Destructive Agents, Finance Firms and Systemic Risk

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Abstract

Popular opinion suggests that malfunctioning, poorly designed incentive schemes in financial firms that encouraged greed and involved excessive salaries were responsible for the excessive risk taking that eventually led to the 2008 financial crash. In this paper we discuss this claim in a theoretical model. We use a modified version of delegated portfolio choice approach with performance contracts. If, in this modified model, we allow for the existence of destructive agents - when maximizing their private utility - each financial firm will take excessive risks. As a result the finance sector develops systemic risk. We define systemic risk as inefficient and excessive risk that is chosen in an endogenous and stable manner by the aggregate market.

JEL classifications: D82, D86, G14
Keywords: delegated portfolio choice, systemic risk, destructive agent, adverse selection

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1 Introduction

In a 2008 article discussing the crash of the financial system, the New York Times argued, “Between 1998 and 2008 Rubin was a top official at Citygroup, where he received a cumulative $150 million in compensation. His main impact on bank policy was to push for the kind of aggressive risk taking that crashed the firm. Rubin believed that Citigroup was falling behind rivals like Morgan Stanley and Goldman, and he pushed to bulk up the bank’s high-growth fixed-income trading, including the CDO business...”\(^1\) Similarly, in 2012 was written in the Harvard Business Review, "... the rise of the alternative assets industry has altered behavior through much of the financial sector. Financial markets based compensation has become the norm in modern American capitalism. Unfortunately, the idea of market based compensation is both remarkably alluring and deeply flawed. The result has been the creation of perhaps the largest and most pernicious bubble of all: a giant financial incentive bubble."\(^2\) Such anecdotal examples stand for more general popular claims such as ‘greedy’ managers are responsible for excessive risk taking and caused the 2008 crash that was followed by one of the largest financial crises ever. In other words, in the public’s opinion, the blame for the crash largely lies with malfunctioning and poorly designed incentive schemes in the financial institutions that encouraged excessive risk taking by payment incentives.

That said, academic research has so far not supported such a clear negative appraisal of intra-firm incentive systems. Therefore, in this paper we examine three central questions raised by these popular claims. (i) Can a principal-agent structure with asymmetric information, moral hazard, and hidden action in financial firms explain the emergence of the financial crisis and "systemic risk"? More specifically, (ii) how do contractual incentives cause excessive risk-taking at individual firm level? And finally, (iii) how do intra-firm incentive systems cause systemic risk at sector level?

The goal of this paper is to understand what happens in terms of risk-taking behavior in financial firms and the finance sector if we just apply simple performance contract schemes widely used in the finance industry. The goal is not to determine what may have been the optimal contract to prevent the financial crisis; rather, we seek to understand what may have encouraged or even caused the crisis with respect to an incomplete understanding of incentives and payment schemes within financial firms.

To answer these questions we suggest a modified version of a "delegated portfolio choice" model. One of the main modifications in our model is that we allow for the existence of agents who deliberately generate wrong information for their own advantage and hence are not only disloyal but even act deliberately harmfully when maximizing their private utility. In analogy to Baumol (1990) we refer to these agents as "destructive agents".\(^3\) If such destructive agents

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\(^3\)At this point we import the concept from entrepreneurship literature. The notion of a "destructive entrepreneur" Baumol (1990) transfer to managers being a destructive agent.
manage their firms' portfolios, each financial firm will take excessive risks and the finance industry will develop systemic risk. In other words, we want to find out if excessive risk (that is, more than efficient risk) is systematically taken when a simple performance contract is the dominant compensation scheme, as can be observed in the financial industry. Wrong kinds of incentives encourage deliberately destructive agents to generate risk externalities and systematically allocate assets against the principal’s interest.

Our point of departure is the delegated portfolio choice model. In the one-period setting, the broader discussion starts with the contribution by Bhattacharya and Pfeiderer (1985), who introduced a model with asymmetric information on a risky asset. A better informed agent needs to be motivated to use this information in the principal’s best interest. Hence, a performance related contract is suggested.

In the debate that followed, the design of the optimal contract became a major point of discussion. In a classic setting an agent has to invest effort into acquiring information about the risky asset. If the agent’s effort to obtain this information is observable and there is no asymmetry of information, choices are jointly determined as they are for one common unit and a first-best solution is derived. In this case Stoughton (1993) shows that if both, agent and principal, are risk-averse a contract simply implies optimal risk-sharing between agent and principal.

The more interesting case, however, remains the one with information asymmetry. While information asymmetry and associated moral hazard problems do not allow for a first-best outcome under linear sharing rules, models such as that of Holmstrom and Milgrom (1987) show conditions under which linear contracts can be best in a second-best world. However, in the delegated portfolio choice context Stoughton (1993) and Admati and Pfeiderer (1997) bring up another important phenomenon, namely the irrelevance result. That is, in standard principal-agent models the payment incentive is related to project performance and hence makes the agent work harder on behalf of the principal. Since standard modeling in delegated portfolio models with linear contracts allow agents not just to invest effort into obtaining information but also into further controlling the portfolio outcome, the principal’s incentive instruments can be neutralized and not directly used to motivate the agent. Hence the simple relationship known from standard principal-agent models no longer works in the delegated portfolio approach, as it is modeled in the literature. Stracca (2006) even concludes in a survey that “more fundamentally, a compensation contract which is optimal (from a second best perspective) in a general class of delegated portfolio management problems is not known even under the assumptions of Holmstrom and Milgrom (1987). Generally speaking, the literature has reached more negative rather than constructive results, and the search for an optimal contract has proved to be inconclusive even in the most-simple settings.”

However, there are some results that can be regarded as major benchmarks in the debate, even if they are derived within a specific setting. Admati and Pfeiderer (1997) suggest that a quadratic contract may solve the problem connected to the irrelevance result, hence the agent can respond to the signal in
a non-linear way. Li and Tiwari (2009) suggest that, for non-linear contracts that promise a fixed payment, a proportional asset-based fee and a benchmark-linked fulcrum fee it is always optimal to include a benchmark-linked option type bonus incentive fee (with the appropriate choice of benchmark).

Not only the question of optimal contracts is considered in the literature; the question of how certain contracts impact managers’ decisions and sometimes principals’ interests is also relevant. With respect to the question of excessive risk-taking, the results are mixed. The problem of symmetric or asymmetric contracts is of particular interest. Starks (1987), for instance, compares a symmetric fulcrum performance fee with an asymmetric bonus contract and argues that the symmetric performance fee is preferable because it can at least align risk attitudes between agent and principal.

Comparing an asymmetric incentive contract with symmetric fulcrum contracts, Das and Sundaram (2002) find in a signaling model that incentive fees may lead to more risky portfolios due to agent self-selection. In this way, asymmetric contracts tend to lure less informed agents into the business.

Carpenter (2000) looks at a risk-averse manager compensated with a call-option contract and obtains opposite effects concerning the risk that a manager is willing to take. Chen and Pennacchi (2002) and Ross (2004) obtain similar results. Ross identifies a number of effects depending on assumptions about the utility function and specific contract design. E.g., an agent with an option-like compensation contract may even choose a level of volatility that is lower than the one they would take if they traded on their own. Under a value-at-risk (VaR) constraint Sheng et al. (2012) show that a linear performance-based contract can motivate agents to acquire information. However, a VaR constraint may increase the moral hazard problem.

Beyond the above discussion, the issue of limited liability has become an important issue in the debate on risk taking. In a rather general approach Grinblatt and Titman (1989) illustrate that under limited liability, agents tend to take on a riskier portfolio. Palomino and Prat (2003) depart from Gollier et al. (1997) and transfer the limited liability issue to a full modeled principal agent setting. 4 Palomino and Prat explicitly mention that the agent can conduct sabotage actions, an approach we think is very realistic and hence will become part of our own modeling later on. In their modeling they are unable to find an optimal linear contract under limited liability.

It is evident that the literature is aware of asymmetric information, moral hazard, and the principal’s enormous difficulties in controlling the agent’s activities. However, in the setting so far the agent acts as a more or less willing instrument and just needs to be sufficiently motivated to put more effort into realizing the principal’s interests. Hence the agent is reduced to a very small set of choices that in no way reflect the enormous range of unobservable actions they could conduct. There are many more dimensions in which they could act unobserved to realize their personal interests than, e.g., the simple choice of

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4 Also Hellwig (1994) and Biais and Casamatta (1999) study moral hazard with both effort and risk. They do not consider a full portfolio choice setting.
effort level in obtaining asset information. Only very few authors mention these other potential activities as part of the agent’s strategy. Since this notion of the agent using information asymmetry to develop their own active strategy and actions is not yet included in the models, we use this idea as a point of departure.

Once again, let us be clear that we do not intend to determine an optimal contract within a certain, even more complex, delegated portfolio setting. The novelty of this paper is that we explicitly model a destructive agent who exploits information asymmetry to deliberately manipulate information to realize their own interests and simultaneously harm the principal under performance contract conditions. We also want to discuss how this kind of agent may be the selected, employed and successful type in the market for managers via adverse selection, and may also be responsible for excessive risk taking in individual finance firms and the finance sector.

