manager is supposed to maximize his divisional profit.

HQ is just able to observe the final profit or loss of the divisions without the information if the profit or loss was generated by the intrafirm trade or by other divisional activities. Therefore, due to this lack of information, HQ cannot simply solve a profit maximization problem by setting a transfer price for the divisions. In our model, HQ delegates the responsibility to determine a transfer price to the divisions, meaning that HQ decides for negotiated transfer pricing as the transfer pricing scheme.

In this paper, we use methods from cooperative game theory to model negotiations on the transfer price and suggest to apply an appropriate modification of the Nash bargaining solution that is capable to deal with the incomplete information setting.

The literature on transfer pricing offers a variety of solutions to the transfer pricing problem, such as cost-based, market-based, or negotiated transfer pricing schemes. Wagenhofer (1994), for example, collects and evaluates these methods in a simple model, focusing on (ex post) efficiency. Since we essentially use the same framework as Wagenhofer (1994), we are able to directly compare the Nash solution to the alternative methods by means of the examples discussed there. A more recent survey on transfer pricing schemes is Göx and Schiller (2006).

We compare our approach to the seminal works on negotiated transfer pricing, Vaysman (1998), Baldenius et al. (1999) and Baldenius (2000), as well as Wagenhofer (1994). The main difference is that in these models, negotiations are modeled by a non-cooperative game between the two divisions and the final transfer price results from an equilibrium outcome. In contrast to that, we follow a purely cooperative approach. While the analysis in the preceding models focuses on efficiency, (Rajan and Reichelstein, 2004, *Management Science*) highlight the role of incentive compatibility and individual rationality as desirable properties for transfer pricing mechanisms. They extend an earlier work by (Harris et al., 1982, *Management Science*). In particular, both papers consider the transfer pricing problem from a mechanism design perspective. Especially the incentive compatibility is crucial, as it guarantees that neither division has an incentive to reveal its private information incorrectly (e.g., on costs or revenues), given the other division reports truthfully. In that sense, incentive compatibility is a device to overcome information asymmetries among the divisions. However, this desirable feature potentially comes at the cost of efficiency losses. This well-known conflict was already formalized in Myerson and Satterthwaite's (1983) impossibility theorem and partially resolved in Matsuo (1989).

A seemingly different approach to deal with negotiations under incomplete information is found in the literature on cooperative bargaining theory, having its roots in works by Harsanyi (1967, 1968a,b, *Management Science*). The presence of incomplete information actually shifts the problem from negotiations over actual prices and quantities (here transfer probabilities) to negotiations over transfer contracts, specifying the transfer details (price, quantity) for any possible combination of private information. A few years later Harsanyi and Selten (1972, *Management Science*) introduce a generalization of Nash's (1950, 1953) proposed

solution for complete information bargaining games, which is undoubtedly the most widely used one in theory and practice. An axiomatic foundation of the generalized Nash bargaining solution (or *Nash solution* in short) was later given in Weidner (1992). Up to now, few papers explicitly use methods from cooperative game theory to solve the transfer pricing problem. Leng and Parlar (2012) compute Shapley value based transfer prices and Haake and Martini (2012) determine bargaining solutions under symmetric information.

The goal of this paper is to propose an alternative solution to the transfer pricing problem, combining incentive compatibility and negotiations. To be more precise, we define a transfer pricing game under incomplete information and apply the generalized Nash bargaining solution. Requiring agreements to be incentive compatible and/or efficient, we highlight the relation between these two desirable properties. As a necessary intermediate result for the applicability of the generalized Nash bargaining solution, we show that the transfer pricing game is regular, meaning that it is possible to guarantee each division a strictly positive expected profit, regardless of their specific private information. From a managerial perspective, one appealing feature of the generalized Nash bargaining solution is that it provides each division with a strictly positive expected profit. Furthermore, as the examples illustrate, the Nash solution tends to keep differences in divisional profits smaller compared to other solutions. In this spirit, it could be considered as a fairness property.

The organization of the paper is as follows. Section 2 discusses the basic model and introduces the transfer pricing bargaining game. The generalized Nash bargaining solution is defined in Section 3. Here, we show regularity (Proposition 2) and derive necessary conditions for a transfer contract that implements the generalized Nash bargaining solution (Propositions 3, 4 and 5). Section 4 then compares the Nash solution with other solutions discussed in Wagenhofer (1994). This is mainly done through Examples 3 and 4. Section 5 concludes.

## 2 The Framework

We consider a firm with two divisions trading one unit of an intermediate product that is produced by division 1 and later sold on an external market by division 2. In our model, division 1 has private information on its production cost, while division 2 privately knows sales revenues. Following Harsanyi (1967) a *type* of division  $i$  summarizes all its privately known characteristics. For simplicity, we assume that division 1's possible type is  $t_1 \in T_1 := \{H, L\}$  with the effect that production costs are either  $C_L$  or  $C_H$  with  $C_L < C_H$ . Similarly, division 2 is of type  $t_2 \in T_2 \in \{H, L\}$  with revenues  $R_L$  or  $R_H$  ( $R_L < R_H$ ), respectively. Observe that trade among the divisions is only efficient whenever revenues exceed costs.

We assume that divisional types are distributed independently, i.e., the a priori probability distribution on the set of type profiles  $T = T_1 \times T_2$  be given by

$$P_{HH} = \varepsilon\delta, P_{HL} = \varepsilon(1 - \delta), P_{LH} = (1 - \varepsilon)\delta, P_{LL} = (1 - \varepsilon)(1 - \delta),$$

( $0 < \varepsilon < 1$ ,  $0 < \delta < 1$ ) where  $P_{t_1 t_2}$  denotes the probability that division  $i$  is of type  $t_i$  ( $i=1, 2$ ).<sup>2</sup> Hence, the high cost seller type  $H$  appears with a probability of  $\varepsilon$  and the high revenue buyer type  $H$  with a probability of  $\delta$ .

Since division managers are typically better informed about divisional characteristics (costs, revenues) than HQ, we consider a model in which the decision on the transfer price and transfer probability is completely left to the two divisions. In our model we assume that the two divisions bargain employing models from cooperative game theory. The negotiation result is a transfer payment  $Y$  that is paid from the buying division 2 to the selling division 1 and a probability  $Q$  with which the unit of the product is actually transferred.<sup>3</sup>

In view of our incomplete information setup, the divisions have to agree on transfer price and probability for any possible type profile, thus rather bargain over type-dependent contracts or mechanisms. Formally, a *mechanism* determines for each type profile a transfer payment and a transfer probability, i.e.,  $\mu^{(Y,Q)} = (Y_t, Q_t)_{t \in T}$ .

Summarizing, we suppose the following time line of events as illustrated in Figure 2.

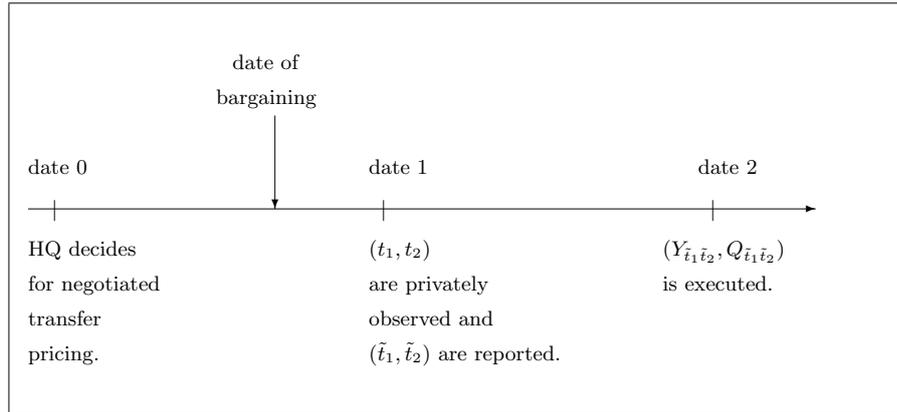


Figure 2: Time line for the negotiated transfer pricing model.

At date 0, the HQ commits negotiated transfer pricing as the chosen transfer pricing scheme. Then the selling and buying division negotiate over (type-dependent) transfer payments and transfer probabilities. At date 2, the managers observe their types privately. The selling division observes its cost type  $t_1 \in T_1 = \{H, L\}$  and the buying division its revenue type  $t_2 \in T_2 = \{H, L\}$ . The two divisions are asked to confidently report their types (not necessarily truthfully). Finally, at

<sup>2</sup>For simplicity, we denote a type profile  $(t_1, t_2)$  in short by  $t_1 t_2$ .

<sup>3</sup>Alternatively, one may think of a constant returns production technology with constant unit costs and linear pricing on the external market. Then  $Q$  can be interpreted as a “quantity” being the fraction of a maximal amount of the good that is traded.

date 3 according to the reported types, denoted by  $(\tilde{t}_1, \tilde{t}_2)$ , the transfer payment  $Y_{\tilde{t}_1\tilde{t}_2}$  and the transfer probability  $Q_{\tilde{t}_1\tilde{t}_2}$  are executed.

The utility of a division is its (additional) profit from the trade of the intermediary product. If the true type profile is  $t = (t_1, t_2) \in T$  and the reported type profile is  $\tilde{t} = (\tilde{t}_1, \tilde{t}_2) \in T$ , then the transfer payment and the transfer probability is  $(Y_{\tilde{t}}, Q_{\tilde{t}})$  and the utilities are given by

$$u_1((Y_{\tilde{t}}, Q_{\tilde{t}}), t_1) = Y_{\tilde{t}} - Q_{\tilde{t}}C_{t_1} \quad \text{and} \quad u_2((Y_{\tilde{t}}, Q_{\tilde{t}}), t_2) = Q_{\tilde{t}}R_{t_2} - Y_{\tilde{t}}.$$

Note that the utility functions  $u_i$  are affine in  $Y_{\tilde{t}}$  and  $Q_{\tilde{t}}$ .

Although HQ is not directly involved in this intrafirm bargaining game, it can nonetheless have influence on feasible mechanisms. For example, HQ could impose a restriction that neither division should have a negative (expected) payoff from the transaction. Indeed, this can be verified without knowing the exact type profile. Thus, HQ would only be willing to accept (interim) individually rational mechanisms. Furthermore, and this is central in our analysis, HQ could only allow for incentive compatible mechanisms, meaning that no division can benefit from misrepresenting its true type, while the other division reports truthfully.

