Basic Income and Labor Supply: The German Case

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Abstract This paper deals with the effects of implementing a basic income on the labor supply side. The German welfare as well as tax and social contributions system are investigated. The results clarify that the abolishment of the so-called unemployment trap due to a basic income policy is a decisive advantage of this approach. In order to demonstrate possible labor supply side reactions to a basic income policy, we use the neoclassical labor supply model and adapt it for our purposes. We compare the effects of implementing a basic income on different types of employees concerning their consumption preferences. We show that, even in the neoclassical labor supply model without intrinsic work motivation, the basic income increases the participation rate in the labor market. Furthermore, current employees are partially incited to increase their labor supply. Therefore, a basic income would not only reduce unemployment but could also expand the magnitude of employment.

Keywords Basic Income · Neoclassical Labor Supply · Unemployment Trap

JEL Classification J2 · J6

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1 Introduction

Germany, a highly developed social market economy, is still confronted with the issues of poverty, inequality, and unemployment. The risk of poverty rate accounted for 15.6 percent and the unemployment rate for 7.7 percent in 2010 (Statistisches Bundesamt 2011, Statistisches Bundesamt 2012). In this context, the ongoing dispute about the effectiveness of the existing conditional social welfare system in Germany brought an already widely known proposal of a basic income as an alternative to Hartz IV in economic as well as in political circles back to life.

The idea of a basic income itself arouses controversial standpoints. On the one hand, it is seen as an effective strategy to cope with poverty and unemployment (Van Parijs 2004, Standing 2008); on the other hand, the basic income approach has been heavily criticized. The advocates of the approach emphasize such advantages of a basic income as its *ex ante* character, simplicity, and transparency, as well as cost reduction in connection with the simplified procedures (Van Parijs 2001, Hohenleitner & Straubhaar 2008). The critics of a basic income mention huge financing costs and a possible decline of the GDP in connection with higher taxation rates necessary to finance the basic income schemes (Hauser 2006). Based on the economic theory of the *homo economicus*, it is also assumed that a basic income reduces the work incentives.¹ For a *homo economicus*, the motivation to perform any working activity is considered rather extrinsic or more precisely pecuniary (Petersen et al. 2011). In other words, work is considered as something rather unpleasant but necessary to provide a certain level of consumption whereas leisure is something desirable. The results drawn from this argumentation are that having a basic income complying with necessary standards of living an individual may refrain from seeking a job, if unemployed, or withdraw from the labor market, if employed. In this context, Hauser (2006) expects a sharp decline of the labor supply especially among such groups as long-term unemployed, women looking after children, elder employees, and possibly young entrants of the labor market. In addition, he claims that higher tax rates in connection with a basic income could possibly induce employees to leave the labor market.

The present paper investigates the possible outcomes of implementing a basic income

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¹The core of this theory is a rational individual acting in such a way as to maximize his or her own utility.
scheme on the labor market. First of all, the existing guaranteed minimum income system, Hartz IV, will be compared to a basic income approach. The Hartz IV system is strongly conditional, namely it depends on the number of family members, the level of earned or unearned income, the ability or willingness to work, etc. Any additionally earned or unearned income will be considered and subtracted from the amount of social transfers. Particularly in the low-wage segment, there would be almost no difference in the income for an individual irrespectively of the fact whether he or she works or just receives social benefits. The important advantage of a basic income approach is that the monetary incentive to participate in the working activity should not be destroyed if individuals are able to keep the full amount of the basic income irrespectively of the income earned (Van Parijs 2004). The theoretical concept of the basic income has been translated into a variety of practical proposals with different amounts of transfers and different financing schemes. With respect to Germany, such models as “Grüne Grundauslage” of the Green Party of Germany, “Solidarisches Bürgergeld” of Dieter Althaus, “Grundeinkommen” of Götz Werner, and “HWWI Model” of Thomas Straubhaar have been developed.\footnote{For further information, see Emmler & Poreski (2006), Althaus (2011), Werner (2012), and Hohenleitner & Straubhaar (2008).} In the present paper, the latter will be applied due to its standardized and transparent financing scheme.