In a nutshell, we suggest a very simple and slightly modified version of a delegated portfolio model. In this model the agent has perfect information (no effort to collect information is needed). The principal can observe only the portfolio return and look at observable indicators, like the expected return and e.g. volatility as risk indicator. However, pure portfolio volatility as observable risk indicator is not sufficient for determining the individual financial firm’s true portfolio risk performance. The reason is simple. For the firm’s portfolio assessment the principal is interested in the idiosyncratic risk performance of the managed portfolio. From the principal’s perspective observable volatility covers idiosyncratic risk components and elements which are driven by other systematic (common) risk conditions. As information asymmetry does not allow the principal to separate between idiosyncratic and systematic risk, the principal’s perception and assessment of portfolio performance is subject to interpretations. Hence, the perception of risk is influenced by the information the principal obtains from all kinds of sources, including from the agent, who is compensated according to portfolio performance. As a result, the agent can use the principal’s imperfect information and their own information advantage to put effort into manipulating the principal’s perception of idiosyncratic risk for their own benefit. The agent chooses a high-return and high-risk portfolio while actively concealing and manipulating information about the true idiosyncratic risk (in a manner that best serves their personal objectives). Due to asymmetric information they appear to perform well and hence will earn a high performance-related income. When principals negotiate such contracts with potential agents we observe an adverse selection in the market for agents. The principal cannot see the true characteristics of agents (like the ability to manipulate information, or the degree of risk aversion) and hence selects bad agents. As a result we obtain an inefficient excessive risk-taking in each firm. What is more, such excessive risk is systematically taken by the entire market. This endogenously evolving excessive risk is identified as "systemic risk."

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5 E.g., Palomino and Prat (2003) speak of sabotage as a potential action on the part of an agent.
2 Finance Firm with Delegated Portfolio Choice

In this section we look at a single finance firm that (i) needs to negotiate and draw up a contract between the principal and the agent that is going to manage the portfolio and (ii) has to select from the market of agents the one that it believes will serve its purposes best.

2.1 Describing the Structure of the Firm

The finance firm's business is to manage a given amount of wealth. Wealth is either owned by the principal itself, or the principal is liable for values deposited by external depositors. We assume no leverage (no bank-specific model) even if it can be included. Profits are attained through returns from a portfolio chosen by the agent. For simplicity, there are no operating or monitoring costs.

The firm's portfolio consists of two kinds of assets, a risk-free asset (government bond) with a certain return $R_B$, and a set of risky capital assets characterized by projects with i.i.d returns, an expected return $ER_K$, and an identical volatility $\sigma_K$ representing the true risk. These assumptions are an extreme simplification, since we assume away covariate and systematic risk components for describing risk in this model. We do that because we would like to discuss the most simple case that can make the point to easily distinguish between "true risk" in this model and the perception of risk by the principal. Returns of risky assets are normally distributed and the portfolio share of this asset is $b$. Hence, the expected return of the portfolio and the true portfolio risk is

$$ER = (1 - b)R_B + bER_K$$
$$\sigma^2 = b^2\sigma_K^2$$  \hspace{1cm} (1)

true portfolio risk.  \hspace{1cm} (2)

Due to asymmetric information the interpretation of $\sigma^2$ will be different for the agent and the principal.

The agent is fully informed and knows that $\sigma_K^2$ is the true idiosyncratic risk that cannot be diversified, and that there is no systematic risk element which might have come from common general economic shocks. Hence the agent also knows that the level of the observable portfolio risk $\sigma^2$ depends only on the choice of portfolio share $b$ of the risky asset.

The principal however has no such information about assets, except the observable portfolio outcome. Under regular conditions the principal would assume that the observed $\sigma^2$ includes idiosyncratic and systematic elements from general common shocks which are not connected to the individual portfolio choice. Hence, the principal needs to identify the idiosyncratic risk to asses the specific performance of the managed portfolio. To illustrate the idea this is an example.

The principal observes returns of the firm’s portfolio and calculates the mean as estimator for the expected return as well as the squared standard deviation
as estimator for the variance and hence the risk. However, if the principal now wants to assess the specific performance of the portfolio they presume that e.g. a recent global downturn of the economy is an important element of the observed deviation of the mean, such that the portfolio might have well performed except for the global shock. Therefore, in order to assess the performance of their specific portfolio they would need information how systematic shocks affected the observed outcome. However, by definition of asymmetric information the principal’s information are imperfect, while the agent has full information. This information asymmetry leads to an opportunity for the agent to conduct unobservable actions.

The principal is risk averse with an Arrow-Pratt coefficient of absolute risk aversion $U > 0$. To assess the portfolio performance the principal is interested (i) in the returns, and (ii) the identification of idiosyncratic risk performance of the managed portfolio, because only portfolio specific risk is subject to active choices; systematic risk components generated by common shocks are out of the control of portfolio managers, and hence is independent of the specific portfolio structure which is subject to evaluation.

Further, the principal can observe only portfolio returns. It neither does know about the true risk of single assets, nor does it even know what set of risky assets exists in the economy, or to what extend common shocks truly affect total performance of the financial system. The principal delegates optimal portfolio choice to an agent. It recruits the agent from a competitive agent market without search costs. In order to appoint a manager it negotiates the terms of the performance contract with each applicant. It offers a contract with a fixed payment $w_f$ and a linear reward $\alpha$ on perceived performance above close competitors’ portfolios performance. During these negotiations both parties are trustable in terms of their observable actions. The principal receives a trustable performance promise by the agent. That is, the principal will later observe the promised expected return and realizes a perceived risk of the portfolio as promised in the contract.

However, for the principal the simple observable portfolio variance is not a perfect indicator of true idiosyncratic portfolio risk as it might include systematic risk. The variance is subject to interpretation within a reported context. However, the reported context and the selection of information for this interpretation is part of the agent’s job. As the principal can only get hold of a very limited set of information, the largest and most detailed amount of information is exclusively owned by the agent. When the principal evaluates the observable volatility it will have to use the information and the interpretation of the agent to identify the idiosyncratic portfolio specific risk. Hence, the principal’s perception of idiosyncratic risk will depend heavily on the agent’s advice.

With reference to the contract and the agent’s credibility concerning risk, the agent is credible in that the principal’s perception of portfolio risk will not deviate from the promises made in the contract. With respect to observable returns and the perception of risk, the agent appears to have met their con-
tractual obligations. As a result the principal will hire an agent based on their promises which, according to their best information, will also be fulfilled.

**The agent** also is risk-averse. The Arrow-Pratt coefficient of absolute risk aversion for the agent is $\eta_u > 0$. Comparing the agent’s and principal’s risk preferences we suppose that the principal is at least as risk averse as the agent, $\eta_U \geq \eta_u$. The agent has perfect information. Upon hiring, they will give a credible promise that they will act according to their contractual obligations. That is, the agent is credible in that the principal’s perception of portfolio performance will not deviate from the promises made in the contract. However, the agent simultaneously manipulates information that is important for risk perception and risk evaluation. The agent takes advantage of asymmetric information. That is, they try to hide true idiosyncratic risk and manipulate portfolio specific perception. The agent conceals the true risk structure with concealing effort $q$, marginal concealing costs $c$, and concealing effect $1/q$ such that only $1/q$ of observable volatility $\sigma_K$ becomes perceived as idiosyncratic risk, even if $\sigma_K$ in fact indicated the true portfolio specific risk. Hence parts of the true portfolio specific risk $\sigma_K$ are wrongly related to systematic risk components which truly do not exist.

**Principal-agent interaction** is determined by asymmetric information with respect to a deviation of the true portfolio specific risk and the principal’s perception of this risk. Due to such an information asymmetry and moral hazard there may be hidden actions, so a destructive agent will use this asymmetry to their advantage. The idea of modeling economic activities not meant to serve productively for a market or costumer is not new. The notion of an economic person that deliberately acts destructively to gain profits was notably brought up by Baumol (1990) in the context of entrepreneurship. In a different context and with a slightly different background idea, rent seeking addresses a similar issue.\(^6\) However, transferring this concept to the principal-agent problem is an obvious step. Profiting from hidden actions even if they deliberately harm others is a behavior that we cannot categorically rule out in business. In our model the agent can allocate effort. While the principal believes that the agent’s effort will improve portfolio performance, the agent makes an effort to conceal the true risk and manipulate observable risk indicators.\(^7\)

### 2.2 Contract Design

**The agent’s decision problem:** The agent has a utility function of the form $u(y) = -e^{-f(y)}$ with income $y$. If the agent is hired by the principal they

\(^6\)Bhagwati (1982) and later Murphy et al. (1991) and (1993) developed the profit and rent seeking concept as non-productive economic activity generating gains for the rent seeker.

\(^7\)We could of course include positive efforts as well and optimally allocate effort between constructive and destructive efforts. However, to focus on the point and keep things as simple as possible, we assume with respect to positive efforts that costs of effort towards positive changes are too high that theses efforts are conducted.
obtain a fixed salary \( w_f \) and a symmetric linear performance related income \( w_p \), so that the total income is \( y = w_f + w_p \). Since portfolio performance is a random variable the performance related salary \( w_p \) is also random and has to be evaluated as such. To determine the "performance"-related reward, we need a benchmark to compare with. Due to the assumption of information asymmetry the principal is a rather uninformed person who does not have detailed information or assessment tools about current financial conditions and financial markets. This fact in particular is the reason why the principal delegates wealth management to the agent. Therefore, the most simple way to evaluate the relative performance of the own portfolio is to look at competitors in the market. Close competitors have similar business models and returns of competing firms are public knowledge. Hence the principal assesses the own portfolio performance relatively to the performance of the closest competitors of the finance firm. The principal rewards the agent if the perception of the own managed portfolio \( R \) performs better than the portfolios of close competitors whose returns \( R_{CC} \) are also normally distributed.

