A Semi-APARCH approach for comparing long-term and short-term risk in Chinese financial market and in mature financial markets[§]

Yuanhua Feng[*1] and Lixin Sun[**2]

[1]Faculty of Business Administration and Economics, University of Paderborn, Germany

[2]Center for Economic Research, Shandong University, China

Abstract

The aim of this paper is to analyze the long-term and short-term risk components in Chinese financial market and to compare them with those in mature financial markets. For this purpose a most recently proposed Semi-APARCH is applied to the Shanghai Index and the Shenzhen Index, and four financial indexes in mature markets. A few important empirical findings are achieved. Firstly, the current long-term risk in Chinese financial market is stable and at a low level. Secondly, the dependence level between long-term risk in Chinese financial market and that in mature financial market is not high. Thirdly, the short-term risk in Chinese financial market differs to that in a mature financial market at least in two ways: 1) The leverage effect in Chinese financial market is much lower than that in a mature financial market. 2) The innovations in Chinese financial returns is nearly heavy-tailed distributed. This is however not the case in a mature market.

Key Words: Chinese financial market, mature financial markets, long-term risk, short-term risk, semiparametric APARCH

JEL Classification Codes: C14, G10

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[*]Yuanhua Feng: E-mail: yuanhua.feng@wiwi.upb.de; Tel. +49-5251-603379; Fax:– 605005

[**]Lixin Sun: E-mail: lxsun@sdu.edu.cn or SunLixin@gsm.pku.edu.cn; Tel. +86-531-88363659

1 Introduction

The aim of this paper is to analyze the long-term and short-term risk components in Chinese financial market and to compare them with those in mature financial markets, where the long-run risk component will be modeled by a smooth volatility trend (also called the scale function) in the return series and the conditional risk dynamics are then analyzed using some suitable ARCH (autoregressive conditional heteroskedasticity, Engle, 1982) or GARCH (generalized ARCH, Bollerslev, 1986) models. Well known approaches for this purpose are for instance the Semi-GARCH (semeparametric GARCH) model of Feng (2004) proposed by introducing a scale function into the standard GARCH model. van Bellegem and von Sachs (2004) discussed the forecasting of financial time series under time varying unconditional variance. A general time varying ARCH model was introduced by Dahlhaus and Subb Rao (2006). Engle and Rangel (2008) proposed a Spline-GARCH model with a nonparametric volatility trend, which is defined as a function of the observation time, i.e. the location, and possible exogenous macroeconomic variables and is estimated by an exponential quadratic spline.

Most recently, Feng (2013) proposed a semiparametric APARCH (Semi-APARCH) by introducing a smooth scale function into the APARCH (asymmetric power ARCH, Ding et al., 1993), which extends the SemiGARCH of Feng (2004) to a wide class of Semi-GARCH-type models including semiparametric generalization of the GJR-GARCH (Glosten et al., 1993), the TS-GARCH (Taylor, 1986 and Schwert, 1989), the NGARCH (Nonlinear GARCH, also called PGARCH, power GARCH, Higgins and Bera, 1992) and the TGARCH (threshold GARCH, Zakoian, 1994) as special cases. The author also developed a fully data-driven algorithm for selecting the bandwidth and proposed the use a nonnegatively constrained local linear estimator of the scale function based on absolute returns, not squared returns. This makes sure that the data-driven algorithm converges under weak moment conditions.

In this paper the Semi-APARCH model is applied to the Shanghai Composite Index (SHI) and the Shenzhen composite Index (SZI) and four selected indexes in mature markets, i.e. the Standard and Poor 500 (S&P), the DAX, the Nikkei 255 (NIK) and the Hong Kong Hang Seng Index (HSI), as well. It is found that the proposed bandwidth selection algorithm works very well in practice. The use of the absolute returns is sometimes clearly better than the use of the squared returns, in particular when estimation at the boundary is considered. first of all, it is found that the current long-term risk in Chinese financial market is low level and stable. To quantify the relationship between a pair of scale functions, we propose to use the concept of the "correlation" of two deterministic function. Numerical results of this criterion provides us an overview on the closeness (or similarity) of a pair of scale functions. Not surprisingly, it is found that the scale functions in the SHI and the SZI run almost parallel to each other, while the other four scale functions are usually closely related to each other. The long-term risk in the two Chinese indexes is also clearly related to that in S&P, NIK and HSI. But there seems no relationship between the long-term risk in SHI and SZI, and that in DAX. This is mainly due to the fact that there was almost no spillover effect of the financial crisis 2002/2003 and the big shock in August and September 2011 in German financial market upon Chinese financial market. In the second stage, the standardized returns are firstly analyzed and compared by means of an APARCH model with t-distributed innovations and a fixed power 1. It is found that the behavior of the conditional risk in Chinese financial market differ to that in a mature market in at least two ways: 1) The leverage effect in Chinese financial market is very low, while it is very high in the other mature markets. In a mature market the leverage coefficient may achieve 1 so that the contribution of a positive return yesterday to today's volatility is nearly zero, and 2) The degree of freedom of the innovations in Chinese financial market is much lower than that in a mature financial market. The fitted t-distribution for the innovations is nearly heavy-tailed. That is the chance of an extreme event in Chinese financial market is much higher than in a mature financial market. A best model is then selected for each index from a large number of suitable models using the BIC (Bayesian information criterion). It is found that, in any case, the use of a fixed power $\delta = 1$ in the APARCH model is better than the use of an automatically selected power, while the best innovation distribution changes from case to case.

Some related studies in the literature are e.g. Zhang and Ma (2006) on the modeling of volatility of the SHI using different GARCH-type models, hou (2007) on the asymmetry effect in the volatility of the SHI and SZI using the nonparametric GARCH model proposed by Bühlmann and McNeil (2002), Yoon et al. (2011) with a VaR analysis for the Shanghai Stock Market using a FIAPARCH model (fractionally integrated APARCH) and Ho and Zhang (2011) on the relationship between financial markets in China, Taiwan and Hong Kong using a multivariate FIAPARCH model and other related models. So far as we know, there is still no study on the volatility in Chinese financial market using Semi-GARCH type models. Hence one of the contributions of the current paper is to fill this gap.

The paper is organized as follows. The Semi-APARCH model is described in Section 2.

Estimation of this model is summarized Section 3. Section 4 reports the empirical results on the long-term risk of the selected indexes. The parametric models are fitted and compared in Section 5. Final remarks in Section 6 conclude the paper.

2 The Semi-APARCH model

The Semi-APARCH model is a special case of the general Semi-GARCH framework introduced by Feng (2013) with a APARCH specification for the stationary part. In this paper we propose to use this class of Semi-GARCH models, because it includes the original Semi-GARCH models and several widely applicable extensions of it. Let r_t, $t = 1, ..., n$, denote the logarithmic returns from an asset. The Semi-APARCH model is defined as follows:

$$r_t = s(\tau)\sigma_t\varepsilon_t, \tag{1}$$

where $s(\tau) > 0$ is a smooth scale function, $\tau_t = t/n$ is the rescaled time, ε_t are i.i.d. random variables with zero mean and unit variance, and σ_t^2 is the conditional variance of the re-scaled process $\xi_t = r_t/s(\tau_t) = \sigma_t\varepsilon_t$. It is assumed that the conditional variance follows the specification

$$\sigma_t^\delta = \alpha_0 + \sum_{i=1}^{p} \alpha_i(|\xi_{t-i}| + \gamma_i\xi_{t-i})^\delta + \sum_{j=1}^{q} \beta_j\sigma_{t-j}^\delta, \tag{2}$$

where is $\alpha_0 > 0$, $\alpha_i \geq 0$, $\beta_j \geq 0$, $-1 \leq \gamma_i \leq 1$ and δ is a suitable positive number. It is assumed that $\sum \alpha_i + \sum \beta_j < 1$. The coefficients γ_i stand for the so called leverage effect, which takes the asymmetric news impact on the volatility into account. Note that the APARCH model is well-defined for $\gamma_i = 1$ or $\gamma_i = -1$. For instance, $\gamma_1 = 1$ indicates a perfect leverage effect. Now the contribution of a positive return on yesterday to today's volatility vanishes. In this case the corresponding ARCH part only reflexes the effect of negative past returns. Models (1) and (2) extend the SemiGARCH model in different ways and provide us a tool to decompose financial risk into a local (unconditional) component $s(\tau_t)$, a conditional component σ_t and that caused by the i.i.d. innovations ε_t. Possible asymmetric innovation distributions can also be used. Note however that the leverage effect and the asymmetry in the innovation distribution are two properties. The former is determined by the past information, the latter not.