In summary, we assume that the two divisions bargain over allocations of expected utility generated by incentive compatible and (interim) individually rational mechanisms taking conflict outcome  $(0, 0)$ .<sup>4</sup>

In what follows we concentrate on the case

$$R_H \geq C_H > R_L \geq C_L.$$

Since  $C_H > R_L$ , and therefore trade for the type profile  $HL$  cannot be profitable for both divisions simultaneously, we assume  $Y_{HL} = 0$  and  $Q_{HL} = 0$ . Hence, for a mechanism  $\mu^{(Y,Q)}$  division  $i$ 's *expected utility*  $U_i(\mu^{(Y,Q)}|t_i)$  from  $\mu^{(Y,Q)}$  when being of type  $t_i$  is given by

$$U_1(\mu^{(Y,Q)}|H) = \delta(Y_{HH} - Q_{HH}C_H) \tag{EU1}$$

$$U_1(\mu^{(Y,Q)}|L) = \delta(Y_{LH} - Q_{LH}C_L) + (1 - \delta)(Y_{LL} - Q_{LL}C_L) \tag{EU2}$$

$$U_2(\mu^{(Y,Q)}|H) = \varepsilon(Q_{HH}R_H - Y_{HH}) + (1 - \varepsilon)(Q_{LH}R_H - Y_{LH}) \tag{EU3}$$

$$U_2(\mu^{(Y,Q)}|L) = (1 - \varepsilon)(Q_{LL}R_L - Y_{LL}). \tag{EU4}$$

---

<sup>4</sup>The elements described constitute a Bayesian bargaining problem

$$\Gamma = (D, (0, 0), T_1, T_2, u_1, u_2, P)$$

with  $D \subseteq \mathbb{R}$ , a convex polyhedron in the sense of Myerson (1979). We refer to it here as the transfer pricing game.

The mechanism  $\mu^{(Y,Q)}$  is (*interim*) *individually rational (IR)* if the following constraints are satisfied

$$\delta(Y_{HH} - Q_{HH}C_H) \geq 0 \quad (\text{IR1})$$

$$\delta(Y_{LH} - Q_{LH}C_L) + (1 - \delta)(Y_{LL} - Q_{LL}C_L) \geq 0 \quad (\text{IR2})$$

$$\varepsilon(Q_{HH}R_H - Y_{HH}) + (1 - \varepsilon)(Q_{LH}R_H - Y_{LH}) \geq 0 \quad (\text{IR3})$$

$$(1 - \varepsilon)(Q_{LL}R_L - Y_{LL}) \geq 0. \quad (\text{IR4})$$

If (IR1) to (IR4) hold with strict inequality, the mechanism  $\mu^{(Y,Q)}$  is *strictly (interim) individually rational*. In an individually rational mechanism each division obtains at least the expected utility it receives in the case of disagreement, independent of the observed type.

The mechanism  $\mu^{(Y,Q)}$  is (*Bayesian*) *incentive compatible (IC)* if the following constraints are satisfied

$$\delta(Y_{HH} - Q_{HH}C_H) \geq \delta(Y_{LH} - Q_{LH}C_H) + (1 - \delta)(Y_{LL} - Q_{LL}C_H) \quad (\text{IC1})$$

$$\delta(Y_{LH} - Q_{LH}C_L) + (1 - \delta)(Y_{LL} - Q_{LL}C_L) \geq \delta(Y_{HH} - Q_{HH}C_L) \quad (\text{IC2})$$

$$\varepsilon(Q_{HH}R_H - Y_{HH}) + (1 - \varepsilon)(Q_{LH}R_H - Y_{LH}) \geq (1 - \varepsilon)(Q_{LL}R_H - Y_{LL}) \quad (\text{IC3})$$

$$(1 - \varepsilon)(Q_{LL}R_L - Y_{LL}) \geq \varepsilon(Q_{HH}R_L - Y_{HH}) + (1 - \varepsilon)(Q_{LH}R_L - Y_{LH}). \quad (\text{IC4})$$

The inequalities (IC1) to (IC4) compare a division's expected utility when reporting its true type (l.h.s.) with its expected utility when pretending to be of the other type (r.h.s.). The expectation is taken w.r.t. probabilities over the other division's type. Note that it is assumed that the other division reports its type truthfully. Phrased differently, a mechanism  $\mu^{(Y,Q)}$  induces a non-cooperative Bayesian game in which the divisions are asked to report their types. Incentive compatibility ensures that honest reporting is a Bayesian Nash equilibrium of this game.

Finally, a mechanism is *ex post efficient (EPE)*<sup>5</sup> if the intermediate product is transferred with probability  $Q_t \in \{0, 1\}$  for all  $t \in T$  and  $Q_t = 1$  whenever strictly mutually beneficial trade is possible and is equal to 0 whenever trade is strictly unprofitable for at least one division. This means that for  $t = (t_1, t_2)$ ,

$$Q_t = \begin{cases} 1 & , \text{if } R_{t_2} > C_{t_1} \\ 0 & , \text{if } R_{t_2} < C_{t_1}. \end{cases}$$

In case  $R_{t_2} = C_{t_1}$ , EPE imposes no additional restriction on  $Q_t$ . In particular, if we have  $R_H > C_H > R_L > C_L$ , then any EPE mechanism satisfies  $Q_{HH} = Q_{LH} = Q_{LL} = 1$  and  $Q_{HL} = 0$ .

The existence of IR and EPE mechanisms is easy to see. However, requiring additional IC restrictions may lead to non-existence as the following result of Matsuo (1989) shows.

<sup>5</sup>Compare Holmström and Myerson (1983) for further notions of efficiency for mechanisms.

**Proposition 1** (Matsuo, 1989). *If we assume  $R_H > C_H > R_L > C_L$ , then there exists an IR, IC, and EPE mechanism if and only if*

$$\varepsilon\delta R_H + (1 - \varepsilon)R_L \geq \delta C_H + (1 - \delta)(1 - \varepsilon)C_L. \quad (1)$$

Following the proof in Matsuo (1989) we obtain the following immediate implications.

**Corollary 1.** *(i) If we assume  $R_H = C_H > R_L > C_L$ , then there exists an IR, IC, and EPE mechanism if and only if*

$$R_L \geq \delta C_H + (1 - \delta)C_L.$$

*(ii) If we assume  $R_H > C_H > R_L = C_L$ , then there exists an IR, IC, and EPE mechanism if and only if*

$$\varepsilon R_H + (1 - \varepsilon)R_L \geq C_H.$$

*(iii) If we assume  $R_H = C_H > R_L = C_L$ , then no IR, IC, and EPE mechanism exists.*

In particular, if mutually beneficial trade is possible only when revenues are high and costs are low, then no EPE mechanism exists that satisfies the IR and IC constraints (part (iii) of the corollary).

### 3 The Generalized Nash Bargaining Solution

The previous section discussed the bargaining problem under incomplete information. Indeed, the two divisions bargain over (type-dependent) mechanisms rather than over a single price and quantity. In view of the IR, IC, and EPE restrictions, we may shape different bargaining problems, depending on which of these restrictions shall be active. To be precise, we consider individual rationality as an indispensable restriction and rather concentrate on the tension between incentive compatibility and (ex post) efficiency. Therefore, we discuss three versions of the bargaining problem in which the divisions bargain over (1) IR and IC, (2) IR, IC, and EPE, and (3) IR and EPE mechanisms. While the latter focuses on first best outcomes, the first one highlights the trade-off between IC and EPE. It turns out that even under mild conditions, existence of mechanisms that satisfy IR, IC and EPE cannot be guaranteed (Proposition 1 and Corollary 1), so the scenario in (2) is not always a feasible option. Observe again that all three conditions can be verified by HQ, as only information on costs, revenues and probabilities are necessary. Hence, HQ may set the set of feasible mechanisms for the bargaining problem and may check whether the final agreement satisfies them. To keep terminology simple, we will say that divisions bargain over IR/IC/EPE mechanisms, whichever are applicable. Formally, denote by  $\mathcal{M}^1$  the set of IR/IC

mechanisms, by  $\mathcal{M}^2$  the set of IR/IC/EPE mechanisms, and by  $\mathcal{M}^3$  the set of IR/EPE mechanisms.

Independent of which version of the transfer pricing bargaining problem we consider, we use the generalized Nash bargaining solution to propose some  $\mu^{(Y,Q)}$  as the outcome of the incomplete information bargaining game from the previous section. For bargaining problems under complete information, the Nash bargaining solution maximizes the product of players' utilities and thus selects an efficient allocation of utility. This idea is transferred to the incomplete information setup in the following way: Treat each type of each division as a single *agent*. An agent's utility is then her division's utility conditioned by the type corresponding to that agent. These conditional utilities are given in (EU1) to (EU4). Note that for a mechanism  $\mu^{(Y,Q)}$ , these conditional utilities can be calculated ex ante and therefore are commonly known. Now, following Harsanyi and Selten (1972), Myerson (1979) and Weidner (1992), the *generalized Nash bargaining solution* selects those mechanisms  $\mu^{(Y,Q)}$  for which the weighted product of agents' utilities is maximal. More precisely, the probability with which a specific type occurs is attached as exponent to the corresponding agent's utility. In the transfer pricing game it selects mechanisms  $\mu^{*(Y,Q)}$  that maximize the objective function

$$F(\mu^{(Y,Q)}) := (U_1(\mu^{(Y,Q)}|H))^\varepsilon \cdot (U_1(\mu^{(Y,Q)}|L))^{(1-\varepsilon)} \cdot (U_2(\mu^{(Y,Q)}|H))^\delta \cdot (U_2(\mu^{(Y,Q)}|L))^{(1-\delta)}.$$

over  $\mathcal{M}^1, \mathcal{M}^2$ , or  $\mathcal{M}^3$ , respectively, depending on which constraints on feasible agreements are imposed. Note that  $F$  is quasiconcave in expected utilities.

Now, the constraints in (IR1) to (IR4) and (IC1) to (IC4) are linear in prices and quantities. Furthermore, EPE mechanisms form an affine subspace in  $\mathbb{R}^8$ , since the only restrictions are  $Q_{HH} = Q_{LH} = Q_{LL} = 1$ . It follows that either  $\mathcal{M}^i$  is a convex polyhedron (in  $\mathbb{R}^6$ ). Since conditional expectation is a linear operator, the set of expected utility allocations from mechanisms in  $\mathcal{M}^i$  is again a convex polyhedron. Hence there is at most<sup>6</sup> one expected utility allocation in which  $F$  is maximized. Phrased differently, the generalized Nash bargaining solution collects all mechanisms in which the product of agents' expected utilities is maximal. Observe that we do not adopt a welfaristic viewpoint by defining the solution containing mechanisms rather than utility allocations. However, although there might be more than one mechanism that maximizes  $F$ , expected utilities for each agent of each division are the same.

