Investment under Threat of Disaster

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Abstract

During the last 40 years the number and severity of economic, natural, and political disasters has significantly increased all over the world. Disasters are characterized by a highly uncertain frequency of occurrence and size of impact. Due to their relatively small probability, for a long time they were not regarded as an essential element of investment decisions. Although this has changed recently, especially in the context of specific applications in finance, a transfer to a general evaluation of disasters has not taken place yet. This paper shows how disastrous events of uncertain occurrence and uncertain size can be included in the most frequently used evaluation method, namely expected net present value (ENPV). We identify an Ito-Lévy Jump Diffusion process as an adequate stochastic process for this kind of phenomenon and determine how to account for such large uncertain events. We also illustrate that disregarding this phenomenon may easily lead to unprofitable investment behavior. Hence, disasters do have a huge impact on investment behavior and should be included into project evaluation.

JEL classifications: D81, G11

Keywords: disaster evaluation, large risk and uncertainty, non-marginal stochastic shocks, investment project evaluation

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During the last 40 years the number and severity of economic, natural, and political disasters has significantly increased all over the world. Disasters are characterized by a highly uncertain frequency of occurrence and size of impact. Due to their relatively small probability, for a long time they were not regarded as an essential element of investment decisions. Although this has changed recently, especially in the context of specific applications in finance, a transfer to a general evaluation of disasters has not taken place yet. This paper shows how disastrous events of uncertain occurrence and uncertain size can be included in the most frequently used evaluation method, namely expected net present value (ENPV). We identify an Ito-Lévy Jump Diffusion process as an adequate stochastic process for this kind of phenomenon and determine how to account for such large uncertain events. We also illustrate that disregarding this phenomenon may easily lead to unprofitable investment behavior. Hence, disasters do have a huge impact on investment behavior and should be included into project evaluation.

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1 Introduction

Disasters\(^1\) are an enormous hazard to economic activities and are defined as rare, devastating ecological, political, technical, or economic events that occur at unpredictable points in time and with massive direct and threatening effects. For instance, shocks like the 9/11 terrorist attacks or political or even revolutionary riots such as the Arab Spring do not occur often, however they have a severe impact on economic conditions. Similarly, in the economic dimension we have observed that events such as the Lehman Brothers bankruptcy in 2008 and the subsequent financial crisis are connected to the potential failure of large banks or even states. In the context of technical or natural disasters, Figure (1) suggests that on a global scale, the number and severity of disasters has increased since the 1970s. Hence, disasters were and will always be a significant phenomenon in economic reality.

![Figure 1, Natural disasters reported in numbers and estimated damages 1975 - 2010](image)

In spite of their low probability, such major stochastic shocks cause uncertainty for the affected individuals. That is, in contrast to other marginal variations, disasters may not only strongly affect current conditions, they may even overthrow them. For this reason disasters are a central element of future developments and hence have to be included when evaluating projects. We show how the threat of disastrous events affects even simple evaluation methods, such as the Expected Net Present Value (ENPV). More precisely, we distinguish between marginal and non-marginal shocks by describing uncertain developments using more general stochastic processes (Ito-Levy\(^2\).

\(^1\)For a more detailed description and classification, see EM-DAT.
Jump Diffusion). For such a process we (i) confirm the standard result that the ENPV does not account for marginal variability. In addition, we (ii) obtain an additional element in the discount factor that summarizes the effects of non-marginal stochastic shocks on the expected project value. In other words, this parameter is an additional discount factor in the evolution of the expected net present value. Hence, while marginal variability has no effect on investment decision based on ENPV, non-marginal stochastic shocks do because, as we show, they may cause an apparently beneficial project to become worthless.

Hence, the questions we answer in this paper are: How can disasters be included into investment decisions based on ENPV? How does the potential occurrence of uncertain disasters affect the decision to realize an investment project? How can an investor cope with the uncertain timing and magnitude of the resulting effect in project evaluation and project decision making?