It is assumed that ξ_t also has unit variance so that the model is uniquely defined. This implies that $E(\sigma_t^2) = 1$. In practice, returns may also have a nonparametric drift function. For simplicity, this will not be considered in the current paper, because our focus is on the estimation of $s(\cdot)$ and σ_t. Moreover, it is well known that under common regularity conditions the effect of the error in a nonparametric estimator of an unknown drift function on the estimation of $s(\tau)$ is asymptotically negligible. Hence all of the results given in this paper hold for a model with a nonparametric drift function, provided that the drift function is estimated by another well-developed data-driven algorithm.

The theoretical reason to use the SemiAPARCH model instead of a the APARCH model is that, if the scale function in model (1) changes over time, then the parametric specification in (2) cannot be estimated consistently from the data without estimating and removing the nonstationary scale function. This will, for instance, cause the well known phenomenon with $\hat{\alpha}_1 + \hat{\beta}_1 \approx 1$ by the most widely used GARCH(1, 1) model (see e.g. Mikosch and Stărică, 2004). On the other hand, consistent estimator of the scale function can be easily conducted without any knowledge on the parametric structure of σ_t and ε_t. After estimating and removing the nonstationary scale function, we obtain an approximately stationary process for further analysis. When the process itself follows a parametric GARCH-type model, i.e. with a constant unconditional variance, the semiparametric framework proposed in this paper still works but with some loss of the efficiency. The practical reason for the use of the SemiAPARCH model is that the scale function $s(\tau)$ reflexes the long-run overall effect of the macroeconomic factors on the volatility change in the financial market, which can be easily predicted and should be mainly used for long-term macroeconomic decision makings.

3 The semiparametric estimation procedure

The models introduced in the last section can be estimated using a semiparametric procedure. At first, $s(\tau)$ can be estimated by some nonparametric regression approach consistently without any parametric assumptions on σ_t and ε_t. Then the conditional variance can be analyzed further using the APARCH model based on the standardized returns. In the following the estimation procedure as proposed by Feng (2013) will be summarized briefly. For more detailed theoretic discussions we refer the reader to that work.

3.1 Estimation of $s(\tau)$

Usually, $s^2(\tau)$ is estimated from r_t^2 first. $\hat{s}(\tau)$ is then obtained by taking the square root of $\hat{s}^2(\tau)$. In this paper we propose to estimate $s(\tau)$ from $|r_t|$. This is a special case to estimate the scale function from $|r_t|^d$ as proposed by Feng (2013). Now, $d = 1$ is fixed. The reason for this choice is that the data-driven algorithm can be carried out under the assumption that the fourth order moment of ξ_t exists, which is also the moment condition required for fitting a standard GARCH model using the quasi conditional maximum likelihood method. Note however that the scale function estimated from $|r_t|$ is not an estimate of $s(\tau)$, but that of another function $g(\tau)$. Let $c_1 = E(|\xi_t|)$, which is not equal to 1. We have $g(\tau) = c_1 s(\tau)$ and $s(\tau) = c_1^{-1} g(\tau)$. Hence, after getting the estimated scale function from $|r_t|$, we have to rescaled it to obtain a consistent estimate of $s(\tau)$. The proposed constrained local linear estimator of the scale function processes as follows. Let $K(u)$ be a kernel function and $b > 0$ be the bandwidth. A local linear estimator of $g(\tau)$ at $0 \leq \tau \leq 1$ is obtained by minimizing

$$Q = \sum_{t=1}^{n} \{|r_t| - a_0 - a_1(\tau_t - \tau)\}^2 K\left(\frac{\tau_t - \tau}{b}\right). \tag{3}$$

This results in $\tilde{g}(\tau) = \hat{a}_0$. The advantage of a local linear estimator is that the bias of it is always of the order $O(b^2)$. This is in particular important for application, because the forecasting of the trend is mainly made based on the estimation at the right end point. A problem is that $\tilde{g}(\tau)$ obtained above may be sometimes negative, in particular when sample size is small and a small bandwidth is used. To ensure the non-negativity, we propose to use the final estimator $\hat{g}(\tau) = |\tilde{g}(\tau, d)|$, which is almost surely positive. See Feng (2013) for the detailed explanation why $\hat{g}(\tau)$ should be used instead of $\tilde{g}(\tau)$ together with some related asymptotic results on the constrained local linear estimator.

Assume now that $E(\xi_t^4) < \infty$, a consistent estimate of $s(\tau)$ can be obtained as follows. Let $\hat{\xi}_{1,t} = r_t / \hat{g}(\tau_t)$. Note that $\hat{\xi}_{1,t} \approx c_{-1}\hat{\xi}$ and that $E(\xi_t^2) = 1$. This leads to a consistent estimate of c_1

$$\hat{c}_1 = \left[\frac{1}{n} \sum_{t=1}^{n} \hat{\xi}_{1,t}^2\right]^{-1/2}. \tag{4}$$

We obtain

$$\hat{s}(\tau_t) = \hat{c}_1^{-1} \hat{g}(\tau_t), \tag{5}$$

which is indeed obtained through rescaling the sample variance of the standardized returns to be 1. Note that, so far $\hat{g}(\tau)$ is consistent, the effect of the error in it on $\hat{c}_1$ is asymptotically negligible. Hence $\hat{c}_1$ is $\sqrt{n}$-consistent. This leads to the conclusion that the MSE of $\hat{s}(\tau)$ obtained above is approximately $c_d^{-2/d}\mathrm{MSE}[\hat{g}(\tau)]$, which is still of the order $O(n^{-4/5})$, provided that $\hat{g}(\tau)$ is obtained by a suitable data-driven algorithm. However, $\hat{s}(\tau)$ is not an efficient nonparametric estimate of $s(\tau)$, because the optimal bandwidth for estimating $g(\tau)$ is different to that for estimating $s^2(\tau)$ from r_t^2.

A fully data-driven algorithm and a simple test of the significance of the scale function were also developed by Feng (2013), where the iterative plug-in idea of Gasser et al. (1991) was adapted to selected b in (3) with a starting bandwidth selected by the cross-validation rule. Furthermore, the so-called exponential inflation method proposed by Beran and Feng (2002) is employed so that the algorithm run quickly. This algorithm will be used for the application in Section 4.

3.2 Fitting an APARCH model to the standardized returns

The results in the above subsection show that the long-term risk can be estimated and removed first, without estimating the short-term (conditional) risk. Now, we can define $\hat{\xi}_t = r_t/\hat{s}(\tau_t)$. The unknown parameters of a chosen APARCH model can be estimated from $\hat{\xi}_t$ by approximate (conditional) QLM method proposed in the literature. A suitable model can also be selected using e.g. the BIC. For practical implementation an suitable package for fitting GARCH models can be used. In this paper we will use the "fGarch" package in in R, because the data-driven algorithm to be proposed in the next section is also carried out in R. Further computational details may be found e.g. in Wurtz et al. (2013).