Hence, if the principal gives a proportional reward \( \alpha \) for ostensibly good performance \( [w_p = \alpha (R - R_{CC})] \) we can describe a symmetric linear performance contract by

\[
w_f + \alpha (R - R_{CC}).
\]

The agent’s utility function can now be written as

\[
u(y) = -e^{-\eta \alpha (w_f + \alpha (R - R_{CC}))}.
\]

Since we assume that the return of the risky asset follows a normal distribution the agent’s expected utility would be\(^8\)

\[
E u = -e^{-\eta \alpha \left[ w_f + \alpha ER - \frac{\eta^2 \alpha^2 \sigma^2}{2} \sigma^2_{RCC} + \frac{\eta^2 \alpha^2 \sigma^2}{2} \right]}.
\]

Further, under perfect information or perfect loyalty the agent could now choose a portfolio share that maximizes their expected utility and simultaneously also the principal’s. However, while the principal believes that the agent is loyal, the agent is aware of information asymmetries and acts destructively. That is, the principal is able to observe portfolio returns, yet they cannot observe the true idiosyncratic risk. They cannot use the observable variance as true risk indicator. They presume that the observable variance contains usual idiosyncratic and systematic risk elements. However, for the assessment of the portfolio performance they need to filter the idiosyncratic component. Hence, with respect to idiosyncratic risk assessment of the portfolio the principal has to rely on interpretations and indicators that are reported by its agent. Hence, the destructive agent can manipulate and reduce the principal’s perception of specific portfolio related risk. The agent can increase the portfolio share \( b \) of

\(^8\)Here, \( ER_M \) is the expected return of the market portfolio and \( \sigma^2_M \) is the variance of the market portfolio indicating market risk. Both, the expected market return and the market risk, is public knowledge.
the risky asset and simultaneously reduce the principal’s perception of idiosyncratic risk by concealing efforts. In the model, the agent’s risk-concealing effort \( q \geq 1 \) affect the principal’s portfolio specific risk perception by \( 1/q \). However, concealing risk comes at a price. Marginal costs increase with the share of the risky asset \( b^2 \), and depend on the agent’s individual concealing cost factor \( c \). If an agent can easily manipulate information with very little cost to themselves, \( c \) is small. By definition, the destructive agent suffers no additional costs when being disloyal or even actively harming the principal. They are purely acting in their own interest with full awareness of information asymmetries. The destructive agent’s total concealing costs are then given by \( cb^2q \). Implementing the destructive agent-related elements in the model, the perceived portfolio specific risk of the principal is reduced by \( 1/q \) of the observable volatility and the concealing effort reduces the agent’s income by \( cb^2q \). The agent’s expected utility can now be maximized by two choices, the choice of the optimal concealing effort \( q \) and the choice of the optimal portfolio share \( b \). Maximization is subject to a contract offered by the principal with a fixed payment \( w_f \) and a symmetric performance factor as well as the rationality constraint.

\[
\max_{b,q} Eu(y) = \max_{b,q} -e^{\eta_u \left[w_f - \eta_u \alpha ER + \frac{\eta_u \alpha^2 \sigma^2}{2} - \alpha ER_{CC} + \frac{\eta_u \alpha^2 \sigma^2}{2} \right]} \quad (3)
\]

\[
\text{s.t. } u(\bar{w}) \leq Eu(y). \quad (r.c.) \quad (4)
\]

The rationality constraint (4) states that the utility of the outside option \( u(\bar{w}) \) is not higher than the expected utility of the contract \( Eu(y) \).

**Principal’s Decision Problem:** The principal has a utility function of the form \( U(Y) = -e^{-F(Y)} \) with income \( Y \). Net income consists of asset returns from the firms portfolio \( R \) minus a fixed salary \( w_f \) and a performance-related salary \( w_p \) paid to the agent who manages the portfolio. The principal compares the perceived performance of the managed portfolio \( R \) with given portfolios of close competitors \( R_{CC} \) and rewards the agent in a linear and symmetric manner for the excess performance, \( \alpha (R - R_{CC}) \). Hence, its income is \( Y = R - w_f - \alpha (R - R_{CC}) \). We can specify the principal’s utility function as

\[
U = -e^{-\eta_U (R - w_f - \alpha (R - R_{CC}))}.
\]

The expected utility for a normally distributed return of the own and competitors’ portfolios is

\[
EU = -e^{-\eta_U \left((1-\alpha)ER + \alpha ER_{CC} - w_f - \frac{\eta_U (1-\alpha)^2 \sigma^2}{2} + \frac{\eta_U \alpha^2 \sigma^2}{2} \right)}.
\]

\( ^9 \)Again, as mentioned before, market performance is public knowledge.
It is worth noting again that due to information asymmetries the principal cannot observe the true idiosyncratic risk of the portfolio. It trusts its agent and relies on the agent’s information. However, the reported portfolio specific risk is manipulated with concealing effort $q$ and concealing effect $1/q$. The principal maximizes expected utility considering the rationality constraint (4) and incentive constraint (6) by choosing the contract components $w_f$ and $\alpha$.

\[
\max_{\alpha, w_f} \text{EU}(\alpha, w_f) = \max_{\alpha, w_f} -\eta_U \left( (1-\alpha)ER + \alpha ER_{CG} - w_f - \frac{\eta_U (1-\alpha)^2 \sigma_b^2}{2q} - \frac{\eta_U \sigma_b^2 \sigma_{CC}}{2q} \right)
\]

\[
\text{s.t.} \quad (b, q) = (b^*, q^*) \in \arg \max_{b, q} \text{EU}(b, q), \quad \text{(i.c.)} \quad (6)
\]

\[
\text{and} \quad u(\bar{w}) \leq E(u(y)) \quad \text{(r.c.)}
\]

**Drawing up a contract:** Having identified the decision problems we can now determine all elements of a contract. That is, when a principal announces a vacancy an agent will apply and negotiate the terms and conditions of their contract with the principal. The result of these negotiations is a potential agreement on a contract as defined in Proposition 1.

**Proposition 1 (contract):** With asymmetric information and hidden action there exists a potential contract under which (i) the agent chooses an optimal portfolio share of the risky asset $b^*$ and an optimal risk concealing effort $q^*$ given the principal’s offer of a fixed salary $w_f$ and a performance share $\alpha$

\[
b^* = b^*(\eta_u, c, ER_K, ER_B, \sigma_K)
\]

\[
q^* = q^*(\eta_u, c, \alpha, \sigma_K).
\]

(ii) the principal chooses an optimal fixed salary and an optimal performance share depending on perceived performance.

\[
\alpha^* = \alpha^*(\eta_U, \eta_u, c, ER_K, ER_B, \sigma_K, \sigma_{CC}), \quad \text{with} \quad \frac{d\alpha}{d\eta_U} < 0 
\]

\[
w_{f}^* = w_{f}^*(\eta_U, \eta_u, c, ER_K, ER_B, \sigma_K, \sigma_{CC}).
\]

For a proof of Proposition 1 see Appendix 1.

**2.3 Adverse selection of agents**

So far we have determined a contract between a principal and a potential agent given the characteristics of both parties. In this model the principal is completely characterized by (i) its risk preferences (risk averse), (ii) its objective to perform better than competitors and (iii) its preference for a symmetric performance contract. Since we assume that all these characteristics are identical for
all principals the modeled principal is representative of the economy. Agents differ in their individual risk preferences $\eta_{uj} \in \mathbb{R}$ and concealing costs $c_j \in \mathbb{R}$. If an agent is able to easily manipulate information and conceal true risks, $c_j$ will be small. An agent who is not willing or unable to manipulate information will have high concealing costs $c_j$. In this model we assume that concealing costs and the degree of risk aversion is distributed among heterogeneous agents such that we can order all $n$ agents according to the degree of risk aversion $[\eta_{u1} < \eta_{uj} < \eta_{un}]$ and/or their concealing costs $c_1 < c_j < c_n$.

When the principal offers a vacancy a number of prospective agents will apply and the principal will negotiate with each of the candidates. A potential contract is negotiated with each applicant and the principal can choose from among the applicants according to their promises and to the apparent utilities these promises would generate. As described above, the agent fulfills the terms of their contract according to the principal’s perceived (but manipulated) information. Then the principal chooses the agent whose contract seems to generate the highest (indirect) utility under the contract. This leads to the following proposition.

**Proposition 2** (principal’s utility from agent’s attributes): When the principal recruits a potential agent (i) ostensible (indirect) utilities from perceived performance increase with agent’s having decreasing risk aversion $\eta_{ua}$, or decreasing concealing costs $c$.

$$\frac{\partial EU^*(\eta_u)}{\partial \eta_u} < 0 \quad \text{for } \eta_u \in (0,EU^*),$$

$$\frac{\partial EU^*(c)}{\partial c} < 0 \quad \text{for sufficiently small } \eta_U$$

(ii) True (indirect) utilities from true firm performance decrease with decreasing risk aversion $\eta_{ua}$, or decreasing concealing costs $c$

$$\frac{\partial EU_{true}^*(\eta_u)}{\partial \eta_u} > 0 \quad \text{for sufficiently large } \sigma_{CC}$$

$$\frac{\partial EU_{true}^*(c)}{\partial c} > 0 \quad \text{for sufficiently small } c.$$

For a proof of Proposition 2 see Appendix 2.

This proposition makes clear that the activities of a destructive agent will deceive the principal and cause harm and hence are inefficient. The more unscrupulous an agent is when concealing risk, the better they seem to perform and the greater the damage to the principal. What is more, the more risk loving an agent is, the more successful they seem to be in the perception of the principal. Technically speaking, the agent deliberately produces a negative external effect for the principal which they can hide. This effect is not adequately rewarded in the contract and is hence a (currently unobservable) negative externality that
affects the principal. The contract fails to generate an efficient exchange of 

service and reward. The agent is rewarded for a service they do not deliver.

However, the misconception about the agent’s real performance has another, 

even worse dynamic effect. Once a principal recruits an agent and negotiates 

the contracts it will be misguided. Because of asymmetric information, it will 

prefer less honest and more unscrupulous or more risk loving agents. This leads 

to the next proposition.