During the last decade there have been attempts to include disasters into investment decisions, yet so far no consensus on how to do so appropriately has been found. However, Zeckhauser (1996) was one of the first to treat disasters as economic events and to define their costs as the sum of losses caused and the costs incurred by actions to reduce those losses. Although the costs of catastrophes have increased over time, reasons for neglecting disasters can still be found in the literature. Firstly, the comparatively low probability of disasters leads to a lack of comprehension so that, according to Kunreuther et al. (2001), individuals tend to ignore disasters and not to demand protection. Secondly, Kunreuther and Kleffner (1992) and Kunreuther (1996) have found that agents often underestimate disaster probabilities and have high discount rates for future benefits. Finally, Kunreuther and Pauly (2004) argue that information costs for obtaining information about real probabilities are regarded as too high compared to the expected loss generated by disasters.

In the years that followed, all three arguments for neglecting disasters in decision making were discussed. In the context of natural disasters Smith et al. (2006) emphasize that people indeed respond to natural disasters and that they do so in three different ways: they may self-protect, buy insurance, or move away from hazard prone areas. That is, households compare the costs and benefits of the three alternatives and make their choice by taking into account the harms caused by disasters, with the chosen reaction depending only on their income level. The first economic modelling of disasters and their inclusion into evaluation methods is provided by Sutter and Poitras (2010) who use the expected utility approach to show that people should account for disasters when deciding which type of house to build. They find that people substitute from manufactured homes in risk-prone areas. While both approaches – that of Smith et al. (2006) and Sutter and Poitras (2010) – are similar in that disasters matter to decision-making, neither addresses the problem that disasters may be underestimated by agents. In response, Lave and Apt (2006) claim that extreme natural events are more frequent than expected, and Viscusi (2009) shows that people do not appropriately evaluate risks from terrorism, natural disasters, and traffic accidents. He finds that death through terrorism is valued twice as high as death as a result of natural disaster but valued equally to death by accident. This paradoxical view of risks may explain decisions that are made in the context of natural disasters. Born and Viscusi (2006) find that insurance companies also
suffer from major catastrophes and face higher losses after a disaster. As a consequence they raise insurance premiums in order to lower loss ratios in the following period. This ex-post adjustment indicate an inadequate evaluation of disasters in the past.

The next question that arises is how disasters have been included into the evaluation of investment projects. Starting with the Cost Benefit Approach (CBA), we can see that while CBA under certainty is well developed\(^2\) the issue of how to evaluate large uncertainty appropriately remains an unresolved puzzle. Graham (1981) was one of the first to extend CBA and to include uncertain outcomes with known probability. His simple expected utility approach makes it possible to determine an action with the best outcome and a value of the corresponding option. Although CBA is simple to apply, there are also major concerns that it is not capable of adequately evaluating large uncertain events and disaster risk. For instance, Lave and Apt (2006) use CBA to determine the optimal size of a dam as flood protection and show that people refuse to buy insurance because of the above-mentioned undervaluation reasons.

Disaster risk evaluation, however, has been a major issue in financial economics, and more recently has also entered business cycle theory. Beginning with Merton (1975) jump processes are used to model rare events that cause non-marginal movements of values. He argues that most of the time an asset follows a Brownian motion and, with a known probability, jumps by random amplitude. Based on this argument different conclusions about option values\(^3\) and business cycles\(^4\) can be made for a set of simple jump processes. The occurrence of disasters was first modelled by simple downward jumps by Cox et al. (2000, 2004). As an extension, Yang and Zhang (2005) and Jang (2007) use a jump diffusion process to model the randomness of disasters, that is, their unknown frequency and impact. More general jump diffusion models are considered by Pham (1997), Kou (2002) and Kou and Wang (2003). Specifically, Pham (1997) and Kou (2002) generalize the Black and Scholes option pricing model in order to account for empirical phenomenons of asset prices such as volatility smiles and jump risks.\(^5\) The recent generalization by Cai and Kou (2011) shows that a mixed-exponential jump diffusion model is able to approximate any other distribution as closely as possible. Furthermore they prove that analytical solutions can be found for Laplace transformations of prices and sensitivity parameters of path-dependent options. Jump models are also used to describe random waiting periods between trades or hedging options. In this sense, Cartea and Meyer-Brandis (2010) utilize, among others, compound Poisson processes for waiting periods that are exponentially distributed and show an effect on option prices, while Kaeck (2012) tests different jump diffusion models and their ability to model hedging options. He shows that the inclusion of jumps is necessary to improve hedging performance. A further application of rare events

\(^2\) See, e.g., Prest and Turvey (1965), Sassone and Schaffer (1978), Layard and Glaister (1994).