Denote the true unknown parameter vector of a chosen GARCH model by θ_0. Let $\hat{\theta}$ be the estimate of θ_0 obtained from $\hat{\xi}_t$ and $\tilde{\theta}$ denote the standard QMLE obtained under the assumption that ξ_t were observable. It is well known that under suitable regularity conditions $\tilde{\theta}$ is $\sqrt{n}$-consistent and asymptotically normal. By adapting the results of Feng (2004), it can be shown that, under suitable regularity conditions: 1) $\mathrm{Var}\,(\hat{\theta}) = \mathrm{Var}\,(\tilde{\theta}) + o(n^{-1/2})$ and 2) Let $B_\theta = E(\hat{\theta} - \tilde{\theta})$. Then B_θ is of the order of magnitude $O[b^2 + (nb)^{-1}]$. This means that the additional variance caused by the errors in $\hat{\xi}_t$ is asymptotically negligible. The $O(b^2)$ term in B_θ is due to $E[\hat{s}^2(\tau_t) - s^2(\tau_t)]$ and the $O[(nb)^{-1}]$ term due to $\mathrm{Cov}\,[\xi_t^2, \hat{s}(\tau_t)]$. If a bandwidth $O(n^{-1/2}) < b < O(n^{-1/4})$ is used, B_θ is asymptotically negligible. Now $\hat{\theta}$ is also $\sqrt{n}$ consistent and asymptotically normal. If the data-driven proposed in the next section is used, the bias term B_θ will be of the order $O(n^{-2/5})$. We see that in the Semi-APARCH models $\sqrt{n}$-consistent parametric estimation is no long possible, if the scale function changes over time. In the special case, when r_t follow a stationary GARCH-type model, a bandwidth of the order $O_p(1)$ will be selected by the proposed data-driven algorithm in next section. Now, the parametric estimation is still $\sqrt{n}$-consistent but is inefficient. This means that some efficiency will be lost, if a semiparametric GARCH-type model is fitted to a stationary GARCH-type process. In the next section a simple stationary test is proposed based on the selected bandwidth. If this test is significant, the proposed semiparametric model will be used. Otherwise, a stationary GARCH-type model should be used.

4 Comparing the long-run risk

The proposed algorithm was applied to several Chinese stock indexes and quite a few stock indexes in mature financial markets. The results show that the proposal works very well in practice. In the following the Shanghai Index (SHI), Shenzhen Index (SZI), Standard and Poor 500 (S&P), DAX 30 (DAX), Nikkei 225 (NIK) and Hang Seng Index (HSI) from January 1997 to August 2013 are chosen to show the detailed empirical results. Before 1997 the two Chinese indexes were clearly under developed. Those data are not used so that the qualification of the analysis is not affected. To simplify the comparative study, observations of the other indexes before 1997 are also excluded. The scale function was estimated with the bandwidth selected using the bisquare kernel as the weight function in (3). For each index, the scale function is estimated with $d = 1$ and $d = 2$, respectively. The selected (relative) bandwidths in all cases are given in Table 1 together with the corresponding number of observations, which varies from one series to another, because the national holidays in those countries are quite different. From Table 1 wee see that the selected bandwidth is quite reasonable and varies from one series to another very clearly. The bandwidth selected for Hang Seng Index seems to be a little bit small. But it still works well. The observations, the returns series, the estimated scale functions with $d = 1$ (solid line) and $d = 2$ (dashed line), and the standardized returns for the Shanghai Index and Shenzhen Index are displayed in Figures 1and 2, respectively, where the confidence bounds given in Figure (c) and the results in Figure (d) in each case are calculated using the scale function estimated with $d = 1$. From Figure 1(c) and 2(c) we can see that the scale function reflexes the long-term risk in a financial market. The sub-period around a very high peak corresponds to a financial crisis and a sub-period, where the scale function stays at a relatively low level, is a phase of the financial market with a stable development. Our empirical results show tha the risk in Chinese financial market before 1999 was very high. It achieved a relatively low level at about the beginning of 1999 and stand at about the same level until middle 2006. Then the risk increased strongly, when the indexes themselves began to increase strongly. This can be understood as the beginning of a (possible) financial crisis. This period with very high financial risk lasted till about September 2009, when the global financial crisis was over. From 2010 the risk achieved a level, which is much lower than that during the financial crisis. Although the estimated overall average level, and the low- and up-confidence-bounds for the estimated scale function are displayed here to see, if the scale function changes over time significantly, it can also be used as criteria to assess, is the risk is very low, at a common level or already too high. In particular, the up-bound can be used as an early warning criterion for a possible financial crisis in the future. For this purpose, the use of a low significance level (i.e. with a big α value) might be more preferable. Detailed discuss on this is however beyond the aim of the current paper. Concerning the current situation in Chinese Financial market, we can see that the risk now is clearly below the average level and stable. Hence, there is no signal that Chinese financial market will experience another high risk period in the next few years.

The same results as shown in Figures 1 and 2 but for S&P, DAX, NIK and HSI, respectively, are shown in Figures 3 through 6. Consider the DAX first. From Figure 4(c) we can see that in German financial market there were three sub-periods with extreme high risk. The first one is between 2002 and 2003 caused by the crisis of the "Neuer Markt". The second one was due to the financial crisis in 2007/2008 and the last happened in August and September 2011 caused by a big shock in the DAX index due to the "Euro crisis". The 2007/2008 financial crisis started first in the US. It became immediately a global financial crisis. We can see that during the sub-period the level of risk is always the highest in all of those indexes, except for DAX, where the level of risk in 2002 and 2003 was even higher. But that financial crisis was relatively a local one. Of course it had a clear spillover effect on the US financial market. This crisis also had an unclear spillover effect on the financial market in Japan. No spillover effect of this financial crisis on the financial markets in China and Hong Kong can be seen. The shock in DAX caused by the "Euro crisis" also exhibited a clear spillover effect on the S&P index and a relatively unclear effect on the HSI. No spillover effect of this shock on the financial markets in China and Japan can be seen. The risk level in the leading financial index S&P 500 was above the average level before 2003, including the spillover effect of the German "Neuer

Markt". Then it experienced a period for several year with very low risk before the 2007/2008 financial crisis. During the financial crisis the risk in the US market became very very high and lasted for about two year. After the financial crisis the US financial market experienced one shock at the end of 2010 and another caused by the "Euro crisis". The long-term risk of NIK before the financial crisis looks quite similar to that of S&P. After the financial crisis the long-term risk of NIK was at a relatively low level. No clear spillover effect of the shock in the US at the end of 2010 and of the "Euro crisis" can be seen. However, the long-term risk of NIK increases very strongly since a few months. It is not yet clear whether this is just a middle-term shock or a pre-crisis period. To decide this, we still need to observe the Japanese financial market further for a few months. The long-term risk of the Hang Seng Index is also similar to that of S&P. However, its scale function changes smoothly over the whole period. There was no long sub-period where the return series seemed to be stationary. This should be the reason, why the selected bandwidth for this series is very small.