Before the generalized Nash bargaining solution can be applied, a technical obstacle has to be removed. As for the Nash bargaining solution in the complete information case, it is crucial that there is at least one mechanism in  $\mathcal{M}^i$ , which gives each agent (type of a division) a strictly positive conditional utility so that the product function  $F$  is not constantly zero. According to Harsanyi and Selten (1972) a transfer pricing game that admits such a mechanism is called *regular*.

<sup>6</sup>Note that depending on the values of  $R_H, R_L, C_H, C_L$  some  $\mathcal{M}^i$  might be empty.

**Proposition 2.** *Suppose that divisions bargain over IR and IC, or over IR and EPE mechanisms, then transfer pricing game is regular. The transfer pricing game over IR, IC, and EPE mechanisms is regular, if and only if inequality (1) is strict. Hence, the generalized Nash bargaining solution is well-defined.*

The proposition not only states that the bargaining problem admits strictly positive profits, but it sharpens Proposition 1 w.r.t. strict individual rationality.

*Proof.* We show regularity in the three cases by defining a mechanism that satisfies the IR constraints with strict inequality and obey the remaining restrictions.

*Case 1:* In the presence of IR and IC constraints, consider the mechanism  $\mu^{(Y,Q)}$  with

$$\begin{aligned}(Y_{HH}, Q_{HH}) &= \left( \frac{3(1-\varepsilon)Y_{LL} + \delta R_H - (1-\varepsilon)(1+\delta)C_L + \delta(1-\varepsilon)C_H}{3\delta}, \frac{1}{3} \right), \\(Y_{LH}, Q_{LH}) &= \left( \frac{3(\delta-\varepsilon)Y_{LL} + \delta R_H + \varepsilon(1+\delta)C_L + \delta(1-\varepsilon)C_H}{3\delta}, 1 \right), \\(Y_{HL}, Q_{HL}) &= (0, 0), \\(Y_{LL}, Q_{LL}) &= \left( \frac{R_L + C_L - 2\delta(R_H - C_H) - 2\delta\varepsilon(C_H - C_L)}{6}, \frac{1}{3} \right).\end{aligned}$$

*Case 2.1:* In the presence of IR, IC, and EPE constraints and additionally  $\varepsilon\delta R_H + (1-\varepsilon)R_L > \delta C_H + (1-\delta)(1-\varepsilon)C_L$ , use the following mechanism  $\mu^{(Y,Q)}$  with

$$\begin{aligned}(Y_{HH}, Q_{HH}) &= \left( \frac{(1-\varepsilon)Y_{LL} + \varepsilon\delta R_H - (1-\delta)(1-\varepsilon)C_L}{\delta}, 1 \right), \\(Y_{LH}, Q_{LH}) &= \left( \frac{(\delta-\varepsilon)Y_{LL} + \varepsilon\delta R_H + (1-\delta)\varepsilon C_L}{\delta}, 1 \right), \\(Y_{HL}, Q_{HL}) &= (0, 0), \\(Y_{LL}, Q_{LL}) &= \left( \frac{(1-\delta)(1-\varepsilon)C_L + \delta C_H - \varepsilon\delta R_H + (1-\varepsilon)R_L}{2(1-\varepsilon)}, 1 \right).\end{aligned}$$

*Case 2.2:* In the presence of IR, IC, and EPE constraints and additionally  $\varepsilon\delta R_H + (1-\varepsilon)R_L = \delta C_H + (1-\delta)(1-\varepsilon)C_L$  we show that no strictly individually rational mechanism exists. To see this we first add constraint (IC2) multiplied by  $(1-\varepsilon)$  to constraint (IC3) multiplied by  $\delta$  and obtain:

$$\begin{aligned}(1-\delta)(1-\varepsilon)Y_{LL} - (1-\varepsilon)C_L + \delta R_H - \varepsilon\delta Y_{HH} \\ \geq (1-\varepsilon)\delta Y_{HH} - (1-\varepsilon)\delta Y_{LL} - (1-\varepsilon)\delta C_L + (1-\varepsilon)\delta R_H.\end{aligned}$$

Rearranging yields

$$(1-\varepsilon)Y_{LL} - (1-\varepsilon)(1-\delta)C_L + \varepsilon\delta R_H - \delta Y_{HH} \geq 0.$$

If constraint (IR1) holds with strict inequality, we have  $Y_{HH} > C_H$ . Hence, we have

$$\begin{aligned} 0 &\leq (1 - \varepsilon)Y_{LL} - (1 - \varepsilon)(1 - \delta)C_L + \varepsilon\delta R_H - \delta Y_{HH} \\ &< (1 - \varepsilon)Y_{LL} - (1 - \varepsilon)(1 - \delta)C_L + \varepsilon\delta R_H - \delta C_H \\ &= (1 - \varepsilon)(Y_{LL} - R_L). \end{aligned}$$

Therefore,  $Y_{LL} > R_L$ , which contradicts the strict inequality of constraint (IR4).

*Case 3:* In the presence of IR and EPE constraints, consider the mechanism defined in Proposition 3, which we will not repeat here.

In the three cases, 1, 2.1 and 3, tedious, yet straightforward calculations show that the given mechanism is strictly individually rational.<sup>7</sup> Therefore, Theorem 3 in Myerson (1979) for regular bargaining problems can be applied showing the existence and uniqueness of agents' expected utilities.  $\square$

Proposition 2 shows that the generalized Nash bargaining solution is actually well-defined. We now investigate how corresponding mechanisms look like. For this, we start with the first best solution as a "benchmark case". This means that we leave out the incentive constraints and have the divisions bargain over IR and EPE mechanisms.

**Proposition 3.** *Suppose that divisions bargain over IR and EPE mechanisms. Then the generalized Nash bargaining solution is attained by the mechanism  $\mu^{*(Y,Q)}$  with*

$$\begin{aligned} (Y_{HH}, Q_{HH}) &= \left( \frac{\delta(R_H + C_H) + (1 - \varepsilon)\delta(C_H - R_L) + (1 - \varepsilon)(R_L - C_L)}{2\delta}, 1 \right), \\ (Y_{LH}, Q_{LH}) &= \left( \frac{R_H + C_L - \varepsilon Y_{HH} - (1 - \delta)Y_{LL}}{1 - \varepsilon + \delta}, 1 \right), \\ (Y_{HL}, Q_{HL}) &= (0, 0), \\ (Y_{LL}, Q_{LL}) &= \left( \frac{-\delta(R_H - C_H) - (1 - \varepsilon)\delta(C_H - R_L) + (1 - \varepsilon)(R_L + C_L)}{2(1 - \varepsilon)}, 1 \right). \end{aligned}$$

The agents' expected utilities are

$$\begin{aligned} U_1(\mu^{*(Y,Q)}|H) &= U_1(\mu^{*(Y,Q)}|L) = U_2(\mu^{*(Y,Q)}|H) = U_2(\mu^{*(Y,Q)}|L) \\ &= \frac{\delta(R_H - C_H) + (1 - \varepsilon)\delta(C_H - R_L) + (1 - \varepsilon)(R_L - C_L)}{2} \\ &= \frac{\delta R_H + (1 - \varepsilon)(1 - \delta)R_L - \varepsilon\delta C_H - (1 - \varepsilon)C_L}{2}. \end{aligned} \quad (2)$$

To prove Proposition 3, we make use of the following technical lemma, whose proof is found Appendix B.

<sup>7</sup>Interestingly, the mechanism in Case 1 is a convex combination of three other mechanisms mentioned in Appendix A, Remark 1. A similar observation holds for the mechanism from Case 2.1, see Appendix A, Remark 2.

**Lemma 1.** Let  $f : \mathbb{R}_+^n \rightarrow \mathbb{R}_+$  be defined by

$$f(x_1, \dots, x_n) = \prod_{i=1}^n x_i^{l_i} \quad \text{with } l_1, \dots, l_n > 0$$

and consider the constrained maximization problem

$$\max_{x_1, \dots, x_n} f(x) \quad \text{s.t.} \quad \sum_{i=1}^n l_i \cdot x_i \leq c$$

with  $c \in \mathbb{R}_+$ . Then there is a unique maximizer  $(x_1^*, \dots, x_n^*)$  and  $x^*$  satisfies  $x_1^* = \dots = x_n^*$ .

*Proof of Proposition 3.* We may apply the lemma to the transfer pricing game  $f$  being the generalized Nash product, where the  $x_i$ 's are the agents' conditional utilities resulting from the mechanism  $\mu^{(Y,Q)}$  and the  $l_i$ 's are the type probabilities. The constraint in the lemma is satisfied with

$$c := \delta R_H + (1 - \delta)(1 - \varepsilon)R_L - \varepsilon\delta C_H - (1 - \varepsilon)C_L,$$

since

$$\begin{aligned} & \varepsilon(U_1(\mu^{(Y,Q)}|H)) + (1 - \varepsilon)(U_1(\mu^{(Y,Q)}|L)) + \delta(U_2(\mu^{(Y,Q)}|H)) + (1 - \delta)(U_2(\mu^{(Y,Q)}|L)) \\ &= \varepsilon\delta(Y_{HH} - Q_{HH}C_H) + (1 - \varepsilon)(\delta(Y_{LH} - Q_{LH}C_L) + (1 - \delta)(Y_{LL} - Q_{LL}C_L)) \\ & \quad + \delta(\varepsilon(Q_{HH}R_H - Y_{HH}) + (1 - \varepsilon)(Q_{LH}R_H - Y_{LH})) + (1 - \delta)(1 - \varepsilon)(Q_{LL}R_L - Y_{LL}) \\ &= \varepsilon\delta Q_{HH}(R_H - C_H) + (1 - \varepsilon)\delta Q_{LH}(R_H - C_L) \\ & \quad + (1 - \varepsilon)(1 - \delta)Q_{LL}(R_L - C_L) \\ & \leq \delta R_H + (1 - \delta)(1 - \varepsilon)R_L - \varepsilon\delta C_H - (1 - \varepsilon)C_L = c \end{aligned} \quad (3)$$

is bounded by  $c$ . Actually, (3) holds because all transfer probabilities are no greater than 1. Therefore, it holds with equality for EPE mechanisms. Moreover, the variables (expected utilities) are assumed to be nonnegative. It follows that the domain of the maximization problem includes all EPE and IR mechanisms. From Lemma 1, we know that the optimal solution exhibits the same coordinates, meaning that all conditional utilities are equal. This constitutes a system of linear equations. Straightforward calculations show that  $\mu^{*(Y,Q)}$  is a solution to that system. Now, as  $\mu^{*(Y,Q)}$  is IR and EPE, and is a maximizer of the maximization problem, it must be the generalized Nash bargaining solution when IR and EPE constraints are active.  $\square$

Interestingly, each agent's utility in (2) is HQ's ex ante expected profit from efficient trading up to a factor of 1/2. The factor 1/2 then shows that this expected profit is divided among the two divisions. However, HQ cannot enforce this profit in practice, as it necessitates truthful reporting. Straightforward calculations, though, reveal that the mechanism in Proposition 3 does not satisfy the incentive constraints, confirming the well-known trade-off between efficiency and incentive compatibility.