\(^3\) See, e.g., Cox et al. (1975, 1979).

\(^4\) More recently disasters have also been seen as important determinants of many other economic variables that drive business cycles. For instance, Gourio (2012) emphasizes that disasters such as Great Recessions depress employment, output, investment, stock prices, and interest rates, and increase expected returns on risky assets. This approach is applied to human capital theory in Bilkic et al. (2012).

\(^5\) Kou and Wang (2003) also discuss various characteristics of jump processes, e.g., first passage times when introducing the double exponential distribution, and derive respective option prices.
in economic problems is shown in the discussion of the equity premium puzzle. Mehra and Prescott (1985) were the first to find high-risk premia in equities and thereby opened up a completely new research field. One explanation of this phenomenon is given by Rietz (1998) who claims that the possibility of an unlikely market crash are responsible for the additional price difference in equities. This explanation would not be taken up by the economic research community for almost 20 years. Then, Barro (2006) picked up Rietz’s idea and found empirical evidence of economic disasters leading to higher risk premia. Since then this topic has been discussed in different variations. For instance, Barro and Ursua (2008) investigate in an empirical approach how wars can result in consumption disasters and thereby show a further application. As an extension of Rietz (1998), Gabaix (2008, 2012) discusses disasters as a determinant of ten macro-economic puzzles, such as the risk-free rate puzzle and excess volatility puzzle. Using calibration methods he shows that disasters can explain some of the economic phenomena. A theoretical contribution in this context is provided by Wachter (2013) who uses jump diffusion processes to explain the equity premium puzzle. Hence, by deriving the premiums for equities under disaster risk she shows that investors obtain large risk premiums when facing disaster risks compared to investors that do not.6

The paper is structured as follows. The first part of the next section discusses the most commonly used process for modelling investment decisions under uncertainty and shows its characteristics regarding the evaluation of uncertainties. The next part extends the geometric Brownian motion for disasters occurring at an uncertain point in time with an unpredictable magnitude and analyzes the effects of the additional uncertainty component on the investment decision. In order to illustrate the effects, we use an example. The third section concludes.

2 Investment Project Evaluation Using Stochastic Processes

Investment project evaluation considers an uncertain future environment and hence requires a characterization of stochastic future outcomes. The simplest characterization of stochastic outcomes is a static probability distribution of values. However, for evaluation methods such as the NPV approach investors look at a full sequence of outcomes of each future period and try to come as close as possible to a characterization of this sequence of periods. Hence they would try to characterize the full time path of stochastic realizations as well as possible. So far, in continuous time, stochastic processes are the only broadly understood form of such stochastic time paths. Therefore, applying stochastic processes, usually a geometric Brownian motion, seems the most appropriate way of approaching this objective and including the time dimension of future events.

Hence, as a first benchmark we introduce the Brownian motion and show how this stochastic process enters the ENPV. In a next step we generalize this simple stochastic process to evaluate uncertain events such as disasters occurring randomly and with an uncertain magnitude. Finally, we use an example to show that going beyond marginal shocks and introducing stochastic non-marginal shocks affects investment decisions even for this simplest evaluation method.

6This result generalizes the findings of Jarrow and Zhao (2006) , where portfolio choices, when downside loss-averse, depend on the presence of disasters.
2.1 Evaluation with the Geometric Brownian Motion

Since Black and Scholes (1973) the geometric Brownian motion is one of the most frequently used stochastic processes for modeling values of derivatives and investment projects. The evolution of benefits $Y$ can be characterized by a stochastic differential equation

$$dY = \alpha Y dt + \sigma Y dW \quad \text{with} \quad Y(0) = y_0$$

where $dW$ is the increment of the standard Wiener Process. Equation (1) defines a stochastic process with continuous trajectories (see Figure (1)) that have a trend $\alpha \in \mathbb{R}$ and marginal fluctuations depicted by the volatility $\sigma > 0$.