Comparing the scale functions for given index obtained by $d = 1$ and $d = 2$, we can see that they are usually about the same. Clear differences between the two estimates often happened at the boundary or during a financial crisis. For instance, the estimated scale function with $d = 1$ for S&P, DAX and NIK during the last financial crisis was clearly below that obtained by $d = 2$, because the volatility of the volatility of these indexes during this sub-period was very high. On the other hand, the volatility of volatility of the two Chinese indexes during the last financial crisis was not too extreme and the scale function estimated by $d = 1$ is even larger than that obtained with $d = 2$. For the HSI these two estimates during the last financial crisis are almost the same. However, due to the fact that the selected bandwidths in this case are small, we can see that the performance of the estimate with $d = 2$ at the left boundary is clearly worse than that obtained using $d = 1$. Hence we will propose the use of $d = 1$ in practice. Further evidence for this will be given in the next section.

To provide a detailed quantitative analysis of the relationship between the scale functions in different financial markets we now propose to calculate the correlation coefficients between each pair of these indexes following the definition in Press (1996). Like the usual correlation coefficient, this criterion also varies from -1 to 1. If this criterion is one, it means that the two scale functions run parallel to each other. This is hence a criterion for the parallelism between two deterministic functions. We will call this the "similarity" between two scale functions. A special problem in our case is that the trading days in different countries differ to each other.

Hence we propose to calculate this criterion using the results on all common trading days of the two countries involved. The results together with the number of common trading days (n_{ij}) between countries i and j are listed in Table 2. We see the similarity between the long-term risks of the two Chinese indexes is almost 1, i.e. these two scale functions run almost parallel to each other, as it is expected. The highest similarity between the scale function of two different countries appears for S&P and NIK with the value 0.834, followed by that of S&P and DAX, 0.794, and of S&P and HSI, 0.753. This indicates clearly that the Standard and Poor 500 Index is the most important financial index of the World. The similarity between the long-term risks in the two Asian mature markets is also high. Those for the other pairs are at some middle level (somehow between 0.4 and 0.6), except for those between the long-term risks of DAX and the two Chinese indexes, where the similarity is nearly zero. Finally, note that the average risk levels of these indexes are about 0.16 for the SHI, 0.175 for the SZI, 0.125 for the S&P Index and 0.15 for the others. We see the average risk in the US financial market is the lowest and it is the highest in the Shenzhen market. According to this criterion the Shanghai financial market is better developed than the Shenzhen financial market, although their long-term risk patterns are very similar to each other.

5 Comparing the conditional behaviors

From Figures 1(d) trough 6(d) we can see that the standardized return series are quite stationary. Furthermore, they still exhibit clear cluster effect, which is not affected by estimating and removing the long-term risk, because the long-term and short-term (conditional) risk components are (almost) orthogonal to each other. To analyze the conditional heteroskedasticity in the standardized returns further APARCH models of orders (1, 1), (1, 2), (2, 1) and (2, 2) are fitted to each of those series using the "fGarch" package in R under the normal-, skewed normal-, $t-$, skewed $t-$, generalized error- and skewed generalized error distribution assumption on the innovations. These models will be denoted by APARCH-n, APARCH-sn, APARCH-t, APARCH-st, APARCH-g and APARCH-sg, respectively. It is found that in most of the cases the mean is insignificant. for simplicity, we will assume that the mean is zero in all return series. It is also found that, following the BIC, the behavior of APARCH-n and APARCH-sn models is clearly worse than that of the other models. Hence, these two classes of models will no longer discussed in detail. Moreover, the best order according to the BIC in any case is the order (1, 1). To save space, detailed discussions will hence only be carried out based on APARCH(1, 1) models.

Firstly, it is found that the APARCH-t(1, 1) model with $\delta = 1$ is a suitable tool to discuss different features of the short-term risk in Chinese financial market comparing with those in mature markets. The reason for the use of $\delta = 1$ will be explained later. Coefficients of the fitted APARCH-t(1, 1) model with $\delta = 1$ together with the corresponding standard deviations are given in Table 3. The fitted formulae of the conditional variances for the Shanghai Index and the Standard and Poor 500 Index are e.g.

$$\sigma_t = 0.053 + 0.095(|\xi_{t-1}| - 0.376\xi_{t-1}) + 0.876\sigma_{t-1} \tag{6}$$

with $\eta_t \sim t(5.118)$ and

$$\sigma_t = 0.043 + 0.076(|\xi_{t-1}| - 1.000\xi_{t-1}) + 0.897\sigma_{t-1} \tag{7}$$

with $\eta_t \sim st(10)$. According to the results in (6) and (7) we can see that there are at least two clear differences between the volatility in Chinese financial market and that in the US financial market. Firstly, the degree of freedom of the innovation distribution in the former case is just about 5 and this is 6.37 for the SZI. This means that the innovation distribution in Chinese financial market is nearly heavy-tailed and the eighth moment of ξ_t does not exists. The chance for an extreme return in Chinese financial market is much larger than that in a mature market. This also indicates that the stable estimate of the scale function with $d = 1$ is more preferable. On the other hand, the degree of freedom of the innovation distribution of the S&P returns is 10. This is also true for the DAX and NIK indexes, and it is 8.31 for the HSI. We see the eighth innovation moment in a mature financial market exists. Because the existence of the eighth moment of ξ_t also depends on the coefficients, it will not be discussed in detail in this paper. However, if GARCH-n(1, 1) models are used as a reference, it can be seen that the conditions for the existence of the eight moments are fulfilled by their coefficients.

Secondly, the leverage parameter for the SHI and SZI is just 0.376 and 0.318, respectively. However, it about 1 for S&P, DAX and HSI and o.735 for NIK. This means that the leverage effect in Chinese financial market is not that strong. The contribution of a negative return on yesterday to today's volatility is about twice as the contribution of a positive return. On the other hand, the estimated parameter $\gamma_1 = 1$ means that a positive return on yesterday does not affect today's volatility at all. Here we experience a perfect leverage effect. This is far not the case in Chinese financial market. For the DAX Index the leverage effect is also almost perfect. For the NIK, the contribution of a negative return on yesterday to today's volatility is about 6.5 times of the contribution of a positive return, which is much higher than that in Chinese financial market. In summary, the short-term risk in a mature financial market has the properties that the innovation distribution is clear not heavy-tailed (in the sense that moments until eighth order exist) and the leverage effect is usually very strong. These two properties are not shared by the short-term risk in Chinese financial market. Note that there is no clear difference between the ARCH and GARCH parameters for the two Chinese indexes and those for the mature indexes.

The BIC criteria for all APARCH-t, APARCH-st, APARCH-g and APARCH-sg models of the order (1, 1) are listed in Table 4, where the results are given for the case with δ as a to be estimated free parameter and for the case with $\delta = 1$ fixed, respectively, and the innovation distributions are indicated by *std*, *sstd*, *ged* and *sged* as the corresponding names of objects used in "fGarch". We see that the best model is always one with fixed δ. If we check these results more exactly, we can see that in any case a model with $\delta = 1$ is better than that with an estimated value of δ. Hence $\delta = 1$ is a good choice. Note however that the use of the fixed value $\delta = 2$ is worse than the results in any of the above mentioned two cases. This is a very nice empirical finding, because we have already proposed to use $d = 1$ for estimating the scale function. But the conclusion that $\delta = 2$ in the APARCH models for the selected indexes is the best choice is independent of the value of d. Even if the returns are calculated by means of the estimated scale function with $d = 2$, $\delta = 1$ should also be used in the APARCH models. According to the results in Table 4 the selected best models following the BIC are that with the skewed generalized error distribution for SHI, S&P and DAX, that with the skewed t-distribution for SZI, that with the t-distribution for NIK and that with the generalized error distribution for HSI. We see in four of the six cases a skewed and in the other two cases a non-skewed innovation distribution was selected. A clear different between these results and those given in Table 3 is that now the skewed generalized error distribution with moment of any order is selected for the innovations of the SHI. Following these results, the innovations of the SHI are no longer heavy-tailed. But the results for the SZI are not changed so much. This indicates again that the financial market in Shanghai is better developed than that in Shenzhen.