**Corollary 2.** *In the case that the divisions bargain over IR and EPE mechanisms, the generalized Nash bargaining solution  $\mu^{*(Y,Q)}$  from Proposition 3 satisfies (IC1) and (IC4), but not (IC2) and (IC3).*

Corollary 2 emphasizes the necessity for incentive constraints. The two missing constraints (IC2) and (IC3) are supposed to guarantee that low cost sellers and high revenue buyers tell the truth. Then in the case of the corollary, the selling division has an incentive to always report high cost, while the buying division always announces low revenues.<sup>8</sup> The result will be an announced type profile  $HL$  and no trade and no payments take place.

We therefore turn to the case where the divisions bargain over IR and IC mechanisms. As mentioned above, ex post efficiency can no longer be guaranteed in the generalized Nash bargaining solution. However, in the most profitable type profile  $LH$ , i.e. low costs and high revenues, trade always occurs.

**Proposition 4.** *Suppose that divisions bargain over IR and IC mechanisms.*

- (i) *Then any mechanism  $\mu^{(Y,Q)}$  with  $Q_{LH} < 1$  is Pareto dominated by some IR and IC mechanism.<sup>9</sup>*
- (ii) *Hence, if the generalized Nash bargaining solution is attained by a mechanism  $\mu^{*(Y,Q)}$ , then  $Q_{LH} = 1$ , i.e. the product is traded with probability 1 when the seller is of low cost type and the buyer is of high revenue type.*

*Proof of Proposition 4.* To show (i), we demonstrate that  $Q_{LH} < 1$  leaves some room for a Pareto improvement. Since for any transfer payment between  $R_H$  and  $C_L$  both the selling and the buying division are always willing to trade, we further specify that  $Q_{LH}$  needs to be 1.

Consider a mechanism  $\mu^{(Y,Q)}$  with  $Q_{LH} < 1$  and define a new mechanism  $\tilde{\mu}^{(\tilde{Y},\tilde{Q})}$  by

$$\begin{aligned} (\tilde{Y}_{HH}, \tilde{Q}_{HH}) &= (Y_{HH}, Q_{HH}), & (\tilde{Y}_{LH}, \tilde{Q}_{LH}) &= (Y_{LH} + \gamma C_H, Q_{LH} + \gamma), \\ (\tilde{Y}_{HL}, \tilde{Q}_{HL}) &= (Y_{HL}, Q_{HL}), & (\tilde{Y}_{LL}, \tilde{Q}_{LL}) &= (Y_{LL}, Q_{LL}), \end{aligned}$$

where  $\gamma$  is chosen such that  $\tilde{Q}_{LH} = 1$ . This mechanism  $\tilde{\mu}^{(\tilde{Y},\tilde{Q})}$  still satisfies the IR and IC constraints and gives both divisions for both their types at least the same expected utility, and at least one division is strictly better off. The expected utilities from the mechanism  $\tilde{\mu}^{(\tilde{Y},\tilde{Q})}$  are

$$\begin{aligned} U_1(\tilde{\mu}^{(\tilde{Y},\tilde{Q})}|H) &= U_1(\mu^{(Y,Q)}|H), \\ U_1(\tilde{\mu}^{(\tilde{Y},\tilde{Q})}|L) &= U_1(\mu^{(Y,Q)}|L) + \delta\gamma(C_H - C_L) > U_1(\mu^{(Y,Q)}|L), \\ U_2(\tilde{\mu}^{(\tilde{Y},\tilde{Q})}|H) &= U_2(\mu^{(Y,Q)}|H) + (1 - \varepsilon)\gamma(R_H - C_H) > U_2(\mu^{(Y,Q)}|H), \\ U_2(\tilde{\mu}^{(\tilde{Y},\tilde{Q})}|L) &= U_2(\mu^{(Y,Q)}|L). \end{aligned}$$

<sup>8</sup>To be precise, they do so, assuming that the other division reports truthfully.

<sup>9</sup>To be precise, there is no mechanism  $\hat{\mu}^{(Y,Q)}$  such that each agent's conditional expected utility is no worse than in  $\mu^{(Y,Q)}$  and some agent is strictly better off.

We obtain for the IR constraints

$$\begin{aligned} U_1(\tilde{\mu}^{(\tilde{Y}, \tilde{Q})}|H) &\geq 0, & U_1(\tilde{\mu}^{(\tilde{Y}, \tilde{Q})}|L) &> 0, \\ U_2(\tilde{\mu}^{(\tilde{Y}, \tilde{Q})}|H) &> 0, & U_2(\tilde{\mu}^{(\tilde{Y}, \tilde{Q})}|L) &\geq 0 \end{aligned}$$

and for the IC constraints

$$\begin{aligned} U_1(\tilde{\mu}^{(\tilde{Y}, \tilde{Q})}|H) &= U_1(\mu^{(Y, Q)}|H) \\ &\geq \delta(Y_{LH} - Q_{LH}C_H) + (1 - \delta)(Y_{LL} - Q_{LL}C_H) \\ &= \delta(\tilde{Y}_{LH} - \tilde{Q}_{LH}C_H) + (1 - \delta)(\tilde{Y}_{LL} - \tilde{Q}_{LL}C_H), \end{aligned}$$

$$\begin{aligned} U_1(\tilde{\mu}^{(\tilde{Y}, \tilde{Q})}|L) &= U_1(\mu^{(Y, Q)}|L) + \delta\gamma(C_H - C_L) \\ &\geq \delta(Y_{HH} - Q_{HH}C_L) \\ &= \delta(\tilde{Y}_{HH} - \tilde{Q}_{HH}C_L), \end{aligned}$$

$$\begin{aligned} U_2(\tilde{\mu}^{(\tilde{Y}, \tilde{Q})}|H) &= U_2(\mu^{(Y, Q)}|H) + (1 - \varepsilon)\gamma(R_H - C_H) \\ &\geq (1 - \varepsilon)(Q_{LL}R_H - Y_{LL}) \\ &= (1 - \varepsilon)(\tilde{Q}_{LL}R_H - \tilde{Y}_{LL}), \end{aligned}$$

$$\begin{aligned} U_2(\tilde{\mu}^{(\tilde{Y}, \tilde{Q})}|L) &= U_2(\mu^{(Y, Q)}|L) \\ &\geq \varepsilon(Q_{HH}R_L - Y_{HH}) + (1 - \varepsilon)(Q_{LH}R_L - Y_{LH}) \\ &\geq \varepsilon(Q_{HH}R_L - Y_{HH} + \gamma(1 - \varepsilon)(R_L - C_H)) \\ &\quad + (1 - \varepsilon)(Q_{LH}R_L - Y_{LH}) \\ &= \varepsilon(\tilde{Q}_{HH}R_L - \tilde{Y}_{HH}) + (1 - \varepsilon)(\tilde{Q}_{LH}R_L - \tilde{Y}_{LH}). \end{aligned}$$

Thus, for any mechanism  $\mu^{(Y, Q)}$  with  $Q_{LH} < 1$  we can construct a mechanism  $\tilde{\mu}^{(\tilde{Y}, \tilde{Q})}$  that Pareto dominates  $\mu^{(Y, Q)}$ .

To prove (ii), we use the axiomatization in Weidner (1992) stating that the generalized Nash bargaining solution is Pareto optimal. Precisely, any mechanism for which the Nash product  $F$  is maximal cannot be Pareto dominated. Hence, by part (i),  $Q_{LH} = 1$ .  $\square$

Finally, we explore which IC are binding in the generalized Nash bargaining solution. It turns out that exactly a low cost selling division and a high revenue buying division have a strict incentive to report their types truthfully, whereas the opposite types are indifferent between truthtelling and misreporting.

**Proposition 5.** *Suppose that divisions bargain over IR and IC mechanisms.*

- (i) *In the generalized Nash bargaining solution  $\mu^{*(Y,Q)}$  the incentive constraints (IC2) and (IC3) are always binding.*
- (ii) *At least one of the two constraints (IC1) and (IC4) is not binding in the generalized Nash bargaining solution.  
More precisely, constraint (IC1) is not binding if and only if  $Q_{LL} > 0$  or  $Q_{HH} < 1$  holds. Similarly, constraint (IC4) is not binding if and only if  $Q_{HH} > 0$  or  $Q_{LL} < 1$ .*

Constraint (IC2) ensures that the low cost seller type should not have an advantage from pretending that it has high production costs. Analogously, the constraint (IC3) prevents the buying division from reporting the low revenue type when its true type is the high revenue type. Assume (IC2) and (IC3) are not satisfied and suppose the selling division has low production costs  $L$  and the buying division has high revenues  $H$  so that the type profile is  $LH$ . If the payment  $Y_{HH}$  is higher than the payment  $Y_{LH}$ , then the selling division might want to report the type  $H$ . Its advantage from misreporting is that it does not need to share the difference  $Y_{HH} - Y_{LH}$  with the buying division. But the selling division runs the risk that if the buying division is of type  $H$  and behaves analogously, this means that it pretends to be of type  $L$ . The result is that no trade and payment takes place, although it was profitable. Technically, the constraints (IC2) and (IC3) ensure that the payments  $Y_{HH}$ ,  $Y_{LH}$  and  $Y_{LL}$  are chosen in a way that this situation does not appear. Constraint (IC1) ensures that the selling division does not have an advantage from pretending that he has low production costs if he has the high production cost type. Intuitively there is no economic reason for the selling division to understate its cost and hence for (IC1) to bind. Analogously if (IC4) is satisfied, then the buying division will not overstate its revenues, which seems to be plausible as well. This intuition is confirmed by the above proposition.