**Proposition 3** (principal’s adverse selection of agent): (i) For a set of competing 

agents that differ only in terms of the degree of risk aversion $\eta_u$ or concealing 

costs $c$, the principal chooses the agent with the 

· lowest risk aversion $\eta_u^{\text{min}}$ and/or the lowest concealing costs $c^{\text{min}}$

$$\eta_u^* = \eta_u^{\text{min}} \quad \text{since} \quad \frac{\partial \text{EU}^*(\eta_u)}{\partial \eta_u} < 0$$

$$c^* = c^{\text{min}} \quad \text{since} \quad \frac{\partial \text{EU}^*(c)}{\partial c} < 0$$

· largest share of risky assets and hence the most risky portfolio $\sigma^2$

$$b^* = b^{\text{max}} = b(c^*, \eta_u^*) \quad \text{since} \quad \frac{\partial b}{\partial c} < 0 \quad \text{and} \quad \frac{\partial b}{\partial \eta_u} < 0$$

$$\sigma^{2\text{max}} = (b^{\text{max}})^2 \sigma_K^2$$

(ii) This choice implies the principal’s lowest expected "true (indirect) utility"

$$\text{EU}^{\text{true}}_{\text{true}} = \text{EU}^{\text{true}}_{\text{true}}(\eta_u^*, c^*) \quad \text{since} \quad \frac{\partial \text{EU}^{\text{true}}_{\text{true}}(\eta_u)}{\partial \eta_u} > 0, \quad \frac{\partial \text{EU}^{\text{true}}_{\text{true}}(c)}{\partial c} > 0$$

For a proof of Proposition 3 see Appendix 3.

This proposition describes an adverse selection in the market for agents. The 
effects of this adverse selection of agents refers back to the risk behavior of the firm.

The lower an agent’s risk aversion $\eta_u$, the less painful risk taking for the agent. Further, the lower concealing costs $c_j$ of an agent, the less costly this agent’s concealing efforts. Agents with low concealing costs do not need to spend much when conducting concealing activities, and hence concealing risk generates easy profits. Furthermore, even if agents are only a "little bit" disloyal, such that their concealing costs are not prohibitive but still quite high, the principal would nevertheless choose the agent with the lowest risk aversion they can find. That is, even if in a rather honest society adverse selection of unscrupulous agents
soon terminates, the selection mechanism would still pick the agent with lowest risk aversion among the less scrupulous agents. As a result, selecting the wrong agent leads to excessive risk in the portfolio of each individual finance firm. Risk-taking is inefficient because a firm’s principal would not choose this portfolio structure if it knew about the true risk of the portfolio. Moreover, since adverse selection is driven by a race to find the agent with the lowest risk aversion or concealing costs, the principal will eventually choose the agent who constructs the most risky portfolio possible. Hence each finance firm will choose the highest possible risk.

3 Financial Market Failure & Systemic Risk

3.1 Market Portfolio and Aggregate Risk:

Having discussed the micro-perspective in a given finance firm and individual firm selection of agents, we now need to look at the aggregate effects. This seems straightforward as we have already established that performance contracts and the behavior associated with them are dominant in or even representative of the considered market. Corollary 4 states the implications of the above micro setting for the aggregate market if firms offer performance contracts to managers who manage their firms’ portfolio choice.

Corollary 4 (aggregate market portfolio and risk): If all firms offer performance contracts with rewards for out-performing close competitors’ portfolios, all principals choose the agent with attributes \( c^{\text{min}} \) and \( u^{\text{min}} \), and hence the market portfolio becomes the most risky portfolio possible in the economy \[ \sigma^2 = (b^{\text{max}})^2 \sigma_K^2 \text{ for all firms} \].

For a proof see Appendix 4.

This corollary simply summarizes individual findings for the aggregate market. Since competitors’ portfolios are the benchmark and principals in each firm will try to beat competitors and sets incentives each firm is driving the market towards a higher risk. As a result, setting the wrong incentives in performance contracts and picking the wrong agents will not just lead to excessive risk in one firm; rather, the entire sector will take risks at an inefficient and excessive level.

3.2 Destructive Agents and Systemic Risk:

Having described the main mechanism that leads to individual and aggregate risk-taking the link to the term systemic risk can be drawn. Systemic risk is not a clearly and uniformly defined term, and it is used in various ways.\(^{10}\) Often it is related to contagion and hence describes an externality occurring

\(^{10}\)For a broad discussion see Hellwig (2009).
often exogenous under certain macro conditions. However, in this model we would like to use a definition of systemic risk that directly relates to the model mechanics introduced above and gives a consistent and endogenous explanation of when it occurs and when it does not.

**Definition 5 (systemic risk):** Systemic risk is an inefficient and excessive risk as stable outcome of an aggregate market, directly resulting from endogenous choices of market participants.

Clearly, this rather narrow definition cannot accommodate the many phenomena the term sometimes relates to. However, it attempts to relate the term to a structure that endogenously promotes excessive risk due to ill-designed firms or market characteristics and mechanisms or wrong incentives that cause bad behavior throughout the whole system. In other words, systemic risk is an excessive risk that endogenously evolves out of badly designed conditions and affects the entire system. Hence we can state the following theorem:

**Theorem 6 (destructive agents and systemic risk):** If destructive agents exist in this delegated portfolio model, performance contracts designed to beat close competitors lead to systemic risk.

For a proof see Appendix 6.

The interpretation of this theorem is straightforward. (i) If we extend the activities of agents from just doing good or better to explicitly and deliberately harming others, conditions such as asymmetric information and moral hazard become an even more serious threat. Inefficiency and market failure are likely to occur. In the discussed case they can even destabilize an entire sector or even the economy at large. Technically, negative externalities are not just side effects that are ignored by the originator; they are generated on purpose as they are the only profit making activity.

(ii) As for the argument of limited liability being a major reason for excessive risk-taking, we reveal an aspect that is not yet in the focus of academic research. Limited liability within a firm due to publicly unobservable information is a major source of wrong incentives for agents. As a publicly unobservable risk burden can be allocated to principals with only a limited effect for agents, they can exploit this limited liability for their own purpose.

(iii) Further, the ownership of managed capital does not matter. As soon as the portfolio is delegated to an agent, the agent may behave destructively and allocate unobservable risk to the capital owner while obtaining rewards for observable returns. This is independent of the leverage. In our model the principal owns one hundred percent of the invested capital. Hence, within this scenario the share of equity does not matter and an increase in this share, as recently suggested in the political debate, will not take risk out of the system.

(iv) High risk taken by all finance firms implies instability if risky events are realized. Badly designed firms and misunderstood incentive schemes within
firms can become a permanent source of instability not only for the firm in question. If this kind of bad firm design were to become the dominant model in a given sector, that entire sector would become a source of instability.

4 Summary and Conclusions

We investigate the popular claim made after the 2008 financial crash that a malfunctioning, poorly designed incentive scheme in financial firms that encouraged greed and involved excessive salaries was responsible for excessive risk taking and eventually led to the crash. In this context three questions arise. (i) Can asymmetric information, moral hazard, and hidden action in finance firms explain excessive risk-taking? That is, does the principal-agent problem matter in this context? More precisely, (ii) how do contractual incentives cause excessive risk-taking at individual firm level? And finally, (iii) how do malfunctioning incentives within finance firms cause systemic risk at sector level?

To answer these questions we use a delegated portfolio choice approach with performance contracts. In addition to the existing discussion, but in analogy to Baumol’s destructive entrepreneur,\textsuperscript{11} we introduce the destructive agent. The destructive agent does not simply adjust to given asymmetric information; they deliberately generate wrong information for their own advantage and at the expense of the principal. Thanks to this actively manipulated and asymmetric information about the true idiosyncratic portfolio risk, the agent appears to perform well and hence is paid a high performance-related salary.

As these activities are unobservable for the principal, an ability to deceive and manipulate information becomes crucial for the ostensible performance of an agent. The lower the agent’s risk aversion, or the better they are at concealing portfolio specific risk information, the better they seem to perform and the better – ostensibly – this is for the principal. An adverse selection process for agents leads to the employment of the greediest applicant. Hence, in the presence of performance contracts and destructive agents, a firm will take excessive risk – more than the principal would like to take. However, the failure of an individual firm translates into excessive risk for the entire sector if all firms apply performance contracts and if they reward outperforming of close competitors’ portfolios. Having defined systemic risk as an inefficient and excessive risk that is endogenously chosen by market participants and a stable phenomenon in the aggregate market, we illustrate how a malfunctioning and badly designed incentive system can endogenously destabilize the financial market system and imply systemic risk.

Hence, if we allow for the existence of deliberately harming agents in our model - while maximizing their private utility - we obtain under the discussed conditions a general market failure with respect to risk taking.