The fitted coefficients together with the corresponding standard deviations for all of the selected models except for that for NIK, are shown in Table 5. The results for the best model for NIK are the same as those given in Table 4 and are hence omitted. We see again that the difference between the γ_1 parameter for the two Chinese indexes and that for the others is the same as before. Now consider the three estimated shape parameters for SHI, S&P and DAX under the same skewed generalized error distribution. Assume that these estimates are independent of each other. It can then be shown that the two shape parameters for S&P and for DAX are not significantly different to each other. But the shape parameter for SHI is significantly different to both of the others. That is, even if the same innovation distribution is selected for the three indexes, there is still a significant difference between the shape parameter of he innovations of SHI standardized returns and those of a mature standardized return series.

6 Final remarks

In this paper we applied a most recently proposed Semi-APARCH model to analyze long-term and short-term risk in Chinese financial market and compared it with that in some mature markets. Some interesting different features of the long-term and short-term risk in Chinese financial market and in mature markets are found. for instance the similarity between the long-term risk in Chinese financial market and in a mature market is not as high as that between the long-term risk in two mature markets. Some shocks in some mature financial markets did not have any spillover effect on risk in Chinese financial market. Furthermore, it is in particular indicated that the long-term risk in Chinese financial market at the moment is low and stable. But the average level is a little bit higher than that in a mature market. It is also found that the leverage effect in Chinese financial market is not so high as in a mature market and that the innovation distribution of the standardized returns is nearly heavy-tailed. So far as we know, this study is the first one to apply a Semi-GARCH type model to Chinese financial indexes and to carry out a systematic qualitative and quantitative comparative study between the long-term and short-term risk in Chinese financial market and in mature markets. There are still a lot of open questions in this context to solve. For instance it is of great interest to discuss the further application of the Semi-APARCH model for improving the calculation of risk measures such as the VaR (value at risk) and the ES (expected shortfall). Macroeconomic variables can also be introduced into the proposed Semi-APARCH model following the MIDAS ideas of Engle et al. (2008). Such a model will allow us to study the macroeconomic sources of volatility in financial markets. The extension of the proposal in this paper to multivariate GARCH (MGARCH) case is also a very important topic. Related research in this direction is e.g. Linton and Hafner (2010). Most recently, a few approaches for calculating the systemic risk measures such as the CoVaR (conditional VaR) and the MES (marginal ES) and for determining the SIFI (system important financial institution) by means of a parametric MGARCH model were proposed in the literature. See e.g. Adrian and Brunnermeier (2011), Banulescuy and Dumitrescu (2012), Benoit et al. (2012) and Girardi and Ergün (2013). It is expected that all of those proposals can be improved clearly by means of a suitable Semi-MGARCH model. This problem

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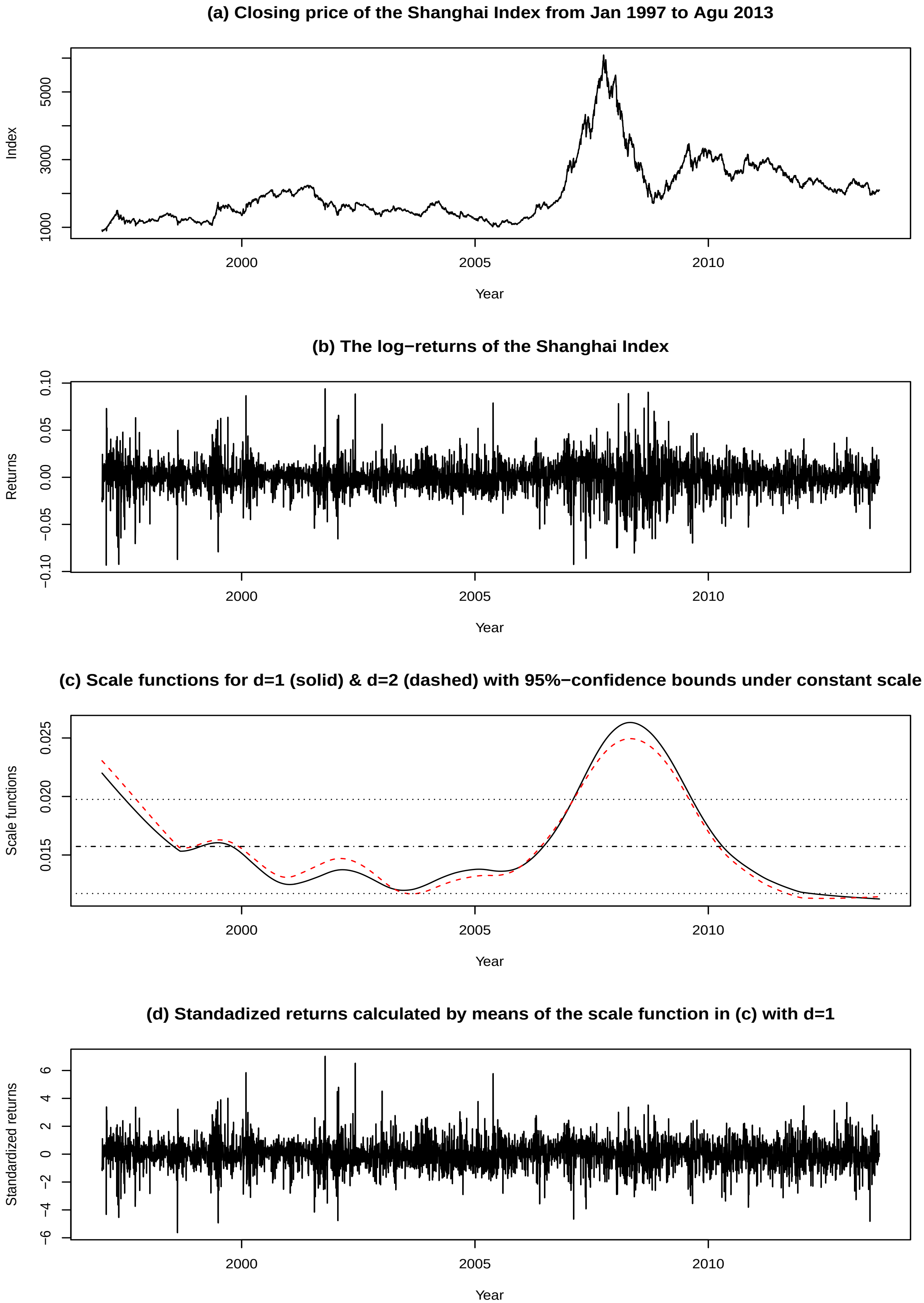

Figure 1: The smoothing results for Shanghai Index from January 1997 to Augst 2013