Observe the similarity between Corollary 2 and Proposition 5. When replacing EPE by IC, those incentive constraints that were not satisfied before are now binding, whereas those that were (possibly weakly) satisfied are now strict (in many situations).

*Proof of Proposition 5.* We first show that constraints (IC2) and (IC3) are binding in the generalized Nash bargaining solution. In the next step, we identify conditions under which both constraints (IC1) and (IC2) (and analogously (IC3) and (IC4)) cannot be binding simultaneously. Let the generalized Nash bargaining solution be attained by a mechanism  $\mu^{*(Y,Q)}$ .

*Step 1:* We first establish that if (IC2) is not binding we have

$$U_1(\mu^{*(Y,Q)}|L) > U_1(\mu^{*(Y,Q)}|H). \quad (4)$$

This can be seen as follows:

$$\begin{aligned}
& U_1(\mu^{*(Y,Q)}|L) - U_1(\mu^{*(Y,Q)}|H) \\
&= \delta(Y_{LH} - Q_{LH}C_L) + (1 - \delta)(Y_{LL} - Q_{LL}C_L) - \delta(Y_{HH} - Q_{HH}C_H) \\
&> \delta(Y_{HH} - Q_{HH}C_L) - \delta(Y_{HH} - Q_{HH}C_H) \\
&= \delta Q_{HH}(C_H - C_L) \geq 0.
\end{aligned}$$

Hereby, the strict inequality comes from the strict inequality of (IC2).

Analogously, if (IC3) is not binding then

$$U_2(\mu^{*(Y,Q)}|H) > U_2(\mu^{*(Y,Q)}|L). \quad (5)$$

We establish that if the incentive constraints (IC2) and (IC3) are not both binding simultaneously, then for given transfer probabilities  $Q_{HH}$ ,  $Q_{LH}$  and  $Q_{LL}$  we can modify the transfer payments  $Y_{HH}$ ,  $Y_{LH}$  and  $Y_{LL}$  in such a way that IR and IC constraints are still met, but the generalized Nash product increases. We distinguish the two cases in which (IC2) or (IC3) are not binding, respectively.

*Case 1:* Suppose (IC2) is not binding. We increase the transfer payment  $Y_{HH}$ , decrease  $Y_{LH}$  and leave  $Y_{LL}$  unchanged. This is done in such a way that the l.h.s. of the incentive constraint (IC3) does not change. Therefore, while increasing  $Y_{HH}$  by  $\frac{k}{\delta\varepsilon}$  we decrease  $Y_{LH}$  by  $\frac{k}{\delta(1-\varepsilon)}$  for small  $k > 0$ . It can be easily seen that the remaining incentive constraints (IC1) and (IC4) as well as the IR constraints (IR1) to (IR4) are not violated for  $k$  small enough. In order to show that the generalized Nash product  $F$  increases, we take the corresponding directional derivative of its logarithm.<sup>10</sup> Formally, this amounts to

$$\begin{aligned}
& \frac{1}{\delta\varepsilon} \frac{\partial \log[F(\mu^{*(Y,Q)})]}{\partial Y_{HH}} - \frac{1}{\delta(1-\varepsilon)} \frac{\partial \log[F(\mu^{*(Y,Q)})]}{\partial Y_{LH}} \\
&= \frac{1}{U_1(\mu^{*(Y,Q)}|H)} - \frac{1}{U_2(\mu^{*(Y,Q)}|H)} - \frac{1}{U_1(\mu^{*(Y,Q)}|L)} + \frac{1}{U_2(\mu^{*(Y,Q)}|H)} \\
&= \frac{U_1(\mu^{*(Y,Q)}|L) - U_1(\mu^{*(Y,Q)}|H)}{U_1(\mu^{*(Y,Q)}|H)U_1(\mu^{*(Y,Q)}|L)} > 0,
\end{aligned}$$

which holds by (4).

*Case 2:* Suppose (IC3) is not binding. Now we increase the transfer payment  $Y_{LH}$ , decrease  $Y_{LL}$  and leave  $Y_{HH}$  unchanged so that (IC2) is unaltered and the remaining IC and IR conditions are still valid. Precisely, increase  $Y_{LH}$  by  $\frac{k}{\delta(1-\varepsilon)}$  and decrease  $Y_{LL}$  by  $\frac{k}{(1-\delta)(1-\varepsilon)}$  for small enough  $k > 0$ . To see that the generalized Nash product  $F$  increases, we again take the directional derivative of its logarithm

---

<sup>10</sup>Recall that maximizing  $F$  or its logarithm results in the same set of maximizing mechanisms.

and use (5), to get

$$\begin{aligned}
& \frac{1}{\delta(1-\varepsilon)} \frac{\partial \log[F(\mu^{*(Y,Q)})]}{\partial Y_{LH}} - \frac{1}{(1-\delta)(1-\varepsilon)} \frac{\partial \log[F(\mu^{*(Y,Q)})]}{\partial Y_{LL}} \\
&= \frac{1}{U_1(\mu^{*(Y,Q)}|L)} - \frac{1}{U_2(\mu^{*(Y,Q)}|H)} - \frac{1}{U_1(\mu^{*(Y,Q)}|L)} + \frac{1}{U_2(\mu^{*(Y,Q)}|L)} \\
&= \frac{U_2(\mu^{*(Y,Q)}|H) - U_2(\mu^{*(Y,Q)}|L)}{U_2(\mu^{*(Y,Q)}|L)U_2(\mu^{*(Y,Q)}|H)} > 0.
\end{aligned}$$

Taking the two cases together, both incentive constraints (IC2) and (IC3) have to be binding for the maximizer of the generalized Nash product under IR and IC constraints.

*Step 2:* Consider first (IC1) and (IC2) and suppose both constraints are binding. Adding (IC1) and (IC2) and rearranging implies

$$[\delta(Q_{LH} - Q_{HH}) + (1-\delta)Q_{LL}](C_H - C_L) = 0 \quad (6)$$

As  $C_H > C_L$  and  $Q_{LH} = 1$  hold in the generalized Nash bargaining solution, the above equation is satisfied if and only if  $Q_{HH} = 1$  and  $Q_{LL} = 0$ . Therefore, if  $Q_{HH} < 1$  or if  $Q_{LL} > 0$ , (IC1) and (IC2) cannot be binding simultaneously. Hence in this case, (IC1) cannot be binding as we already established in Step 1 that (IC2) binds. Conversely, if  $Q_{HH} = 1$  and  $Q_{LL} = 0$  hold, then it is easily verified that (IC1) is satisfied if and only if (IC2) holds. Since (IC2) is binding in the Nash solution, this establishes the first equivalence in part (ii) of the proposition.

Analogous arguments demonstrate the second equivalence on constraints (IC3) and (IC4). Note that if both are binding, their sum amounts to

$$[\varepsilon(Q_{LH} - Q_{LL}) + (1-\varepsilon)Q_{HH}](R_H - R_L) = 0,$$

which is the analogue to (6).

We summarize the above observations. At least one of the two constraints (IC1) and (IC4) is not binding in the generalized Nash bargaining solution. Put differently, at least two incentive constraints, namely (IC2) and (IC3), and at most three incentive constraints, namely either (IC2), (IC3), (IC1) or (IC2), (IC3), (IC4), are binding in the generalized Nash bargaining solution.

We close the proof with the remark that the subsequent examples (Examples 2 and 1) demonstrate that the equivalence in part (ii) is not trivial in the sense that exactly two or three IC constraints might be binding.  $\square$

When comparing (ex post) efficiency and incentive compatibility, we have seen (Corollary 2) that the generalized Nash solution of the bargaining game over IR and EPE mechanisms is in conflict with the IC constraints. The previous Proposition 5 shows in particular that some IC constraint must be strict, indicating that IC might only be realizable at the expense of efficiency. We discuss two

examples showing that no such implication can be drawn. Example 1 calculates the generalized Nash bargaining solution for the three relevant sets of constraints.

**Example 1.** Suppose  $R_H = 6$ ,  $R_L = 3$ ,  $C_H = 4$ ,  $C_L = 2$ ,  $\varepsilon = \frac{4}{5}$  and  $\delta = \frac{4}{5}$ . We calculate the generalized Nash bargaining solution in three different scenarios. By Proposition 1 there exist mechanisms that satisfy IR, IC and EPE.

*Scenario 1:* When the divisions bargain over IR and IC mechanisms (that are not necessarily EPE) we obtain for the generalized Nash bargaining solution

$$\begin{aligned} (Y_{HH}, Q_{HH}) &= (4.93, 1), & (Y_{LH}, Q_{LH}) &= (5.61, 1), \\ (Y_{HL}, Q_{HL}) &= (0, 0), & (Y_{LL}, Q_{LL}) &= (-1.76, 0.49) \end{aligned}$$

with a generalized Nash product of 0.8109.

Observe that the trade probability in the profile  $LL$  is strictly less than 1, showing that the mechanism is not EPE. Furthermore, by Proposition 5, constraints (IC1) and (IC4) cannot be binding in the generalized Nash bargaining solution.

*Scenario 2:* When, in addition to Scenario 1, the divisions bargain over IR, IC, and EPE mechanisms we compute for the generalized Nash bargaining solution

$$\begin{aligned} (Y_{HH}, Q_{HH}) &= (4.89, 1), & (Y_{LH}, Q_{LH}) &= (5.2, 1), \\ (Y_{HL}, Q_{HL}) &= (0, 0), & (Y_{LL}, Q_{LL}) &= (0.76, 1) \end{aligned}$$

with a generalized Nash product of 0.7968. Note that the Nash product here must be smaller than the one in Scenario 1, since the set of mechanisms over which the product is maximized in Scenario 2 is smaller than the set from Scenario 1.

*Scenario 3:* Finally, if the divisions bargain over IR and EPE mechanisms, then the generalized Nash bargaining solution amounts to

$$\begin{aligned} (Y_{HH}, Q_{HH}) &= (5.223, 1), & (Y_{LH}, Q_{LH}) &= (4.2, 1), \\ (Y_{HL}, Q_{HL}) &= (0, 0), & (Y_{LL}, Q_{LL}) &= (-1.9, 1). \end{aligned}$$

Since by Corollary 2 two incentive constraints are not met, the generalized Nash product is the highest in this case, namely 0.9604.