\textsuperscript{11}See Baumol (1990).
References


5 Appendix

5.1 Appendix 1: Proof of Proposition 1: Drawing up a contract

5.1.1 (i) Proof of the agent’s optimal choice

The utility function of an agent is given by:

\[ u(y) = -e^{-\eta_u(w_f + \alpha(R - R_{CC}))} \]

Definition of assets gives:

\[
\begin{align*}
R &= (1 - b)R_B + bR_K \\
E[R] &= :ER = E[(1 - b)R_B + bR_K] = (1 - b)ER_B + bER_K \\
Var[R] &= \sigma_R^2 = Var[(1 - b)R_B + bR_K] = (1 - b)^2Var[R_B] + b^2Var[R_K]
\end{align*}
\]

If \( R \sim N(ER_K, \sigma_K) \) and \( R_{CC} \sim N(ER_{CC}, \sigma_{CC}) \) are normally distributed we obtain for the expected utility

\[
E[u(y)] = : Eu = E\left[-e^{-\eta_u w_f} \cdot E\left(e^{-\alpha \eta_u R + \alpha \eta_u R_{CC}} \right)\right] = -e^{-\eta_u w_f + \alpha ER - \frac{\eta_u \alpha^2 \sigma_K^2}{2} - \alpha ER_{CC}}
\]

When the agent manipulates information we have to add the factor of \( \frac{1}{q} \) as manipulation of variance perception and the manipulation costs \( cq^2 \). As a result the agent maximizes expected utility:

\[
Eu = -e^{-\eta_u \left[w_f - cq^2 + \alpha ER - \frac{\eta_u \alpha^2 \sigma_K^2}{2q} - \alpha ER_{CC} + \frac{\eta_u \alpha^2 \sigma_{CC}^2}{2q}\right]}
\]

Continuity of the exponential function leads to the equivalent maximization problem:

\[
\max_{b,q} F(b, q) = \max_{b,q} \left[w_f - cq^2 + \alpha ER - \frac{\eta_u \alpha^2 \sigma_K^2}{2q} - \alpha ER_{CC} + \frac{\eta_u \alpha^2 \sigma_{CC}^2}{2q}\right]
\]

s.t. \( u(\tilde{w}) \leq E(u(y)) \)

For this problem the FOCs are:

\[
0 = \frac{\partial F(b, q)}{\partial b} = 2cq + \alpha ER_B - \alpha ER_K + \eta_u \frac{b^2 \sigma_K^2}{q}
\]

\[
0 = \frac{\partial F(b, q)}{\partial q} = -cb^2 + \eta_u \frac{\alpha^2 b^2 \sigma_K^2}{2q^2}
\]
As a result, it follows:

\[ c = \eta_u \frac{\alpha^2 \sigma_K^2}{2q^2} = c^* \]
\[ q^2 = \eta_u \frac{\alpha^2 \sigma_K^2}{2c}, \quad \Rightarrow \quad q = \frac{\sqrt{\eta_u \alpha \sigma_K}}{\sqrt{2c}} = q^* \]

and

\[ 0 = 2cb - \frac{\sqrt{\eta_u \alpha \sigma_K}}{\sqrt{2c}} + \alpha ER_B - \alpha ER_K + \eta_u \frac{\alpha^2 b \sigma_K^2}{\sqrt{\eta_u \alpha \sigma_K}} \]
\[ \Leftrightarrow \quad b = \frac{ER_K - ER_B}{2 \sqrt{2c \eta_u \sigma_K}} = b^* \]

Now we need to prove that \( b^* \) and \( q^* \) are really maxima, therefore we take a look at the second derivatives and the Hesse-Matrix.

\[
\frac{\partial^2 F(b, q)}{\partial b^2} = \left[ -2cq - \frac{\eta_u \alpha^2 \sigma_K^2}{q} \right] = \frac{-2cq - \eta_u \alpha^2 \sigma_K^2}{q}
\]
\[
\frac{\partial^2 F(b, q)}{\partial q^2} = \left[ -\frac{\eta_u \alpha^2 b^2 \sigma_K^2}{4q^4} \right] = \frac{-\eta_u \alpha^2 b^2 \sigma_K^2}{4q^4}
\]
\[
\frac{\partial^2 F(b, q)}{\partial b \partial q} = \frac{\partial^2 F(b, q)}{\partial q \partial b} = \left[ -2cb + \frac{\eta_u \alpha^2 b \sigma_K^2}{q^2} \right] = -2cb + \frac{\eta_u \alpha^2 b \sigma_K^2}{q^2}
\]

Hence it follows:

\[
\frac{\partial^2 F(b^*, q^*)}{\partial b^2} = -2c - \frac{\sqrt{\eta_u \alpha \sigma_K}}{\sqrt{2c}} - \frac{\eta_u \alpha^2 \sigma_K^2}{2 \sqrt{2c \eta_u \sigma_K}} = -2c - \frac{\sqrt{\eta_u \alpha \sigma_K}}{\sqrt{2c}}
\]
\[
\frac{\partial^2 F(b^*, q^*)}{\partial q^2} = \frac{-\eta_u \alpha^2 \left( \frac{(ER_K - ER_B)}{2 \sqrt{2c \eta_u \sigma_K}} \right)^2 \sigma_K^2}{4 \left( \frac{\sqrt{\eta_u \alpha \sigma_K}}{\sqrt{2c}} \right)^3} = -\frac{\sqrt{2c (ER_K - ER_B)^2}}{16 \sqrt{\eta_u \alpha \sigma_K}^3}
\]
\[
\frac{\partial^2 F(b^*, q^*)}{\partial b \partial q} = \frac{\partial^2 F(b^*, q^*)}{\partial q \partial b} = -2c \frac{(ER_K - ER_B)}{2 \sqrt{2c \eta_u \sigma_K}} + \frac{\eta_u \alpha^2 \left( \frac{(ER_K - ER_B)}{2 \sqrt{2c \eta_u \sigma_K}} \right)^2}{2 \sqrt{2c \eta_u \sigma_K}}
\]
\[
= -\frac{2c (ER_K - ER_B)}{2 \sqrt{2c \eta_u \sigma_K}} + \sqrt{2c (ER_K - ER_B)} = 0
\]

\[
H(b^*, q^*) = \begin{pmatrix} -2 \frac{\sqrt{2c \eta_u \alpha \sigma_K}}{2 \sqrt{2c \eta_u \sigma_K}} & 0 \\ 0 & -\frac{\sqrt{2c (ER_K - ER_B)^2}}{16 \sqrt{\eta_u \alpha \sigma_K}^3} \end{pmatrix}
\]

and taken as a conclusion from the definition of a definite matrix, \( H(b^*, q^*) \) is negative definite.
5.1.2 (ii) Proof of the principal’s optimal choice

With the principal’s utility function

\[ U = -e^{-\eta_U(R-w_f-\alpha(R-RC))} \]

we analogously obtain the principal’s expected utility

\[ EU = -e^{-\eta_U(R-\alpha ER_{CC})} \cdot \left( e^{-\eta_U((1-\alpha)ER + \alpha ER_{CC})} - \frac{\eta_U(1-\alpha)^2b^2\sigma_K^2 - \eta_Ua^2\sigma_{CC}^2}{2q} \right) \]

Including the term that manipulates the perception of information in the principal’s utility \( \frac{1}{q} \), we obtain the principal’s expected utility, which he wants to maximize:

\[ EU = -e^{-\eta_U((1-\alpha)ER + \alpha ER_{CC}) - w_f} \cdot \left( e^{-\eta_U((1-\alpha)^2b^2\sigma_K^2 - \eta_Ua^2\sigma_{CC}^2)} \right) \]

Continuity of the exponential function leads to the equivalent maximization problem:

\[
\max_{\alpha, w_f} EU(\alpha, w_f) = \max_{\alpha, w_f} \left( (1 - \alpha) \, ER + \alpha ER_{CC} - w_f \right) - \frac{\eta_U(1-\alpha)^2b^2\sigma_K^2 - \eta_Ua^2\sigma_{CC}^2}{2q} \\
\text{s.t. } u(\bar{w}) \leq E(u(y)) \\
\text{and } (b, q) = (b^*, q^*) \in \arg \max_{b,q} EU(b, q)
\]

Therefore the following Lagrange-function is given by

\[
L^U = \left( (1 - \alpha) \, ER + \alpha ER_{CC} - w_f \right) - \frac{\eta_U(1-\alpha)^2b^2\sigma_K^2 - \eta_Ua^2\sigma_{CC}^2}{2q} \\
+ \lambda_U \left[ u(\bar{w}) - E(u(y)^*) \right]
\]

\[
= \left( (1 - \alpha) \, ER_B + (1 - \alpha) \, b \, (ER_K - ER_B) + \alpha ER_{CC} - w_f \right) - \frac{\eta_U(1-\alpha)^2b^2\sigma_K^2 - \eta_Ua^2\sigma_{CC}^2}{2q} \\
+ \lambda_U \left[ u(\bar{w}) + e^{-\eta_a \left[ w_f - c q b^2 + \alpha(1-b)ER_B + \alpha b ER_K - \frac{\eta_Ua^2\sigma_{CC}^2}{2q} \right]} \right]
\]

\[
= \left( (1 - \alpha) \, ER_B + \left( -5\alpha + 6 - \frac{1}{a} \right) \frac{(ER_K - ER_B)^2}{8\sqrt{2c\eta_a}\sigma_K} + \alpha ER_{CC} - w_f - \frac{\eta_Ua^2\sigma_{CC}^2}{2q} \right) \\
+ \lambda_U \left[ u(\bar{w}) + e^{-\eta_a \left[ w_f + \frac{2a(ER_K - ER_B)^2}{8\sqrt{2c\eta_a}\sigma_K} + \alpha ER_B - \frac{\eta_Ua^2\sigma_{CC}^2}{2q} \right]} \right]
\]
The FOCs are
\[ \frac{\partial L}{\partial \alpha} = 0 = \frac{\partial L}{\partial w_f} = 0 = \frac{\partial L}{\partial \lambda_u} \]

Therefore it follows:
\[ 0 = \frac{\partial L}{\partial w_f} = -1 + \lambda_U \left( -\eta_u \left( -Eu^* \right) \right) \Rightarrow \frac{1}{\eta_u Eu^*} = \lambda_U \]
\[ 0 = \frac{\partial L}{\partial \lambda_U} = u(\bar{w}) - E(u(y))^* \Rightarrow u(\bar{w}) = Eu^* \]

\[ 0 = \frac{\partial L}{\partial \alpha} = \left( -ER_B + \left( -5 + \frac{1}{a^2} \right) \frac{(ER_K - ER_B)^2}{8\sqrt{2c_{\eta_u}^2 \sigma_K}} + ER_{CC} - \eta_u \alpha \sigma_{CC}^2 \right) \]
\[ + \lambda_U \left( \frac{2(ER_K - ER_B)^2}{8\sqrt{2c_{\eta_u}^2 \sigma_K}} + ER_B - E[R_{CC}] + \eta_u \alpha \sigma_{CC}^2 \right) \cdot (-\eta_u \left( -Eu^* \right) ) \]
\[ 0 = -3a^2 \frac{(ER_K - ER_B)^2}{8\sqrt{2c_{\eta_u}^2 \sigma_K}} + \frac{(ER_K - ER_B)^2}{8\sqrt{2c_{\eta_u}^2 \sigma_K}} - (\eta_u - \eta_u) \alpha^3 \sigma_{CC}^2 \]