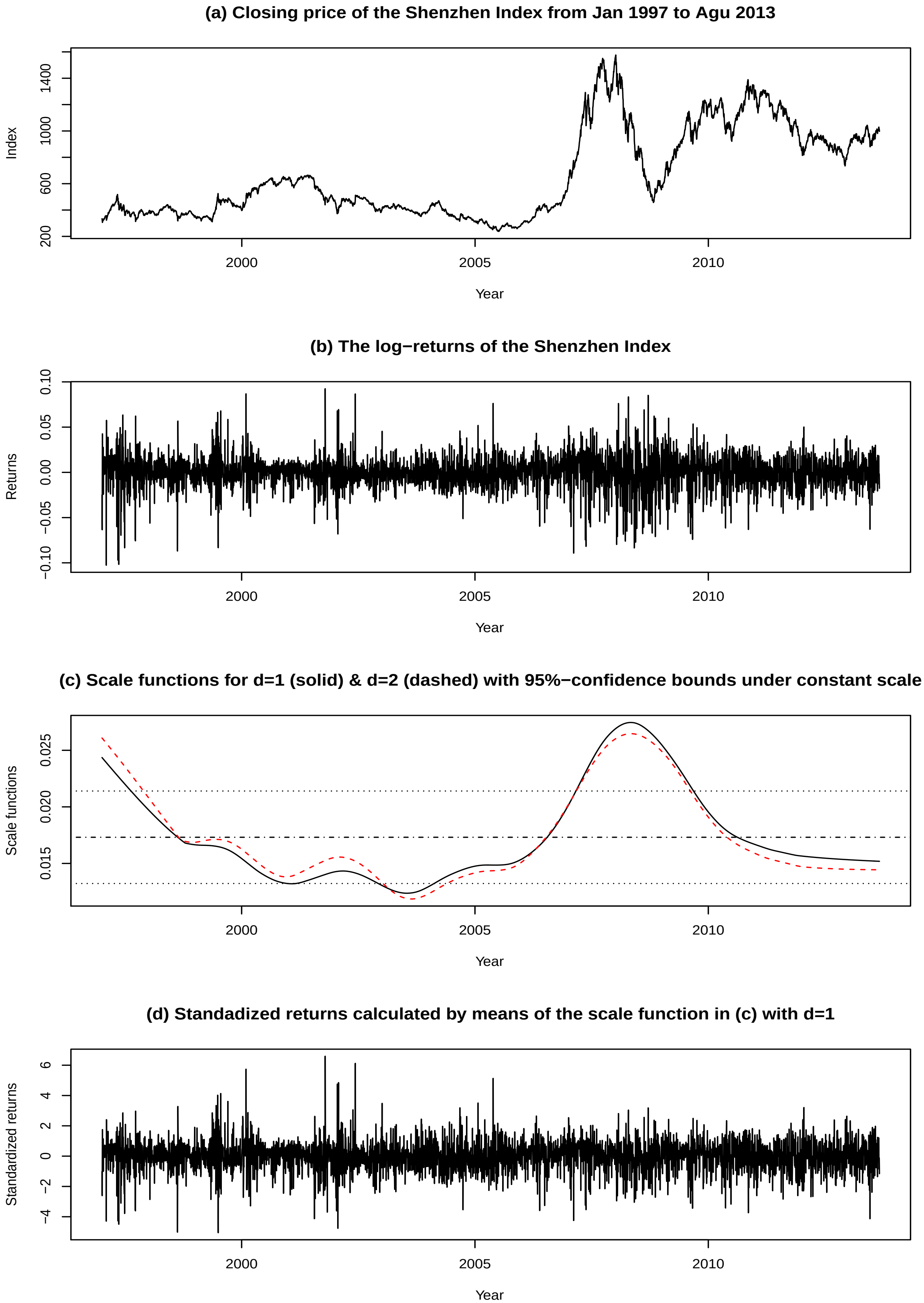

Figure 2: The smoothing results for Shenzhen Index from January 1997 to Augst 2013

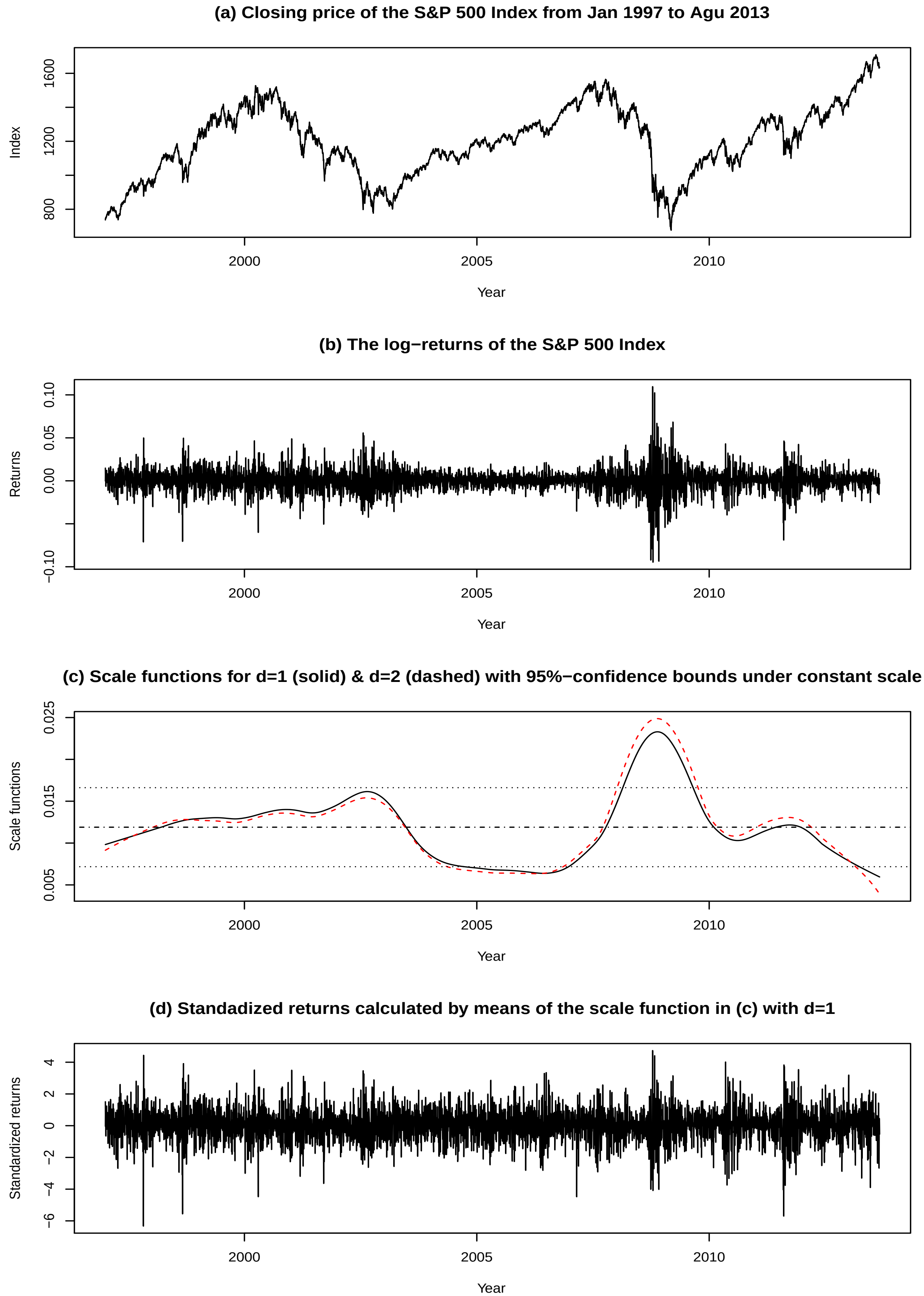

Figure 3: The smoothing results for Standard and Poor 500 Index from January 1997 to Augst 2013