For $\alpha > 0$ the project value increases on average and has the expected value\(^7\) of

$$EY(t) = EY(0)e^{\alpha t},$$

which is the dashed curve in Figure (1). Furthermore, the larger the constant volatility, $\sigma$, the more the project value fluctuates around its expected value $EY(t)$, regardless whether this deviation is positive or negative. Note that for the geometric Brownian motion fluctuations $\sigma$ are modeled continuously so that only marginal differences between one point in time and another are described. However, with $\sigma = 0$ (1) simplifies to an ordinary differential equation with solution $Y(t) = Y(0)e^{\alpha t}$.

Now suppose that an agent considers investing in an investment project with a value that evolves according to (1). To evaluate this project with the net present value approach he has to determine the expected net present value by accumulating the discounted benefits per period and reducing it by the project costs.

**Proposition 1** Let $Y$ be defined in (1). Then the expected net present value (ENPV) of future

\(^7\)For a detailed derivation of the expectation, see Dixit and Pindyck (1994).
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benefits described by $Y$ is

$$ENPV_1 = -I_0 + E \left( \int_0^\infty Y e^{-r(t-T)} dt \right) = -I_0 + \frac{Y(T)}{r-\alpha}, \quad r > \alpha,$$

(2)

with $r$ being the risk free interest rate and $I_0$ being investment costs.

**Proof.** See Appendix 1. ■

According to (2), the expected net present value consists of the project value at time $T$ discounted by the difference between the risk-free interest rate $r$ and the drift $\alpha$ and reduced by the investment costs $I_0$. Hence, the investor will carry out the project only if $ENPV_1 \geq 0$. The decision does not depend on the volatility $\sigma$ so that marginal shocks have no effect on the investment decision. In other words, using the ENPV technique and taking the geometric Brownian motion for describing the stochastic income path of an investment projects does not account for any shocks, not even for marginal shocks from one moment to the next. This is consistent with the interpretation of the ENPV method as a risk-neglecting approach because the volatility describes the stochastic deviation from the mean evolvement and hence is taken as an indicator of risk. Although Black and Scholes’ approach was a benchmark in the evolution of evaluation theories, Merton (1975) was the first to criticize this drawback of the geometric Brownian motion. Merton (1975) points out that prices do not move according to the geometric Brownian motion, nor does trade take place continuously. He argues that stock prices can never be represented by continuous stochastic processes because uncertainty produced by incoming important news can lead to an immediate non-marginal upward or downward movement in prices that may affect investment decisions.

### 2.2 Evaluation with Jump Processes

To model disasters as large stochastic events that are non-marginal stochastic shocks in the benefit stream, we introduce the discontinuous counterpart of the geometric Brownian motion. Specifically, this is a stochastic process that is constructed by continuous and discontinuous Lévy processes.\(^8\) In this paper, we follow the approach of Pham (1997) and Kou and Wang (2003) and combine a jump process with the geometric Brownian motion. The resulting geometric Ito-Lévy Jump Diffusion process is defined by the stochastic differential equation

$$d\tilde{Y} = \tilde{Y} \bar{\sigma} dt + \tilde{Y} \sigma d\tilde{W} + \tilde{Y} \int_\mathcal{U} zN(t, dz), \quad \tilde{Y}(0) = y_0$$

(3)

with $\bar{\sigma}, \bar{\sigma} > 0$ and constant, and $d\tilde{W}$ denoting the increment of the Wiener process. $N$ stands for a Poisson process with intensity $\lambda$ and the integral $\int_\mathcal{U} zN(t, dz)$ itself describes a compound Poisson process that accumulates all jumps due to exogenous shocks. The intensity of the compound Poisson

\(^8\)A more detailed description of Lévy processes, especially of the prerequisites for definitions, can be found in, e.g., Cont and Eberlein (2010), Oksendal and Sulem (2007), Andersen et al. (2009) or Protter (1990).
process can also be rewritten as \( v(dz)dt = \lambda h(dz)dt \) where \( h \) the distribution of the jump step heights. \( v(dz) \) is known as the Lévy measure. The last term can be interpreted as a stochastic process that models positive or negative reactions of benefits to offered opportunities and threats. The direction and magnitude of one jump is represented by the step height \( \triangle \tilde{Y}(t) := z = \tilde{Y}(t) - \tilde{Y}(t^-) \in U \) with \( U \) being a Borel set that does not contain 0. According to the representation of the stochastic process (3) the project value evolves like a geometric Brownian motion and jumps upwards or downwards at random points in time. Figure (2) is an example of a path of a geometric Ito-Lévy Jump Diffusion process.