Using the derivative \( \frac{\partial L}{\partial \lambda_U} \), we obtain an implicit equation that has to be solved for \( w_f \):

\[ u(\bar{w}) = Eu^* \]
\[ \Leftrightarrow \ln(-u(\bar{w})) = -\eta_u w_f - \frac{2\sqrt{\eta_u} (ER_K - ER_B)^2}{8\sqrt{2c_{\eta_u}^2 \sigma_K}} + \alpha \eta_u ER_B + \alpha \eta_u ER_{CC} + \frac{\eta_u^2 \alpha^2 \sigma_{CC}^2}{2} \]
\[ \Leftrightarrow 0 = -\eta_u w_f - \frac{2\sqrt{\eta_u} (ER_K - ER_B)^2}{8\sqrt{2c_{\eta_u}^2 \sigma_K}} + \alpha \eta_u ER_B + \alpha \eta_u ER_{CC} + \frac{\eta_u^2 \alpha^2 \sigma_{CC}^2}{2} - \ln(-u(\bar{w})) \]

Because no explicit solution for \( \alpha \) can be derived we need to show that implicit solutions exist.

\[ F := \begin{pmatrix} F_1 \\ F_2 \end{pmatrix} := \begin{pmatrix} -3a^2 \frac{(ER_K-ER_B)^2}{8\sqrt{2c_{\eta_u}^2 \sigma_K}} + \frac{(ER_K-ER_B)^2}{8\sqrt{2c_{\eta_u}^2 \sigma_K}} - (\eta_u - \eta_u) \alpha^3 \sigma_{CC}^2 \\ -\eta_u w_f - \frac{2\sqrt{\eta_u} (ER_K-ER_B)^2}{8\sqrt{2c_{\eta_u}^2 \sigma_K}} + \alpha \eta_u ER_B + \alpha \eta_u ER_{CC} + \frac{\eta_u^2 \alpha^2 \sigma_{CC}^2}{2} - \ln(-u(\bar{w})) \end{pmatrix} \]

Therefore, it follows \( F = 0 \) with notice to the FOCs. The matrix

\[ \frac{\partial F}{\partial (\alpha, w_f)} = \begin{pmatrix} -6a \frac{(ER_K-ER_B)^2}{8\sqrt{2c_{\eta_u}^2 \sigma_K}} - 3(\eta_u - \eta_u) \alpha^2 \sigma_{CC}^2 & 0 \\ -\frac{\sqrt{\eta_u} (ER_K-ER_B)^2}{4\sqrt{2c_{\eta_u}^2 \sigma_K}} + \eta_u ER_B + \eta_u ER_{CC} + \eta_u^3 \alpha^2 \sigma_{CC}^2 & -\eta_u \end{pmatrix} \]
is invertible, because
\[
\det(\frac{\partial F}{\partial (\alpha, w_f)}) = 6\alpha \sqrt{\eta_u (ER_K - ER_B)^2} + 3(\eta_U - \eta_u)\eta_u \alpha^2 \sigma_{CC}^2 \neq 0
\]
thus the implicit function theorem can be applied and we obtain:
\[
\alpha^* = \alpha^* (\eta_U, \eta_u, c, ER_K, ER_B, \sigma_K, \sigma_{CC}),
\]
with \[
\frac{\partial \alpha}{\partial \eta_U} = -\frac{\alpha^2 \sigma_{K}^2}{(6(ER_K - ER_B)^2 + 3(\eta_U - \eta_u)\alpha^2 \sigma_{CC}^2)} < 0, \quad \text{for } \eta_U \geq \eta_u
\]
and
\[
\frac{\partial \alpha}{\partial \sigma_{CC}} = \frac{(3\alpha^2 - 1)(ER_K - ER_B)^2}{16\sqrt{2\eta_u \alpha \sigma_{K}^2}}
\]
\[
\frac{\partial \alpha}{\partial c} < 0 \text{ if } \alpha < \frac{1}{\sqrt{3}} \text{ and } \frac{\partial \alpha}{\partial c} > 0 \text{ if } \alpha > \frac{1}{\sqrt{3}}
\]
\[
w_f^* = w_f^* (\eta_U, \eta_u, c, ER_K, ER_B, \sigma_K, \sigma_{CC}).
\]

**Conclusion:**
Then Proposition 1 leads to the optimal choices:
\[
b^* = b^* (\eta_u, c, ER_K, ER_B, \sigma_K) = \frac{ER_K - ER_B}{2\sqrt{2\eta_u \sigma_K}}
\]
\[
q^* = q^* (\eta_u, c, \alpha, \sigma_K) = \sqrt{\eta_u \alpha \sigma_{K}} \\sqrt{2c}
\]
\[
\alpha^* = \alpha^* (\eta_U, \eta_u, c, ER_K, ER_B, \sigma_K, \sigma_{CC})
\]
\[
w_f^* = w_f^* (\eta_U, \eta_u, c, ER_K, ER_B, \sigma_K, \sigma_{CC})
\]
where \(\alpha^*\) becomes the smaller, the larger the risk aversion of the principal \(\eta_U\).

\[\square\]

**5.2 Appendix 2: Proof of Proposition 2: principal’s utility and agent’s attributes**

**5.2.1 Ostensible (indirect) utility reaction:**

a) Now it is to show for the ostensible (indirect) utilities, \[\frac{\partial EU^*(\eta_u)}{\partial \eta_u} < 0\], where \(EU^*\) is given as:
\[
EU^* = -e^{-\eta_u (1-\alpha^*)ER + \alpha^* ER_{CC} - w_f^* - \eta_u (1-\alpha^*)^2(u^*)^2 \sigma_{K}^2 - \eta_u (\alpha^*)^2 \sigma_{CC}^2}
\]

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To obtain this result we use the envelope theorem and determine the derivative of the Lagrange function at $\alpha^*$ and $w_f^*$.

**Proof.**

\[
\frac{\partial L'}{\partial \eta_u} = \left( -5\alpha + 6 - \frac{1}{\alpha} \right) \frac{(ER_K - ER_B)^2}{8\sqrt{2\sigma_K}} \left( \frac{1}{2} \frac{1}{\eta_u \sqrt{\eta_u}} \right) + \lambda_U (-Eu^*) \left[ -w_f - \frac{2\alpha(ER_K - ER_B)^2}{8\sqrt{2\sigma_K}} \left( \frac{1}{2} \frac{1}{\eta_u} \right) - \alpha ER_B + \alpha ER_{CC} - \frac{2\eta_u \alpha^2 \sigma^2_{CC}}{2} \right]
\]

\[
= \left( 5\alpha - 6 + \frac{1}{\alpha} \right) \frac{(ER_K - ER_B)^2}{16\sqrt{2\sigma_K \eta_u \sigma_K}} \frac{1}{\eta_u} + \frac{1}{\eta_u} \left[ w_f - \frac{2\alpha(ER_K - ER_B)^2}{16\sqrt{2\sigma_K \eta_u \sigma_K}} + \alpha ER_B - \alpha ER_{CC} + \eta_u \alpha^2 \sigma^2_{CC} \right]
\]

\[
= \frac{1}{\eta_u} \left[ (3\alpha - 6 + \frac{1}{\alpha}) \frac{(ER_K - ER_B)^2}{16\sqrt{2\sigma_K \eta_u \sigma_K}} + [w_f + \alpha ER_B - \alpha ER_{CC}] + \alpha^2 \sigma^2_{CC} \right]
\]

gives us the derivative of the utility function with concealing costs:

\[
\frac{\partial EU^*}{\partial \eta_u} = \frac{1}{\eta_u} \left[ (3\alpha - 6 + \frac{1}{\alpha}) \frac{(ER_K - ER_B)^2}{16\sqrt{2\sigma_K \eta_u \sigma_K}} + [w_f + \alpha ER_B - \alpha ER_{CC}] + \alpha^2 \sigma^2_{CC} \right]
\]

At first, we want to analyse the algebraic sign of the first obtained derivative, and in order to keep the arguments clear, we define some abbreviations

\[
x := \left( 3\alpha - 6 + \frac{1}{\alpha} \right) \frac{(ER_K - ER_B)^2}{16\sqrt{2\sigma_K \eta_u \sigma_K}}
\]

\[
y := [w_f + \alpha ER_B - \alpha ER_{CC}]
\]

\[
z := \alpha^2 \sigma^2_{CC}
\]

We note that these abbreviations take on the following algebraic signs: $x < 0$, $y > 0$ and $z > 0$. We can now shorten $\frac{\partial EU^*}{\partial \eta_u}$ to a function $f$, defined as follows:

\[
f(\eta_u) := \frac{x}{\eta_u \sqrt{\eta_u}} + \frac{y}{\eta_u} + z
\]

this function is now only dependent on the variable of interest, $\eta_u$, and possesses the same characteristics as $\frac{\partial EU^*}{\partial \eta_u}$.