The following table displays agents' utilities in the generalized Nash solution for the three scenarios:

Utilities/Scenario	1: IR, IC	2: IR, IC, EPE	3: IR, EPE
$U_1(\mu^{(Y,Q)} H)$	0.7413	0.7121	0.98
$U_1(\mu^{(Y,Q)} L)$	2.3413	2.3121	0.98
$U_2(\mu^{(Y,Q)} H)$	0.9367	1.0479	0.98
$U_2(\mu^{(Y,Q)} L)$	0.6440	0.4479	0.98
Nash product	0.8109	0.7968	0.9604

□

**Example 2.** For parameters  $R_H = 6$ ,  $R_L = 3$ ,  $C_H = 6$ ,  $C_L = 2$ ,  $\varepsilon = \frac{1}{5}$  and  $\delta = \frac{1}{5}$  the generalized Nash bargaining solution is given by

$$\begin{aligned} (Y_{HH}, Q_{HH}) &= (2.19, 0), & (Y_{LH}, Q_{LH}) &= (2, 1), \\ (Y_{HL}, Q_{HL}) &= (0, 0), & (Y_{LL}, Q_{LL}) &= (2.55, 1). \end{aligned}$$

Again observe two points contrary to Scenario 1 in the previous example. First, the generalized Nash bargaining solution is EPE in this case. In the type profile  $HH$ , revenues equal costs so that any trade in that profile is efficient. Second, inserting the data of the mechanism into the IC constraints reveals that (IC2), (IC3), and (IC4) are binding, showing that the equivalence in part (ii), Proposition 5 includes three binding IC constraints, too.

Finally, we state the basic insight from the examples on IC vs. EPE in our last proposition.

**Proposition 6.** *Suppose that divisions bargain over IR and IC mechanisms and suppose EPE mechanisms exist. However, in the generalized Nash bargaining solution EPE is not necessarily satisfied.*

## 4 Comparison with Other Transfer Pricing Mechanisms

In the previous sections we discussed an alternative solution to the intrafirm transfer pricing problem under incomplete information, using a concept that is well-fundged in axiomatic bargaining theory. By means of two examples from Wagenhofer (1994) we now compare the generalized Nash bargaining solution to other widely-used transfer pricing methods. Wagenhofer (1994) describes different (standard) transfer pricing models, such as cost-based, market-based, and two different negotiated transfer pricing mechanisms.

In the first one the buying division is granted all the bargaining power realized by a ‘take it or leave it offer’ from the buying division. This means that it makes an offer for a transfer payment that the selling division can either accept or reject. By the revelation principle, the mechanism can be reduced to a direct one in which both divisions announce their types. However, this mechanism is not IC for all parameters, since the buying division can be in a situation in which its expected profit can be increased by understating revenues.

The second mechanism is the ‘equal-split sealed-bid’ mechanism with equal bargaining powers. Here both divisions simultaneously announce costs and revenues, i.e. their types. If the selling division’s announced costs are lower than the buying division’s revenues, then they trade (with probability 1) and share the difference equally. Otherwise no trade and payments take place. This mechanism is also not IC, since, e.g., the selling division tends to overstate its costs and the

buying division to understate its revenues in certain parameter constellations.<sup>11</sup> As a result, there is a positive probability that trade does not take place, although it would have been profitable.

Example 3 uses the same parameter setup as Example 1 and collects conditional divisional utilities for the various pricing methods. We juxtapose the results in Wagenhofer (1994) with our findings above. Then, Example 4 slightly varies the parameters with the effect that no IR, IC, and EPE mechanisms exist. Recall that HQ's ex ante expected utility from a mechanism  $\mu^{(Y,Q)}$  is given by

$$\varepsilon U_1(\mu^{(Y,Q)}|H) + (1 - \varepsilon)U_1(\mu^{(Y,Q)}|L) + \delta U_2(\mu^{(Y,Q)}|H) + (1 - \delta)U_2(\mu^{(Y,Q)}|L).$$

**Example 3.** Assume as in Example 1:  $R_H = 6$ ,  $R_L = 3$ ,  $C_H = 4$ ,  $C_L = 2$ ,  $\varepsilon = \frac{4}{5}$  and  $\delta = \frac{4}{5}$ . Recall that the condition in Proposition 1 is satisfied so that IR, IC, and EPE mechanisms exist. Table 1 (on p. 22) displays conditional expected utilities in the solution for different transfer pricing mechanisms (cf. Table 1 in Wagenhofer, 1994).

Here, the cost-based and the ‘take it or leave it offer’ mechanism and, of course, the generalized Nash bargaining solution over EPE mechanisms, implement the first best solution while the ex ante cost-based mechanism, the ‘equal-split sealed-bid’ mechanism and the generalized Nash bargaining solution (over IR and IC mechanisms) do not.

Comparing the Nash solution (with or without EPE) to the other solutions, we emphasize two points that are of interest from HQ's perspective:

- First, none of the conditional expectations in the Nash solution is zero. This means that regardless of the type a division observes at the interim stage, it still has a positive expected benefit from continuing the project. This is, e.g., not the case for a high cost selling division in the ‘take it or leave it offer’ mechanism.
- Second, thanks to maximization of a (weighted) product of such utilities, there appears to be a tendency that the range of conditional utilities is smaller in the Nash solution, implying a smaller gap between divisional utilities. In that sense, the Nash solution better *balances* divisional interests and can in that sense be considered “more fair” than other solutions.

From HQ's point of view, ex post efficiency should be a primary goal. Apart from that, the two arguments above provide good reason to use the generalized Nash bargaining solution (over IR, IC and EPE mechanisms) to find an efficient and fair solution.  $\square$

In the final example we modify the probability  $\varepsilon$  with which the seller observes high costs.

<sup>11</sup>Wagenhofer (1994, Proposition 6) shows that the ‘equal-split sealed-bid’ mechanism implements the first best solution if  $(1 - \varepsilon)(R_H - R_L) \leq \varepsilon(R_H - C_H)$  and  $\delta(C_H - C_L) \leq (1 - \delta)(R_L - C_L)$  hold.

**Example 4.** Let, as in the previous examples,  $R_H = 6$ ,  $R_L = 3$ ,  $C_H = 4$ ,  $C_L = 2$ , and  $\delta = \frac{4}{5}$ , but now assume  $\varepsilon = \frac{1}{5}$ . For this parameter setting no IR, IC, and EPE mechanism exists, as the conditions of Proposition 1 are not met. Straightforward calculations show that the generalized Nash bargaining solution (over IR and IC mechanisms) is given by

$$\begin{aligned} (Y_{HH}, Q_{HH}) &= (3.47, 0.67), & (Y_{LH}, Q_{LH}) &= (4.39, 1), \\ (Y_{HL}, Q_{HL}) &= (0, 0), & (Y_{LL}, Q_{LL}) &= (-0.65, 0.18) \end{aligned}$$

Note that  $Y_{HH}$  and  $Y_{LL}$  represent the expected transfer payments when costs and revenues are high, or low, respectively. In both cases the product is not traded with probability 1, showing the lack of EPE. Hence, we may compute the actual transfer payment (for one unit of the product) by  $\frac{Y_{HH}}{Q_{HH}} = 5.2$ , and  $\frac{Y_{LL}}{Q_{LL}} = -3.6$ , respectively.

In particular, when both divisions report high costs and high revenues ( $HH$ ), then, according to our assumptions of the model, we could think of a practical implementation of the mechanism in the following two ways: If the intermediary product is divisible, then only a fraction of  $\frac{2}{3}$  of the available quantity is sold. In case that the product is not divisible and using risk neutrality of the two divisions, then they trade with probability  $\frac{2}{3}$  together with a payment of 5.2. Consequently, with probability  $\frac{1}{3}$  no trade and payments take place, which results in an overall expected payment of 3.47.

Similarly, in the case  $LL$  (both report “low”), the product is traded with probability 0.18 for a payment of 3.6 from the seller to the buyer. This negative payment is due to the fact that (a) IC constraints are in place and (b) in the generalized Nash bargaining solution the differences between expected utilities of the division’s types shall be kept small in order to maximize their product. Nonetheless, each division has a positive (interim) expected utility, regardless of its type, as it is seen below.

Table 2 (on p. 22) shows expected utilities for different transfer pricing mechanisms as discussed in Wagenhofer (1994) together with the generalized Nash bargaining solution over IR and IC mechanisms. The first best solution provides an expected profit of 3.04 to HQ. Therefore, none of the displayed mechanisms reaches the first best solution. Similar to the observations made in Example 3, all types’ expected utilities are strictly positive and show the smallest range. Here, the generalized Nash bargaining solution is again not worse (even strictly better) in terms of corporate profit compared to the other negotiated pricing method, namely the ‘take it or leave it offer’.  $\square$

<i>Expected utilities</i>	Cost-based ex ante	Cost-based interim	‘take it or leave it offer’	‘equal-split sealed-bid’	gen. Nash over IR, IC, EPE	gen. Nash over IR, IC
$U_1(\mu^{(Y,Q)} H)$	1.60	0	0	0.80	0.71	0.74
$U_1(\mu^{(Y,Q)} L)$	3.20	1.60	1.60	2.40	2.31	2.34
$U_2(\mu^{(Y,Q)} H)$	0	2.00	2.00	1.00	1.05	0.93
$U_2(\mu^{(Y,Q)} L)$	0	0.20	0.20	0	0.45	0.64
<i>Ex ante exp. util. HQ</i>	1.92	1.96	1.96	1.92	1.96	1.94

Table 1: Expected Utilities in Example 3:  $R_H = 6$ ,  $R_L = 3$ ,  $C_H = 4$ ,  $C_L = 2$ ,  $\varepsilon = \frac{4}{5}$ ,  $\delta = \frac{4}{5}$ .

<i>Expected utilities</i>	Cost-based ex ante	Cost-based interim	Market-based	‘take it or leave it offer’	gen. Nash over IR, IC, EPE	gen. Nash over IR, IC
$U_1(\mu^{(Y,Q)} H)$	0	0	0	0	–	0.64
$U_1(\mu^{(Y,Q)} L)$	1.60	1.60	1.44	0	–	1.71
$U_2(\mu^{(Y,Q)} H)$	2.0	2.00	2.30	3.20	–	1.40
$U_2(\mu^{(Y,Q)} L)$	0	0	0	0.80	–	0.96
<i>Ex ante exp. util. HQ</i>	2.88	2.88	2.99	2.72	–	2.80

Table 2: Expected Utilities in Example 4:  $R_H = 6$ ,  $R_L = 3$ ,  $C_H = 4$ ,  $C_L = 2$ ,  $\varepsilon = \frac{1}{5}$ ,  $\delta = \frac{4}{5}$ .