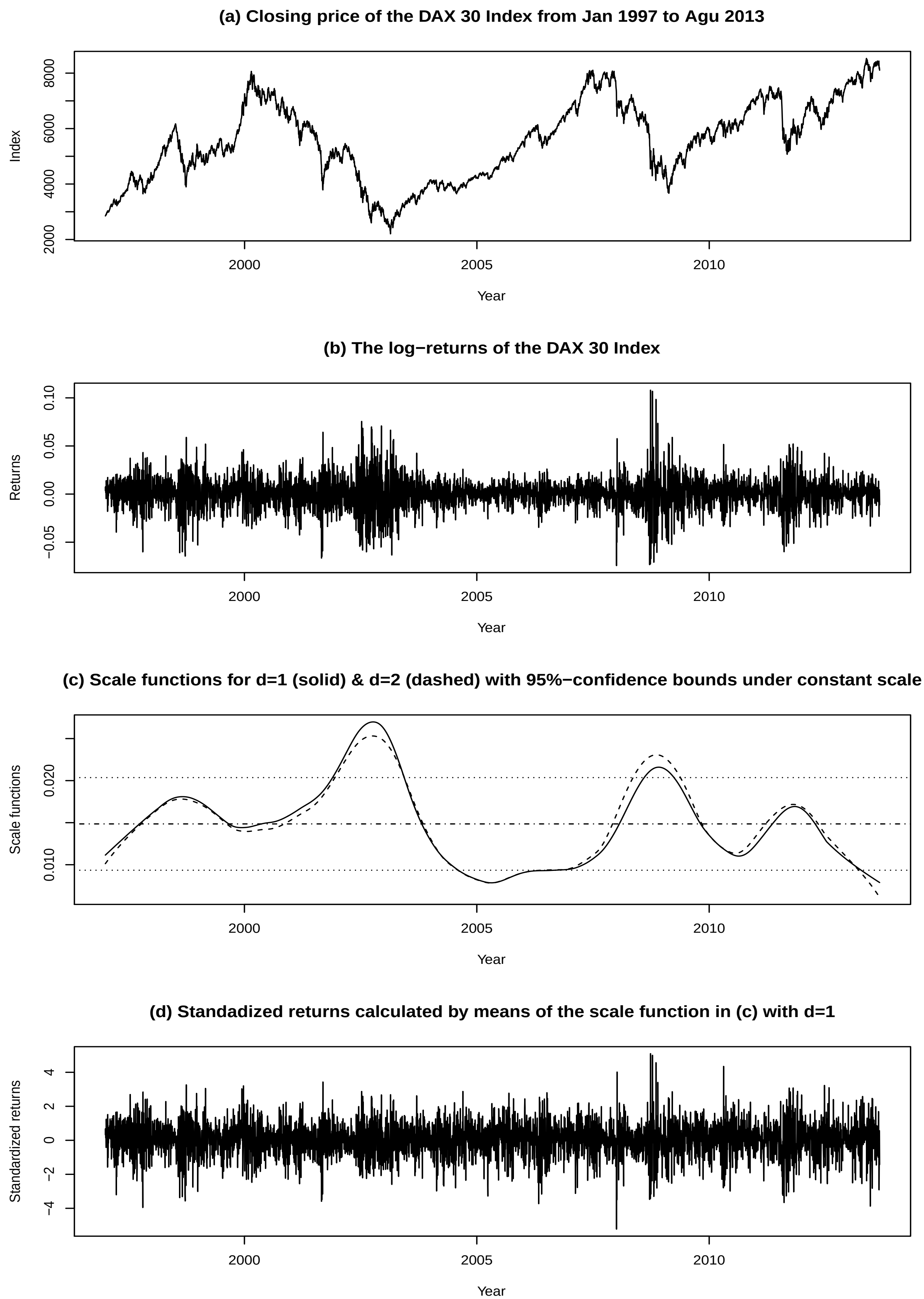

Figure 4: The smoothing results for DAX 30 Index from January 1997 to Augst 2013

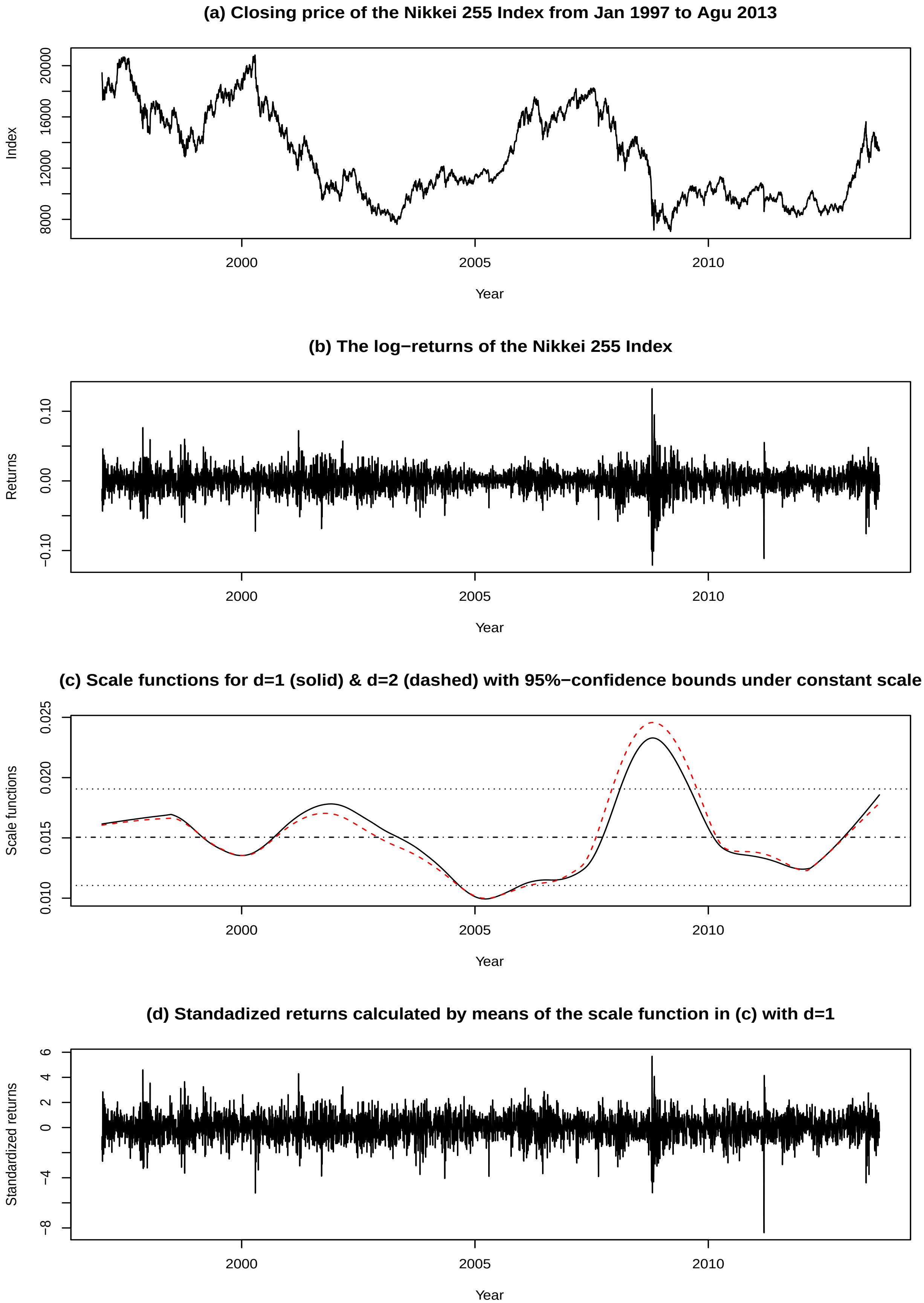

Figure 5: The smoothing results for Nikkei 225 Index from January 1997 to Augst 2013