The Ito-Lévy Jump Diffusion process has two shock components, \( \tilde{\sigma} \) for marginal shocks and \( \int_U z\tilde{N}(t,dz) \) for non-marginal stochastic shocks. Non-marginal shocks are characterized most generally. In particular with this model non marginal jumps may happen at an uncertain time and have a large but uncertain magnitude. That is, we do not know when and with what impact these large shocks occur. However, we may have information about the intensity \( \lambda \) and the distribution of jump height \( h \).

With this combination of continuous volatility and a jump process we can now describe the complete random path of income where marginal and non-marginal shocks in the benefit stream are present. As Knight (1921) argues, when future outcomes are unknown they can either be risky or uncertain. While for Knight risk implies that probability distributions can be stated, uncertainty describes a situation where no such specification can be made. With this modeling we try to move one step towards Knight’s notion of uncertainty. We implement more complex stochastics in this model, and hence introduce a degree of uncertainty and randomness that cannot be covered by a simple probability distribution. We have no information on the likelihood and timing of the next shock and the severity with which the next shock will strikes. That is, with this kind of stochastic modelling future developments are so uncertain that it is impossible to know
the probability that a disaster of size \( x \) will occur at time \( t \). Moreover, we cannot even state the probability of a certain disaster happening during a specified time period from now on. In other words, if an engineer states that the probability that the just finished dam will collapse during the next 100 years is \( \frac{1}{10^{100}} \) and hence negligible, he still uses a probabilistic model of risk. In contrast, using more complex stochastics like Ito-Lévy Jump processes this kind of statement would be impossible since we cannot even give a probability of this disaster happening during the next 100 years. As a result, simple probabilistic statements, in particular when considering large sometimes even “overthrowing” events, are a misleading and insufficient description of the degree of randomness under these circumstances.

However, as we illustrate, we can still evaluate this rather uncertain future development using the simple ENPV technique. Again, we consider an agent who takes a decision on a project with a benefit stream \( \tilde{Y} \).

In order to see how the two risk measures affect the investment decision we can compute the expected net present value of the project. For this we first need the expected value of the process.

**Proposition 2** Let \( \tilde{Y} \) be defined as (3). Assume that the condition \( \int e^{u \tilde{Y}} v(dz) < \infty \) of finite exponential moments holds, so that the moments of the stochastic process in (3) are also finite. Then the expectation value of \( \tilde{Y} \) is determined by

\[
E\tilde{Y}(t) = E\tilde{Y}(0) \exp \left[ t \left( \tilde{\alpha} + \int_{f^{-1}(U)} zv(dz) + \int_U \ln(1 + z) - zv(dz) \right) \right] \tag{4}
\]

\[
= E\tilde{Y}(0) \exp \left[ t \left( \tilde{\alpha} + \lambda \int_{f^{-1}(U)} zh(dz) + \lambda \int_U \ln(1 + z) - zh(dz) \right) \right].
\]

**Proof.** See Appendix 2. \( \blacksquare \)

In contrast to the geometric Brownian motion, the expected value in (4) does not only depend on the drift but also on the direction and magnitude given by the disasters in the benefit stream. In particular, (4) is not automatically an increasing function for \( \tilde{\alpha} > 0 \) since the average growth rate (also known as the overall drift) is determined by \( \int_{f^{-1}(U)} zv(dz) + \int_U \ln(1 + z) - zv(dz) \) which can either be positive or negative depending on whether more upward or more downward jumps occur. Hence, if we consider disasters where only negative jumps occur \( \int_{f^{-1}(U)} zv(dz) + \int_U \ln(1 + z) - zv(dz) \) will be negative and we obtain a function \( \tilde{Y} \) that increases according to \( \tilde{\alpha} > 0 \) and jumps downward due to threatening events. In order to simplify the notion we replace \( \int_{f^{-1}(U)} zh(dz) + \int_U \ln(1 + z) - zh(dz) \) by a disaster variable \( \delta < 0 \) from now on and write \( E\tilde{Y}(t) = E\tilde{Y}(0) \exp [t (\tilde{\alpha} + \lambda \delta)] \).