## 5 Summary and Conclusion

In this paper, we analyzed the generalized Nash bargaining solution applied to a (transfer) pricing problem under incomplete information between a buyer and a seller. As the existence of ex post efficient and incentive compatible mechanisms can only be guaranteed for certain parameter configurations, we considered different scenarios. This means that we examined the solution for cases in which buyer and seller negotiate over mechanisms that are individually rational and in addition either incentive compatible or ex post efficient or both, if possible. Such properties of a mechanism (IR, IC, EPE) may be verified and hence imposed by HQ itself. To demonstrate applicability, we established regularity of the bargaining problem regardless of whether incentive and/or efficiency constraints are in place. Moreover, we investigated how mechanisms for the Nash solution look like. However, even if individually rational, incentive compatible and ex post efficient mechanisms exist, the generalized Nash bargaining solution may fail to be ex post efficient. This illustrates the well-known trade-off between incentive compatibility and ex post efficiency. Nonetheless, for the transfer pricing problem under incomplete information, we find it important that only mechanisms are taken into account that provide no incentive to misreport private information, given that the opponent reports truthfully.

Compared to other (negotiated) transfer pricing mechanisms such as the ‘take it or leave it offer’ or the ‘equal-split sealed-bid’ mechanism, we find that the generalized Nash bargaining solution has two appealing properties that emphasize the idea of fairness behind this solution concept. First, both buyer and seller expect a strictly positive profit at the interim stage, i.e., when knowing the actual costs and revenues. So there is absolutely no incentive for them to quit the mechanism at that stage. Second, since in the Nash solution the weighted product of expected interim utilities is maximized, the range between such expected utilities shall be kept smaller compared to alternative mechanisms. Put in other words, the generalized Nash bargaining solution tries to balance divisional profits, while incentive constraints are still in place. In that sense a “fair” profit division is generated. Two examples (Examples 3 and 4) are included to illustrate these points.

A couple of extensions to our model are interesting directions for future research. First, the assumption of two possible types might be replaced by an interval of types as done in Baldenius (2000) or Edlin and Reichelstein (1995), for example. Myerson and Satterthwaite (1983) investigate bargaining problems between one buyer and one seller with bounded intervals as possible seller and buyer types. They give a criterion for the existence of ex post efficient mechanisms that can be compared with the Proposition 1 from Matsuo (1989) for two possible seller and buyer types. Second, it would be interesting to analyze the influence of specific investments prior (or past) to the bargaining stage, as in Edlin and Reichelstein (1995) or Baldenius (2000), for example, on the (ex ante) efficiency, hold-up problems and divisional profit distributions.

## References

- Baldenius, T. (2000). Intrafirm trade, bargaining power, and specific investments. *Review of Accounting Studies*, 5:27–56.
- Baldenius, T., Reichelstein, S., and Sahay, S. (1999). Negotiated versus cost-based transfer pricing. *Review of Accounting Studies*, 4(2):67–91.
- Edlin, A. S. and Reichelstein, S. (1995). Specific investment under negotiated transfer pricing: An efficiency result. *The Accounting Review*, 70:275–291.
- Göx, R. F. and Schiller, U. (2006). An economic perspective on transfer pricing. In Christopher S. Chapman, A. G. H. and Shields, M. D., editors, *Handbooks of Management Accounting Research*, volume 2, pages 673–695. Elsevier.
- Haake, C.-J. and Martini, J. T. (2012). Negotiating transfer prices. *Group Decision and Negotiation*, pages 1–24.
- Harris, M., Kriebel, C. H., and Raviv, A. (1982). Asymmetric information, incentives and intrafirm resource allocation. *Management Science*, 28(6):604–620.
- Harsanyi, J. C. (1967). Games with incomplete information played by “Bayesian” players, I-III. Part I. The basic model. *Management Science*, 14(3):159–182.
- Harsanyi, J. C. (1968a). Games with incomplete information played by “Bayesian” players, I-III. Part II. Bayesian equilibrium points. *Management Science*, 14(5):320–334.
- Harsanyi, J. C. (1968b). Games with incomplete information played by “Bayesian” players, I-III. Part III. The basic probability distribution of the game. *Management Science*, 14(5):486–502.
- Harsanyi, J. C. and Selten, R. (1972). A generalized nash solution for two-person bargaining games with incomplete information. *Management Science*, 18:80–106.
- Holmström, B. and Myerson, R. B. (1983). Efficient and durable decision rules with incomplete information. *Econometrica*, 51:1799–1819.
- Leng, M. and Parlar, M. (2012). Transfer pricing in a multidivisional firm: A cooperative game analysis. *Operations Research Letters*, 40(5):364–369.
- Matsuo, T. (1989). On incentive compatible, individually rational and ex post efficient mechanisms for bilateral trading. *Journal of Economic Theory*, 49:189–194.
- Myerson, R. B. (1979). Incentive compatibility and the bargaining problem. *Econometrica*, 47:61–73.

- Myerson, R. B. and Satterthwaite, M. A. (1983). Efficient mechanisms for bilateral trading. *Journal of Economic Theory*, 29:265–281.
- Nash, J. F. (1950). The bargaining problem. *Econometrica*, 18:155–163.
- Nash, J. F. (1953). Two-person cooperative games. *Econometrica*, 21:128–140.
- Rajan, M. V. and Reichelstein, S. (2004). A perspective on "asymmetric information, incentives and intrafirm resource allocation". *Management Science*, 50(12):1615–1623.
- Vaysman, I. (1998). A model of negotiated transfer pricing. *Journal of Accounting and Economics*, 25(3):349–384.
- Wagenhofer, A. (1994). Transfer pricing under asymmetric information. *European Accounting Review*, 3:71–103.
- Weidner, F. (1992). The generalized nash bargaining solution and incentive compatible mechanisms. *International Journal of Game Theory*, 21:102–129.

## Appendix

### A Remarks on the Proof of Proposition 2

**Remark 1** (Regularity: IR and IC mechanisms). The mechanism of the proof for Proposition 2

$$\begin{aligned}
(Y_{HH}, Q_{HH}) &= \left( \frac{3(1-\varepsilon)Y_{LL} + \delta R_H - (1-\varepsilon)(1+\delta)C_L + \delta(1-\varepsilon)C_H}{3\delta}, \frac{1}{3} \right), \\
(Y_{LH}, Q_{LH}) &= \left( \frac{3(\delta-\varepsilon)Y_{LL} + \delta R_H + \varepsilon(1+\delta)C_L + \delta(1-\varepsilon)C_H}{3\delta}, 1 \right), \\
(Y_{HL}, Q_{HL}) &= (0, 0), \\
(Y_{LL}, Q_{LL}) &= \left( \frac{R_L + C_L - 2\delta(R_H - C_H) - 2\delta\varepsilon(C_H - C_L)}{6}, \frac{1}{3} \right),
\end{aligned}$$

is the convex combination (with equal coefficients  $\frac{1}{3}$ ) of the following three mechanisms:

- $\mu^{(Y^1, Q^1)}$  with the transfer payments and probabilities:

$$\begin{aligned}
(Y_{HH}^1, Q_{HH}^1) &= \left( \frac{(1-\varepsilon)Y_{LL}^1 + \delta R_H}{\delta}, 1 \right), \\
(Y_{LH}^1, Q_{LH}^1) &= \left( \frac{(\delta-\varepsilon)Y_{LL}^1 + \delta R_H}{\delta}, 1 \right), \\
(Y_{HL}^1, Q_{HL}^1) &= (0, 0), \\
(Y_{LL}^1, Q_{LL}^1) &= (-\delta(R_H - C_H), 0),
\end{aligned}$$

For  $R_H > C_H$  we observe that  $\mu^{(Y^1, Q^1)}$  is strictly individually rational and incentive compatible as the according constraints reduce to:

(IR1)	$\delta\varepsilon(R_H - C_H) > 0$
(IR2)	$\delta\varepsilon(R_H - C_H) + \delta(C_H - C_L) > 0$
(IR3)	$\delta(1 - \varepsilon)(R_H - C_H) > 0$
(IR4)	$\delta(1 - \varepsilon)(R_H - C_H) > 0$
(IC1)	$0 \geq 0$
(IC2)	$0 \geq 0$
(IC3)	$0 \geq 0$
(IC4)	$R_H - R_L \geq 0$

- $\mu^{(Y^2, Q^2)}$  with the transfer payments and probabilities:

$$\begin{aligned} (Y_{HH}^2, Q_{HH}^2) &= \left( \frac{(1 - \varepsilon)Y_{LL}^2 - (1 - \varepsilon)C_L}{\delta}, 0 \right), \\ (Y_{LH}^2, Q_{LH}^2) &= \left( \frac{(\delta - \varepsilon)Y_{LL}^2 + \varepsilon C_L}{\delta}, 1 \right), \\ (Y_{HL}^2, Q_{HL}^2) &= (0, 0), \\ (Y_{LL}^2, Q_{LL}^2) &= \left( \frac{R_L + C_L}{2}, 1 \right). \end{aligned}$$

For  $R_H = C_H$  and  $R_L > C_L$  we observe that  $\mu^{(Y^2, Q^2)}$  is strictly individually rational and incentive compatible as the according constraints reduce to:

(IR1)	$\frac{(1 - \varepsilon)(R_L - C_L)}{2} > 0$
(IR2)	$\frac{(1 - \varepsilon)(R_L - C_L)}{2} > 0$
(IR3)	$\frac{(1 - \varepsilon)(R_H - R_L) + (1 - \varepsilon)(R_H - C_L)}{2} > 0$
(IR4)	$\frac{(1 - \varepsilon)(R_L - C_L)}{2} > 0$
(IC1)	$(C_H - C_L) \geq 0$
(IC2)	$0 \geq 0$
(IC3)	$0 \geq 0$
(IC4)	$0 \geq 0$