A closer look at the limits

\[
\lim_{\eta_u \to -\infty} f(\eta_u) = \lim_{\eta_u \to -\infty} \frac{x}{\eta_u \sqrt{\eta_u}} + \frac{y}{\eta_u} + z = z > 0
\]

\[
\lim_{\eta_u \to 0} f(\eta_u) = \lim_{\eta_u \to 0} \frac{x}{\eta_u \sqrt{\eta_u}} + \frac{y}{\eta_u} + z = -\infty < 0
\]

shows that $f$ owns at least one root in $(0, \infty)$. With further arguments of polynomial algebra, we can deduce that $f$ owns exactly one root $N_f$ in $(0, \infty)$, and inserting the terms for $x$, $y$ and $z$ yields a root $N_{EU^*}$ of $\frac{\partial EU^*}{\partial \eta_u}$ in $(0, \infty)$. 25
Hence, there exists an interval, where the first derivative of the $EU^*$ function is negative. ■

b) Now it is to show for the ostensible (indirect) utilities, $\frac{\partial EU^*(c)}{\partial c} < 0$.

To obtain this result we use the envelope theorem and determine the derivative of the Lagrange function at $\alpha^*$ and $w_f^*$.

$$
\frac{\partial C^U}{\partial c} = \frac{1}{2} \left( -5\alpha^* + 6 - \frac{1}{\alpha^*} \right) \frac{(ER_K - ER_B)^2}{8\sqrt{2} \eta_u \sigma_K} e^{-\frac{c}{2}} + \lambda_U \left[ -\frac{w_f^* + 2\alpha^*(ER_K - ER_B)^2}{8\sqrt{2} \eta_u \sigma_K} e^{-\frac{c}{2}} + \alpha ER_B - E[\alpha R_{CC}] + \frac{\eta_u (\alpha^*)^2 \sigma_{CC}^2}{2} \right] \eta_u \frac{2\alpha^* (ER_K - ER_B)^2}{8\sqrt{2} \eta_u \sigma_K} e^{-\frac{c}{2}}
$$

$$
= \frac{1}{2} \left( -5\alpha^* + 6 - \frac{1}{\alpha^*} \right) \frac{(ER_K - ER_B)^2}{8\sqrt{2} \eta_u \sigma_K} e^{-\frac{c}{2}} + \lambda_U \left[ -\left( -Eu^* \right) \frac{2\alpha^* (ER_K - ER_B)^2}{8\sqrt{2} \eta_u \sigma_K} c^{-\frac{3}{2}} \right]
$$

$$
\frac{\partial EU^*(c)}{\partial c} = -\frac{1}{2} \left( -3\alpha^* + 6 - \frac{1}{\alpha^*} \right) \left( -3\alpha^* + 6 - \frac{1}{\alpha^*} \right) < 0 \quad \text{for sufficiently large } \alpha^* \leq 1
$$

$$
\left( -3\alpha^* + 6 - \frac{1}{\alpha^*} \right) > 0 \quad \text{if } \alpha^* > 1 - \sqrt{\frac{2}{3}} \approx 0.1835
$$

Hence we obtain for the ostensible (indirect) utilities $EU^*(c)$ with $\frac{\partial EU^*(c)}{\partial c} < 0$ for sufficiently large $\alpha^* \leq 1$.

5.2.2 True expected (indirect) utility reaction:

a) Even if ostensible (indirect) utilities decrease with $c$, true utility increase with increasing $\eta_u$, thus we need to show that $\frac{\partial EU^*_{true}(\eta_u)}{\partial \eta_u} > 0$, again by using the envelope theorem for

$$
EU^*_{true} = -\eta_u \left( (1-\alpha^*)ER + \alpha^* ER_{CC} - w_f^* - \frac{\eta_u (1-\alpha^*)^2 \sigma_{CC}^2}{2} - \frac{\eta_u (\alpha^*)^2 \sigma_{CC}^2}{2} \right)
$$

Whereas we can obtain the derivative of the utility function without concealing costs from the following Lagrange function:

$$
\frac{\partial C^U_{true}}{\partial \eta_u} = \lambda_U \left[ -w_f^* + c q b^2 - \alpha (1 - b) ER_B - ab ER_K + \frac{2 \eta_u \alpha^2 b^2 \sigma_{CC}^2}{2q} + \alpha ER_{CC} - \frac{2 \eta_u \alpha^2 \sigma_{CC}^2}{2} \right] = \frac{1}{\eta_u} \left[ w_f^* - c q b^2 + \alpha (1 - b) ER_B + ab ER_K - \frac{\eta_u \alpha^2 b^2 \sigma_{CC}^2}{q} - \alpha ER_{CC} + \frac{\eta_u \alpha^2 \sigma_{CC}^2}{q} \right]
$$

$$
= \frac{1}{\eta_u} \left[ w_f^* - c q b^2 + \alpha (1 - b) ER_B + ab ER_K - \alpha ER_{CC} \right] + \alpha^2 \left[ \sigma_{CC}^2 - \frac{b^2 \sigma_{CC}^2}{q} \right]
$$
and by using the Envelope Theorem once more, we get

$$\frac{\partial EU^{*}_{\text{true}}}{\partial \eta_u} = \frac{1}{\eta_u} \left[w_f - cq b^2 + \alpha (1 - b) ER_B + ab ER_K - \alpha ER_{CC}\right] + \alpha^2 \left[\sigma^2_{CC} - \frac{b^2 \sigma^2_{K}}{q}\right]$$

**Proof.** The algebraic sign of the derivative $\frac{\partial EU^{*}_{\text{true}}}{\partial \eta_u}$ needs a slightly different approach, but first, we also take a look at the limits at the boundaries for $\eta_u$:

$$\lim_{\eta_u \to -\infty} \frac{\partial EU^{*}_{\text{true}}}{\partial \eta_u} = \alpha^2 \left[\sigma^2_{CC} - \frac{b^2 \sigma^2_{K}}{q}\right]$$

$$\lim_{\eta_u \to 0} \frac{\partial EU^{*}_{\text{true}}}{\partial \eta_u} = \infty$$

Since we can manipulate the size of $\sigma^2_{CC}$, the first limit can be taken as positive, and therefore the analysis of the algebraic sign once again centers around finding roots for $\frac{\partial EU^{*}_{\text{true}}}{\partial \eta_u}$, it holds:

$$\frac{\partial EU^{*}_{\text{true}}}{\partial \eta_u} = \frac{1}{\eta_u} \left[w_f - cq b^2 + \alpha (1 - b) ER_B + ab ER_K - \alpha ER_{CC}\right] + \alpha^2 \left[\sigma^2_{CC} - \frac{b^2 \sigma^2_{K}}{q}\right] = 0$$

$$\Rightarrow \eta_u = \left[-w_f + cq b^2 + \alpha (1 - b) ER_B + ab ER_K - \alpha ER_{CC}\right] \frac{\alpha^2 \left[\sigma^2_{CC} - \frac{b^2 \sigma^2_{K}}{q}\right]}{}$$

We label this root as $N_{EU^{*}_{\text{true}}}$, and take a closer look at the algebraic sign:

$$N_{EU^{*}_{\text{true}}} = \left[-w_f + cq b^2 + \alpha (1 - b) ER_B + ab ER_K - \alpha ER_{CC}\right] \frac{\alpha^2 \left[\sigma^2_{CC} - \frac{b^2 \sigma^2_{K}}{q}\right]}{}$$

$$= \frac{\left[2\alpha (ER_K - ER_B)^2\right] + 2\alpha ER_B + \frac{\eta_u \alpha^2 \sigma^2_{CC}}{2} - \ln(-u(\tilde{w}))}{\eta_u} - \frac{\alpha (ER_K - ER_B)^2}{8 \sqrt{2 \eta_u \sigma_{CC}}} - \frac{2\alpha (ER_K - ER_B)^2}{8 \sqrt{2 \eta_u \sigma_{CC}}}$$

$$= \frac{\alpha^2 \left[\sigma^2_{CC} - \frac{(ER_K - ER_B)^2}{4 \sqrt{2 \eta_u \sigma_{CC}}}\right]}{}$$

$$= \frac{\left[3 \alpha (ER_K - ER_B)^2\right] + 2\alpha ER_B + \frac{\eta_u \alpha^2 \sigma^2_{CC}}{2} - \ln(-u(\tilde{w}))}{\eta_u}$$

$$\frac{\alpha^2 \left[\sigma^2_{CC} - \frac{(ER_K - ER_B)^2}{4 \sqrt{2 \eta_u \sigma_{CC}}}\right]}{}$$

$$= \frac{\left[3 \alpha (ER_K - ER_B)^2\right] + 2\alpha ER_B + \frac{\eta_u \alpha^2 \sigma^2_{CC}}{2} - \ln(-u(\tilde{w}))}{\eta_u}$$

$$\frac{\alpha^2 \left[\sigma^2_{CC} - \frac{(ER_K - ER_B)^2}{4 \sqrt{2 \eta_u \sigma_{CC}}}\right]}{}$$