In a next step we compute the expected net present value for a project that evolves according to a geometric Ito-Lévy Jump Diffusion.
Proposition 3 Let $\tilde{Y}$ be defined in (3). Then the expected net present value of future benefits described by $\tilde{Y}$ is

$$ENPV_2 = -I_0 + \int_T^\infty e^{-r(t-T)}\tilde{Y}(t)dt$$

$$= -I_0 + \frac{\tilde{Y}(T)}{(r - \lambda \delta - \bar{\alpha})}$$

for $r > \lambda \delta + \bar{\alpha}$, $h$ being the distribution of step heights of the jump process and $\delta$ the disaster variable.

Proof. See Appendix 2.

From (5) we can see that the disaster-facing investor evaluates the disasters only by looking at the expected net present value of the project. $ENPV_2$ depends on the benefit value in $T$ and the sum of the integrals $\lambda \delta$. Since $\lambda \delta$ itself depends on the intensity and step height of the jumps, more disasters mean more threats for the investor, leading to a negative value of both integrals. In this case negative jumps act as an additional depreciation parameter that in turn decreases the ENPV. As a result, the existence of disastrous threats, the frequency and extent to which they may occur determine the evaluation of the expected project value. Marginal shocks in contrast, indicated by volatility $\sigma$, do not affect the ENPV evaluation framework.

2.3 Effects of Disasters on the Net Present Value

Having determined the ENPV of a project we see that large uncertain shocks matter, whereas volatility does not. In this section we look at the derivatives to analyze how these large shocks affect the investment decision.

By using the expected present value method, which allows for a discontinuous project earnings described by Ito-Lévy Jump Diffusions, we are able to determine the effect of an increasing number of disasters and of increasing volatility on the expected net present value. Hence, the derivatives of the expected net present value according to the jump intensity $\lambda$ and according to volatility $\bar{\sigma}$ are:

$$\frac{\partial ENPV_2}{\partial \lambda} = -\frac{\delta \tilde{Y}(T)}{(r - \lambda \delta - \bar{\alpha})^2} < 0, \quad \frac{\partial ENPV_2}{\partial \bar{\sigma}} = 0$$

An increasing frequency of disasters will lead to a positive numerator, making the derivative of the expected net present value negative. Specifically, if the investment project is prone to more disasters so that disasters occur more often, the ENPV will decrease and the project will lose in value. In other words, major damage caused by natural, technological, or other disasters can happen so frequently that the real investment project may not pay off any longer. In contrast, small variations in the project value have no effect on the investment decision of the investor. As a consequence, an investor would overestimate the project value if he insufficiently accounted for the possibility occurrence of disasters. He would not see the potential loss that is connected to large uncertain shocks and would instead invest in a potentially worthless project.

For a proof see Appendix 3.
Figure 3: Effects of an increasing frequency and magnitude of disasters

In order to show the effect of an increasing number of disasters we provide a graphical illustration in addition to our general analytical findings. We assume that the jump sizes $z$ have the double exponential distribution similar to Kou and Wang (2003)$^{10}$

$$h(z) = p\eta_1 e^{-\eta_1 z}1_{\{z \geq 0\}} + q\eta_2 e^{\eta_2 z}1_{\{z < 0\}},$$

where $p$ is the probability of a positive jump and $q$ of a negative jump, respectively, with $p + q = 1$. $\frac{1}{\eta_1}$ and $\frac{1}{\eta_2}$ denote the means of the two exponential distributions. Each exponential distribution can be interpreted as a distribution of the waiting period until a positive or a negative jump occurs. In other words, during this waiting period the occurrence of fundamental opportunities and disasters affects the decision to invest.

To illustrate,$^{11}$ we assume the investment cost $I_0$ to be equal to 1, the risk free interest rate $r$ equal to 5% and the average growth rate $\alpha$ of benefits equal to 2%. Furthermore, the probability of a downward jump is assumed to be 100% compared to a 0% probability of upward jumps. Since we want to analyze the effect of disasters we let threats be more likely than opportunities. In a period we assume that at $\frac{1}{\eta_1} = \frac{1}{8}$, the mean waiting period for an opportunity is four times longer than for a threat with $\frac{1}{\eta_2} = \frac{1}{4}$. Hence, we disregard positive jumps and double the frequency and impact of negative jumps stepwise while beginning with a 5% impact of a downward jump and a mean arrival rate of 10%. Figure (3) shows that more negative jumps with a higher devastating impact rapidly decrease the ENPV, compared to the dashed line where no jumps occur. For instance, doubling $z$ and $\lambda$ from 10% (in point A) to 20% (in point B) decreases the ENPV by 32% and then to a point when the project value even turns negative (in point C). In this case the threatening impact of disasters on real investment projects outweighs their benefits so that in some cases they should not be carried out.