- $\mu^{(Y^3, Q^3)}$  with the transfer payments and probabilities:

$$\begin{aligned} (Y_{HH}^3, Q_{HH}^3) &= \left( \frac{(1 - \varepsilon)Y_{LL}^3 + \delta(1 - \varepsilon)(C_H - C_L)}{\delta}, 0 \right), \\ (Y_{LH}^3, Q_{LH}^3) &= \left( \frac{(\delta - \varepsilon)Y_{LL}^3 + (1 - \varepsilon)\delta C_H + \varepsilon\delta C_L}{\delta}, 1 \right), \\ (Y_{HL}^3, Q_{HL}^3) &= (0, 0), \\ (Y_{LL}^3, Q_{LL}^3) &= (-\varepsilon\delta(C_H - C_L), 0). \end{aligned}$$

For  $R_H = C_H$  and  $R_L = C_L$  we observe that  $\mu^{(Y^3, Q^3)}$  is strictly individually rational and incentive compatible as the according constraints reduce to:

(IR1)	$\delta(1 - \varepsilon)^2 (C_H - C_L) > 0$
(IR2)	$\delta(1 - \varepsilon)^2 (C_H - C_L) > 0$
(IR3)	$\delta\varepsilon(1 - \varepsilon) (C_H - C_L) + (1 - \varepsilon) (R_H - C_H) > 0$
(IR4)	$\delta\varepsilon(1 - \varepsilon) (C_H - C_L)$
(IC1)	$\delta (C_H - C_L) \geq 0$
(IC2)	$0 \geq 0$
(IC3)	$(1 - \varepsilon) (R_H - C_H) \geq 0$
(IC4)	$(1 - \varepsilon) (C_H - C_L) \geq 0$

**Remark 2** (Regularity: IR, IC and EPE mechanisms). The mechanism of the proof for Proposition 2

$$\begin{aligned} (Y_{HH}, Q_{HH}) &= \left( \frac{(1 - \varepsilon)Y_{LL} + \varepsilon\delta R_H - (1 - \delta)(1 - \varepsilon)C_L}{\delta}, 1 \right), \\ (Y_{LH}, Q_{LH}) &= \left( \frac{(\delta - \varepsilon)Y_{LL} + \varepsilon\delta R_H + (1 - \delta)\varepsilon C_L}{\delta}, 1 \right), \\ (Y_{HL}, Q_{HL}) &= (0, 0), \\ (Y_{LL}, Q_{LL}) &= \left( \frac{(1 - \delta)(1 - \varepsilon)C_L + \delta C_H - \varepsilon\delta R_H + (1 - \varepsilon)R_L}{2(1 - \varepsilon)}, 1 \right) \end{aligned}$$

is a convex combination (with equal factors  $\frac{1}{2}$ ) of the following two mechanisms:

- $\mu^{(Y^4, Q^4)}$  with the transfer payments and probabilities:

$$\begin{aligned} (Y_{HH}^4, Q_{HH}^4) &= (C_H, 1), \\ (Y_{LH}^4, Q_{LH}^4) &= \left( \frac{\varepsilon(1 - \delta)R_H + (1 - \delta)(1 - \varepsilon)C_L + (\delta - \varepsilon)C_H}{1 - \varepsilon}, 1 \right), \\ (Y_{HL}^4, Q_{HL}^4) &= (0, 0), \\ (Y_{LL}^4, Q_{LL}^4) &= \left( \frac{(1 - \delta)(1 - \varepsilon)C_L + \delta C_H - \delta\varepsilon R_H}{1 - \varepsilon}, 0 \right). \end{aligned}$$

We observe using  $\mu^{(Y^4, Q^4)}$  the constraints (IR2) to (IR4) hold with strict inequality while (IR1) is equal to zero.

Summing up the constraints reduce to:

(IR1)	$0 \geq 0$
(IR2)	$\delta (C_H - C_L) > 0$
(IR3)	$\varepsilon\delta R_H + (1 - \varepsilon)R_H - \delta C_H - (1 - \delta)(1 - \varepsilon)C_L > 0$
(IR4)	$\varepsilon\delta R_H + (1 - \varepsilon)R_L - \delta C_H - (1 - \delta)(1 - \varepsilon)C_L > 0$
(IC1)	$(1 - \delta) (C_H - C_L) \geq 0$
(IC2)	$0 \geq 0$
(IC3)	$0 \geq 0$
(IC4)	$\varepsilon (R_H - R_L) \geq 0$

$\mu^{(Y^5, Q^5)}$  with the transfer payments and probabilities:

$$\begin{aligned} (Y_{HH}^5, Q_{HH}^5) &= \left( \frac{\delta\varepsilon R_H + (1-\varepsilon)R_L - (1-\delta)(1-\varepsilon)C_L}{\delta}, 1 \right), \\ (Y_{LH}^5, Q_{LH}^5) &= \left( \frac{\delta\varepsilon R_H + (\delta-\varepsilon)R_L + (1-\delta)\varepsilon C_L}{\delta}, 1 \right), \\ (Y_{HL}^5, Q_{HL}^5) &= (0, 0), \\ (Y_{LL}^5, Q_{LL}^5) &= (R_L, 0). \end{aligned}$$

We observe using  $\mu^{(Y^5, Q^5)}$  the constraints (IR1) to (IR3) hold with strict inequality while (IR4) is equal to zero.

Summing up the constraints reduce to:

(IR1)	$\varepsilon\delta R_H + (1-\varepsilon)R_L - \delta C_H - (1-\delta)(1-\varepsilon)C_L > 0$
(IR2)	$\varepsilon\delta R_H + (1-\varepsilon)R_L + \delta C_L - (1-\delta)(1-\varepsilon)C_L > 0$
(IR3)	$(1-\varepsilon)(R_H - R_L) > 0$
(IR4)	$0 \geq 0$
(IC1)	$(1-\delta)(C_H - C_L) \geq 0$
(IC2)	$0 \geq 0$
(IC3)	$0 \geq 0$
(IC4)	$\varepsilon(R_H - R_L) \geq 0$

Therefore, taking a convex combination (for example with  $\frac{1}{2}$ ) of  $\mu^{(Y^4, Q^4)}$  and  $\mu^{(Y^5, Q^5)}$  yields a mechanism that is strictly individually rational.

## B Proof of Lemma 1

*Proof of Lemma 1.* Inserting the constraint

$$x_n = \frac{c - \sum_{i=1}^{n-1} l_i \cdot x_i}{l_n}$$

into the objective function leads to

$$f(x_1, \dots, x_n) = \prod_{i=1}^{n-1} x_i^{l_i} \cdot \left( \frac{c - \sum_{i=1}^{n-1} l_i \cdot x_i}{l_n} \right)^{l_n}.$$

The function does not depend on  $x_n$ . We obtain for  $j \neq n$  the first order condition

$$\begin{aligned} & \frac{\partial f(x_1, \dots, x_n)}{\partial x_j} \\ &= l_j x_j^{l_j-1} \cdot \prod_{i=1, i \neq j}^{n-1} x_i^{l_i} \cdot \left( \frac{c - \sum_{i=1}^{n-1} l_i \cdot x_i}{l_n} \right)^{l_n} + \prod_{i=1}^{n-1} x_i^{l_i} \cdot \left( \frac{-l_j}{l_n} \right) \cdot l_n \cdot \left( \frac{c - \sum_{i=1}^{n-1} l_i \cdot x_i}{l_n} \right)^{l_n-1} \\ &= l_j x_j^{l_j-1} \cdot \prod_{i=1, i \neq j}^{n-1} x_i^{l_i} \cdot \left( \frac{c - \sum_{i=1}^{n-1} l_i \cdot x_i}{l_n} \right)^{l_n-1} \left( \frac{c - \sum_{i=1}^{n-1} l_i \cdot x_i}{l_n} - x_j \right) \\ &\stackrel{!}{=} 0 \end{aligned}$$

Thus, in order to obtain a maximum we need to have for all  $j \neq n$

$$x_j = \frac{c - \sum_{i=1}^{n-1} l_i \cdot x_i}{l_n}$$

Since the right hand side will be the same for each  $x_j$  and is equal to  $x_n$ , we have for the maximizer  $(x_1^*, \dots, x_n^*)$  of  $f(x_1, \dots, x_n)$  under the constraint  $\sum_{i=1}^n l_i \cdot x_i = c$  that  $x_1^* = \dots = x_n^*$  holds.  $\square$

## Recent discussion papers

2013-07	Sonja Brangewitz Claus-Jochen Haake	Cooperative Transfer Price Negotiations under Incomplete Information
2013-06	Mark Schopf	Preserving Eastern or Offshore Oil for Preventing Green Paradoxes?
2013-05	Mark Schopf Hendrik Ritter	Unilateral Climate Policy: Harmful or even Disastrous?
2013-04	Volker Seiler	Comment on Ameriks, Caplin, Leahy & Tyler (2007): Measuring Self-Control Problems
2013-03	B. Michael Gilroy Anastasia Heiman Mark Schopf	Gibt es eine optimale Frauenquote?
2013-02	Yuanhua Feng Chen Zhou	Forecasting financial market activity using a semiparametric fractionally integrated Log-ACD
2013-01	Berno Buechel Nils Röhl	Robust Equilibria in Location Games
2012-11	B. Michael Gilroy Mark Schopf Anastasia Semenova	Grundeinkommen und Arbeitsangebot: Die Perspektive Deutschlands
2012-10	N. Bilkic B. Carreras Painter T. Gries	Unsustainable Sovereign Debt - is the Euro Crisis only the Tip of the Iceberg?
2012-09	B. Michael Gilroy Heike Schreckenberg Volker Seiler	Water as an Alternative Asset
2012-08	B. Michael Gilroy Mark Schopf Anastasia Semenova	Basic Income and Labor Supply: The German Case
2012-07	Tim Krieger	Es geht nicht ohne internationalen Ordnungsrahmen: Zum Verhältnis von Markt und Staat im Angesicht der Finanzkrise <i>[forthcoming in: Amos international: Internationale Zeitschrift für christliche Sozialethik (2012), 3]</i>
2012-06	Thomas Gries Margarete Redlin	Trade Openness and Economic Growth: A Panel Causality Analysis
2012-05	Mark Schopf Hendrik Ritter	Reassessing the Green Paradox <i>[republished as CIE Working Paper 2013-05 "Unilateral Climate Policy: Harmful or even Disastrous?"]</i>
2012-04	Yuanhua Feng David Hand Keming Yu	A Multivariate Random Walk Model with Slowly Changing Drift and Cross-correlation Applied to Finance
2012-03	Sarah Brockhoff Tim Krieger Daniel Meierriecks	Looking Back on Anger: Explaining the Social Origins of Left-Wing and Nationalist-Separatist Terrorism in Western Europe, 1970-2007