In both bracket terms, $\sigma^2_{CC}$ can now be manipulated in its size, so that both terms are positive, and thus $N_{EU^{*}_{\text{true}}}$ < 0 and $\frac{\partial EU^{*}_{\text{true}}}{\partial \eta_u}$ does not own a root in $(0, \infty)$, and therefore it follows:

$$\frac{\partial EU^{*}_{\text{true}}}{\partial \eta_u} > 0 \forall \eta_u > 0$$
b) Even if ostensible (indirect) utilities decrease with \( c \), true utility increase with increasing \( c \), thus we need to show that \( \frac{\partial U_{true}^U(c)}{\partial c} > 0 \), again by using the envelope theorem for

\[
L_{true}^U = \left( 1 - \alpha \right) ER_B + \left( 1 - \alpha \right) b(ER_K - ER_B) + \alpha ER_{CC} - w_f - \eta_U \left( 1 - \alpha \right) \frac{\eta_U^2 u^2 \sigma_{K}^2}{2} - \eta_U \alpha^2 \sigma_{CC}^2 \right)
\]

\[
+ \lambda_U \left[ u(\bar{w}) + e^{-\eta_u}\left[ w_f - \eta_U^2 u^2 \sigma_{K}^2 \right] \right]
\]

\[
= \left( 1 - \alpha \right) ER_B + \frac{1 - \alpha}{\eta_U(u^2 \sigma_{K}^2)} + \alpha ER_{CC} - w_f \right)
\]

\[
+ \lambda_U \left[ u(\bar{w}) + e^{-\eta_u}\left[ w_f - \eta_U \alpha^2 \sigma_{CC}^2 \right] \right]
\]

\[
= \left( 1 - \alpha \right) ER_B + \frac{1 - \alpha}{\eta_U(u^2 \sigma_{K}^2)} - \frac{\eta_U \alpha^2 \sigma_{CC}^2}{2}
\]

\[
+ \lambda_U \left[ u(\bar{w}) + e^{-\eta_u}\left[ w_f + \alpha \eta_U \alpha^2 \sigma_{CC}^2 \right] \right]
\]

\[
\frac{\partial L_{true}^U}{\partial c} = \left[ \frac{(1 - \alpha)(ER_K - ER_B)^2}{2\eta_u \sigma_{K}^2} \left( -\frac{1}{2} c^{-\frac{3}{2}} - \eta_U \left( 1 - \alpha \right) (ER_K - ER_B)^2 \right) \right]
\]

\[
+ \lambda_U \left[ (-Eu^*) \left( -\eta_u \right) \left( \frac{\alpha (ER_K - ER_B)^2}{4\eta_u \sigma_{K}^2} \left( -\frac{1}{2} c^{-\frac{3}{2}} \right) \right) \right]
\]

\[
= \left[ \frac{(1 - \alpha)(ER_K - ER_B)^2}{4\eta_u \sigma_{K}^2} \left( c^{-\frac{3}{2}} - \eta_U \left( 1 - \alpha \right) (ER_K - ER_B)^2 \right) \right]
\]

\[
+ \lambda_U \left[ \left( -Eu^* \right) \left( -\eta_u \right) \left( \frac{\alpha (ER_K - ER_B)^2}{8\eta_u \sigma_{K}^2} \left( c^{-\frac{3}{2}} \right) \right) \right]
\]

\[
= \left[ \frac{(1 - \alpha)(ER_K - ER_B)^2}{4\eta_u \sigma_{K}^2} \left( c^{-\frac{3}{2}} - \eta_U \left( 1 - \alpha \right) (ER_K - ER_B)^2 \right) \right]
\]

\[
+ \left[ \frac{\alpha (ER_K - ER_B)^2}{8\eta_u \sigma_{K}^2} \left( c^{-\frac{3}{2}} \right) \right]
\]

\[
= \left[ \frac{(ER_K - ER_B)^2}{8\eta_u \sigma_{K}^2} \left( c^{-\frac{3}{2}} \right) \left( \frac{1}{4} + \frac{\alpha}{8} \right) \right]
\]

\[
= \frac{(ER_K - ER_B)^2}{8\eta_u \sigma_{K}^2} \left( c^{-\frac{3}{2}} \right) \left[ \frac{(1 - \alpha)^2}{\sigma_K^2} \right]
\]

\[
= \frac{(ER_K - ER_B)^2}{8\eta_u \sigma_{K}^2} \left( c^{-\frac{3}{2}} \right) \left[ \frac{(1 - \alpha)^2}{\sigma_K^2} \right]
\]

\[
= \frac{(ER_K - ER_B)^2}{8\eta_u \sigma_{K}^2} \left( c^{-\frac{3}{2}} \right) \left[ \frac{(1 - \alpha)^2}{\sigma_K^2} \right]
\]
\[
\frac{\partial EU^*(c)}{\partial c} > 0 \text{ if } \frac{(a-2)\sigma^2 + \eta_u(1-\alpha)^2}{\sqrt{2}\eta_u} > 0 \text{ for sufficiently small } c.
\]

Note,
\[
\lim_{c \to 0} \left[ \frac{(a-2)\sigma^2 + \eta_u(1-\alpha)^2}{\sqrt{2}\eta_u} \right] = \frac{\eta_u(1-\alpha)^2}{\sqrt{2}\eta_u} > 0
\]

\[
\lim_{c \to 0} \frac{\partial EU^*(c)}{\partial c} = \lim_{c \to 0} \left( \frac{(ER_K - ER_B)^2}{8\sqrt{2}\eta_u} \right) \frac{1}{c^2} \left[ \frac{(a-2)\sigma^2 + \eta_u(1-\alpha)^2}{\sqrt{2}\eta_u} \right] = \infty
\]

Given that \(\frac{\partial \alpha}{\partial c} < 0\), if \(\alpha < \frac{1}{\sqrt{3}}\), and under consideration of the restraints for \(\alpha\) derived from \(\frac{\partial EU^*(c)}{\partial c} < 0\), optimal choices for \(c^*\) can be found between \(1 - \sqrt[3]{2} < \alpha^* < \frac{1}{\sqrt{3}}\).

### 5.3 Appendix 3: Proof of Proposition 3 principal’s adverse selection of agents

The continuous set of competing agents is given by

\(agents := \{j \in \mathbb{N} \mid j \text{ describes an agent with concealing cost } c_j \in \mathbb{R}\}\)

Proposition 2 says that \(\frac{\partial EU^*(c)}{\partial c} < 0\) is effective. Hence, the principal will choose the agent \(j \in agents\), that has the lowest concealing cost \(c_{\min} = c_j = c^*\).

Simultaneously the same argument holds for the risk aversion \(\eta_u\), because of \(\frac{\partial EU^*(\eta_u)}{\partial \eta_u} < 0\), thus the principal will choose the agent \(j \in agents\), that has the lowest risk aversion \(\eta^{\min}_{u_j} = \eta_u = \eta^*\).

Furthermore, Proposition 1 says that the agent’s choice for \(q\) looks like:

\[
q^* = \frac{\sqrt{\eta_u\alpha\sigma_K}}{\sqrt{2}c}, \quad b^* = \frac{ER_K - ER_B}{2\sqrt{2}\eta_u\sigma_K}
\]

Therefore \(\frac{\partial q^*}{\partial c} = -\frac{\sqrt{\eta_u\alpha\sigma_K}}{2\sqrt{2}c^{1.5}} < 0\) is valid, and for agent \(j\) it follows:

\[
q_j = q^* = q^{\max} = q(c^*)
\]

Analogously, proposition 1 implies: \(b^* = b^{\max} = b(c^*, \eta_u^*)\) since

\[
\frac{\partial b}{\partial c} = -\frac{ER_K - ER_B}{4\sigma_K \sqrt{2}\eta_u c^{1.5}} < 0
\]

as well as

\[
\frac{\partial b}{\partial \eta_u} = -\frac{ER_K - ER_B}{4\sigma_K \sqrt{2}\eta_u^{1.5}} < 0
\]
\[ \sigma^2 = (b^{\max})^2 \sigma^2_k \]

And last but not least Proposition 2 shows that the equality \( EU_{\text{true}}^{\min} = EU_{\text{true}}^{\ast}(c^\ast) \) holds because of \( \frac{\partial EU_{\text{true}}^{\ast}(c)}{\partial c} > 0 \), which holds for a small \( c \) like \( c^{\min} = c_j \).

\[ \square \]

5.4

5.5 Appendix 4: Proof of Corollary 4

If the market situation presents itself as described above, principals will choose the agent with the lowest concealing costs \( c^{\min} \) and highest concealing effort \( q^{\max} \), as can be seen in the behavior of the derivatives of \( EU^*(c) \) and \( q \). Proposition 2 says that \( \frac{\partial EU^*(c)}{\partial c} \) is strictly negative, \( \frac{\partial EU^*(c)}{\partial c} < 0 \), and therefore it follows \( c^\ast = c^{\min} \). Likewise it follows, \( \frac{\partial q}{\partial c} \) is strictly negative, \( \frac{\partial q}{\partial c} < 0 \), as is proven in Proposition, and therefore \( q^\ast = q^{\max} \).

Furthermore, under these conditions, they will receive the largest share of risky assets, and with notice to \( b^\ast = b^{\max} \), proven in Proposition 3, the portfolio will become the most risky portfolio \( \sigma^2 = (b^{\max})^2 \sigma^2_k \).

\[ \square \]

5.6 Appendix 5: Proof of Theorem 5

Let \( A \neq \emptyset \) be the set of destructive agents, which is not empty, and \( P \neq \emptyset \) the set of principals in a market with asymmetric information and hidden action. In this case every destructive agent \( j \in A \) would choose an optimal portfolio \( (b^*_j, q^*_j) \) and every principal \( i \in P \) would choose an optimal \( (\alpha^*_i, w^*_f, i) \) such as it is described in Proposition 1. Therefore every principal \( i \in P \) recruits an agent \( j \in A \), with lowest concealing costs \( c^{\min}_j \) and highest concealing effort \( q^{\max}_j \), such as it is shown in Proposition 3. As a result of that, every principal \( i \in P \) would choose the agent \( i \in A \), with the largest share of risky assets and hence the most risky portfolio, such as it is also shown in Proposition 3. Hence, this behavior leads to an inefficient and excessive risk for every principal \( i \in P \), because he recruits his agent \( j \in A \), so that he receives the lowest expected true utility \( EU_{\text{true},i}^{\min} \) (Proposition 3). Since all principals make the same decision for the recruitment, this leads to an inefficient and excessive risk, which is endogenously and stably chosen by the aggregate market. According to the definition, this risk is described as systematic risk.

\[ \square \]