However, as a general result we obtain rules for how changes in these parameters translate

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$^{10}$A possible generalization is to use a mixed exponential distribution, provided in Cai and Kou (2011).

$^{11}$For the detailed computation results see table (1) in Appendix 3.
into project values in the EPNV approach. Fortunately, these rules are rather simple, appearing almost like rules of thumb, given that the occurrence and impact of disasters can be described by systematic parameters such as frequency of occurrence and extent of damage. These rules depend on the random distributions we can apply for assessing the value of effects of rare but still existing disastrous events.

3 Summary and Conclusion

This paper addresses the impact of disasters on the value of investment projects. As the number of disasters has significantly increased during the last decades, their impact is a growing threat to long term investment projects. Disasters are large events with a highly uncertain occurrence and impact. However, within the most commonly used methods of project evaluation there is no framework for evaluating effects of disasters. This paper provides a theoretical concept for evaluating disastrous uncertain shocks using the simple ENPV approach. We argue that simple probabilistic statements, in particular when considering large or even “overthrowing” events, are misleading and an insufficient description of the degree of randomness. Hence, we implement a more complex stochastics using a stochastic Ito-Lévy Jump Diffusion process. With such a stochastic process we do not need to state the probability of that a certain disaster will happen during a specified future time period. However, we can still evaluate this rather uncertain future using the simple ENPV technique. We show that in contrast to the well-known fact that marginal shocks, indicated by volatility, do not affect the ENPV, large stochastic shocks do. Parameters characterizing the frequency and the size of these large shocks are elements of the discount factor. We show that a higher frequency and impact of disasters rapidly decreases the value of a project even to negative values. Disregarding potential disasters leads to an overestimation of projects.
4 Appendix

4.1 Appendix 1: Geometric Brownian Motion

Let $Y$ be a geometric Brownian motion defined by the following stochastic differential equation

$$dY = \alpha Y dt + \sigma Y dW$$

with $Y(0) = Y_0$.

where $\alpha$ and $\sigma$ are non-negative constants and $dW$ describes the increment of the standard Wiener Process. From Dixit and Pindyck (1994) and Oksendal (2004) we know that it has the expected value

$$EY(t) = EY(0)e^{\alpha t}.$$ 

Proof of Proposition 1. The variance of the net present value of the future benefit stream is

$$ENPV_1 = -I_0 + E\left(\int_T^{\infty} Ye^{-r(t-T)} dt\right)$$

$$= -I_0 + \left[\frac{1}{\alpha - r}e^{-r(t-T)}e^{\alpha t}Y\right]_T^\infty$$

$$= -I_0 + \frac{Y(T)}{r - \alpha}; \quad r > \alpha.$$ 

\[\square\]

4.2 Appendix 2: Ito-Lévy Jump Diffusion Process

Let $Y$ be an Ito-Lévy Jump Diffusion Process which is defined by the following stochastic differential equation

$$d\bar{Y} = \bar{Y} \alpha dt + \bar{Y} \sigma dW + \bar{Y} \int_U z \tilde{N}(t, dz), \quad Y(0) = y_0$$

with $\bar{\alpha}, \bar{\sigma} > 0$ and constant. Again $d\bar{W}$ denotes the increment of the Wiener process. $\tilde{N}$ stands for the Poisson process with intensity $\lambda$. The SDE has the solution

$$\bar{Y}(t) = \bar{Y}(0) \exp \left[\left(\alpha - \frac{1}{2}\sigma^2\right) t + \sigma \int_0^t \left[\ln(1 + z) - z\right] v(dz) ds\right]$$

$$+ \int_0^t \ln(1 + z) \tilde{N}(ds, dz).$$

Proof of Proposition 2. Since the Wiener and the Poisson processes are independent, we can
obtain the expectation value of $Y$ by separating the components:

$$E\tilde{Y}(t) = E\tilde{Y}(0) \cdot Ee^{(\tilde{a} - \frac{1}{2}\tilde{\sigma}^2)t + \tilde{\sigma}W(t)} \cdot E \left[ \exp \left( \int_0^t \int U \ln(1 + z) N(ds, dz) \right) \right]$$

$$\cdot E \left[ \exp \left( \int_0^t \int U \ln(1 + z) - z v(dz)ds \right) \right].$$