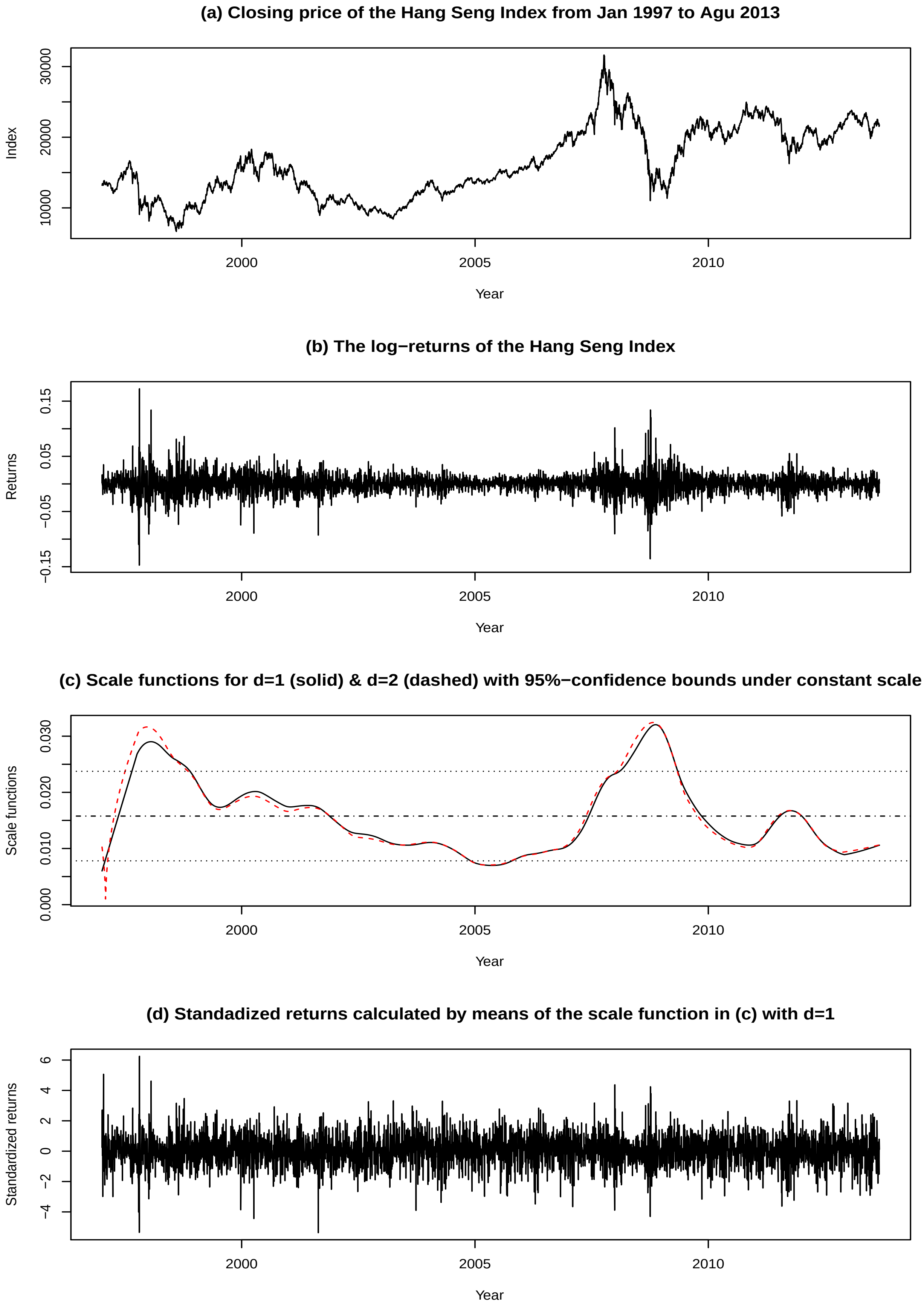

Figure 6: The smoothing results for Hang Seng Index from January 1997 to Augst 2013

Table 1: Numbers of observations and selected (relative) bandwidths in all cases

	SHI	SZI	S&P	DAX	NIK	HSI
n	4028	4028	4193	4239	4091	4153
$d=1$	0.101	0.101	0.075	0.069	0.089	0.045
$d=2$	0.100	0.107	0.075	0.071	0.093	0.047

Table 2: Correlations between the estimated scales functions

Case	SHI/SZI	SHI/S&P	SHI/DAX	SHI/NIK	SHI/HSI	SZI/S&P	SZI/DAX	SZI/NIK
corr.	0.962	0.505	0.081	0.542	0.609	0.443	-0.001	0.535
(n_{ij})	(4028)	(3894)	(3947)	(3833)	(3921)	(3894)	(3947)	(3833)
Case	SZI/HSI	S&P/DAX	S&P/NIK	S&P/HSI	DAX/NIK	DAX/HSI	NIK/HSI	
corr.	0.579	0.794	0.834	0.753	0.619	0.489	0.733	
(n_{ij})	(3921)	(4135)	(3958)	(4045)	(4012)	(4115)	(3924)	

Table 3: Estimated coefficients of the ARPARCH-$t(1, 1)$ models for all indexes

	SHI		SZI		S&P		DAX		NIK		HSI	
	coeff	s.e.	coeff	s.e.	coeff	s.e.	coeff	s.e.	coeff	s.e.	coeff	s.e.
α_0	0.053	0.011	0.055	0.011	0.043	0.006	0.054	0.008	0.054	0.009	0.068	0.012
α_1	0.095	0.011	0.112	0.0121	0.076	0.006	0.073	0.008	0.075	0.009	0.051	0.006
γ_1	0.376	0.073	0.318	0.060	1.000	0.006	0.995	0.131	0.735	0.113	1.000	0.011
β_1	0.876	0.016	0.859	0.017	0.897	0.010	0.888	0.012	0.886	0.013	0.892	0.016
shape	5.118	0.431	6.370	0.621	10.00	1.395	10.00	1.242	10.00	1.206	8.306	1.076

Table 4: BIC of selected APARCH(1, 1) models for all indexes

	δ free				$\delta = 1$				selected	estimated
cond-dis	sdt	ssdt	ged	sged	sdt	ssdt	ged	sged	cond-dis	δ
SHI	2.6962	2.6958	2.6956	2.6942	2.6942	2.6937	2.6936	2.6921	sged	1
SZI	2.6977	2.6859	2.7022	2.6884	2.6957	2.6838	2.7002	2.6864	sstd	1
S&P	2.6845	2.6789	2.6843	2.6766	2.6825	2.6769	2.6823	2.6746	sged	1
DAX	2.7154	2.7116	2.7139	2.7096	2.7139	2.7102	2.7126	2.7083	sged	1
NIK	2.7229	2.7232	2.7240	2.7238	2.7217	2.7219	2.7229	2.7227	std	1
HSI	2.7624	2.7641	2.7553	2.7569	2.7605	2.7622	2.7535	2.7552	ged	1

Table 5: Estimated coefficients of the selected models for all indexes ecept for NIK

	SHI		SZI		S&P		DAX		HSI	
	coeff	s.e.	coeff	s.e.	coeff	s.e.	coeff	s.e.	coeff	s.e.
α_0	0.053	0.011	0.052	0.010	0.046	0.006	0.056	0.008	0.070	0.0128
α_1	0.097	0.011	0.117	0.012	0.078	0.007	0.072	0.008	0.052	0.006
γ_1	0.330	0.069	0.302	0.055	1.000	0.006	0.962	0.126	1.000	0.012
β_1	0.874	0.016	0.860	0.016	0.894	0.010	0.887	0.012	0.889	0.016
skew	0.932	0.015	0.850	0.019	0.881	0.017	0.903	0.018	—	—
shape	1.231	0.037	6.658	0.670	1.548	0.050	1.628	0.053	1.405	0.046