We know that the expectation value of (1) is the same as for the geometric Brownian motion

$$EY(0)e^{\tilde{a}t}.$$

For (2) we use that

$$\int_0^t \int U \ln(1 + z) N(ds, dz)$$

is compound Poisson distributed and has the characteristic function\textsuperscript{12}

$$E \left[ \exp \left( iu \int_0^t \int U \ln(1 + z) N(ds, dz) \right) \right] = \exp \left( t \int_U (e^{iu} - 1) v_f(dz) \right)$$

where $v_f = v \circ f^{-1}$ with $f = \ln(1 + z)$. By choosing $u = -i$ we obtain

$$E \left[ \exp \left( \int_0^t \int U \ln(1 + z) N(ds, dz) \right) \right] = \exp \left( t \int_U (e^z - 1) v_f(dz) \right)$$

$$= \exp \left( t \int_U (e^z - 1) (v(dz) \circ f^{-1}) \right)$$

$$= \exp \left( t \int_{f^{-1}(U)} [(e^z - 1) \circ \ln(1 + z)] v(dz) \right)$$

$$= \exp \left( t \int_{f^{-1}(U)} z v(dz) \right).$$

In order to ensure the existence of the expectation value we have to assume that all moments are

\textsuperscript{12}See Theorem 2.3.7 (i) in Applebaum (2009).
finite. For (3) we obtain:

\[ E \left[ \exp \left( \int_0^t \ln(1 + z) - zv(dz) dt \right) \right] = E \left[ \exp \left( \int_U t \ln(1 + z) - zv(dz) \right) \right] \]

\[ = \exp \left( t \int_U \ln(1 + z) - zv(dz) \right). \]

Therefore, the expectation value of \( Y \) is

\[ E\hat{Y}(t) = \hat{Y}(0)e^{\hat{\alpha}t} \cdot \exp \left( \int_{f^{-1}(U)}^t zv(dz) \right) \cdot \exp \left( -t \int_U zv(dz) \right) \]

\[ = Y(0) \exp \left[ t \left( \hat{\alpha} + \int_{f^{-1}(U)} zv(dz) - \int_U zv(dz) \right) \right]. \]

**Proof of Proposition 3.** The expected net present value using \( \tilde{Y} \) is determined as

\[ ENPV_2 = -I_0 + E \int_T^{\infty} e^{-r(t-T)}\tilde{Y}(t) dt \]

\[ = -I_0 + e^{-r(T-T)}\tilde{Y}(T) \exp \left[ (t-T) \left( \hat{\alpha} + \int_{f^{-1}(U)} zv(dz) + \int_U \ln(1 + z) - zv(dz) \right) \right] dt \]

\[ = -I_0 + \tilde{Y}(T) \frac{r - \lambda \int_{f^{-1}(U)} zh(dz) + \lambda \int_U \ln(1 + z) - zh(dz) - \hat{\alpha}}{r - \lambda \int_{f^{-1}(U)} zh(dz) + \lambda \int_U \ln(1 + z) - zh(dz) + \hat{\alpha}}. \]

\[ r > \lambda \int_{f^{-1}(U)} zh(dz) + \lambda \int_U \ln(1 + z) - zh(dz) + \hat{\alpha}. \]

**4.3 Appendix 3: Effects of Disasters on the ENPV**

**Proof of Proposition 4:**

\[ \frac{\partial ENPV_2}{\partial \sigma} = 0 \]

\[ \frac{\partial ENPV_2}{\partial \sigma \lambda} = -\delta \frac{\tilde{Y}(T)}{(r - \lambda \delta - \hat{\alpha})^2} < 0 \]
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<th>$P(T)[mio]$</th>
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Table 1: Example for the effects of disasters on project value
References


113-140.