The third section considers a basic income within the framework of the neoclassical labor supply model from both, analytical and graphical, perspectives. Petersen et al. (2011) have compared the labor supply in the neoclassical environment first without any kind of welfare system and then after the implementation of a basic income, concluding that a basic income should lead to a reduction in the labor supply. However, our findings, derived from the comparison of the existing social welfare system and a basic income approach, do not only demonstrate that a basic income does not necessarily cause an increase in unemployment but, on the contrary, may stimulate the labor market participation.

We conclude by summarizing the results from both aspects of our analysis, the practical, i.e. the comparison of Hartz IV and the basic income proposal of the HWWI, and the theoretical, i.e. the consideration of the basic income in the neoclassical framework.

\footnote{HWWI means Hamburger WeltWirtschaftsInstitut.}
The current German social welfare system has been heavily criticized for its counterproductive effects on the labor market. The biggest concerns are connected with the regulations for the additional earnings (Sinn 2004). Recipients of the social benefits involved in working activities are confronted with a reduction in the amounts of transfers. As a consequence, the work performed in addition to the social benefits does not necessarily increase the amount of the final income. In this context, the unemployment trap leads to lower work incentives and thus to reduced labor supply (Schramm 2008).

In what follows, we compare the Hartz IV system to the basic income proposal of the HWWI. For the sake of comparability, variant 1B of the approach is considered, consisting of a monthly net payment of 600 euro and a monthly voucher for health and nursing insurance of 200 euro. According to the proposal, the basic income does assist in tackling the problem of the state deficit. Consequently, the necessary income tax is determined under the assumption that the future state deficit is equal to zero.

In the current system, the difference between the gross wage and the net income is explained by the income and the solidarity tax as well as by the social contributions for health, nursing, pension, and unemployment insurance. A Hartz IV recipient faces transfer withdrawal rates from 80 to 100 percent. In the basic income proposal of the HWWI, there is a flat income tax of 61 percent and a lump sum tax of 2400 euro per capita and year for social contributions. In what follows, we limit our analysis to a single parent with two children. Child benefits amount to 4416 euro, Hartz IV plus housing benefit amount to 18168 euro, and the net basic income amount to 21600 euro.

In Figure 1, the net income as a function of the gross wage for a single parent with two children is illustrated. In the case of the “Hartz IV Recipient”, the curve starts at a net income of 18168 euro and is nearly flat. It touches the blue one at a gross wage (net income) of 26524 (22128) euro. Earning a greater gross wage, one is worse off being a recipient than a denier. In the “HWWI Proposal”, the curve starts slightly above the red one at a net income of 21600 euro and increases linearly. It cuts the blue one at a gross wage.

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4See Bundesagentur für Arbeit (2012c); (2 · 184 · 12 euro).
5See Bundesagentur für Arbeit (2012b) and Senatsverwaltung für Gesundheit und Soziales (2012); maximum child benefits (1 · 374 · 12 + 2 · 299 · 12 euro), adequate housing in capital (6504 euro).
6See Hohenleitner & Straubhaar (2008, 33); variant 1B (3 · 600 · 12 euro).
wage (net income) of 99798 (60521) euro. In the “Hartz IV Denier” case, the curve starts at a net income of 4416 euro and increases sharper than the green one.

![Figure 1: German Unemployment Trap](image)

In the current system, there is obviously an unemployment trap. On the one hand, there is nearly no work incentive within the Hartz IV system. One could obtain an additional net income of 3960 euro by earning a gross wage greater or equal to 18000 euro. On the other hand, for a single parent with two children, there is nearly no chance to leave the Hartz IV system. He or she would need a net income greater or equal to 22128 euro which is above the mean (21500 euro) and the median (19424 euro) net income of single parents.\(^7\) Not surprisingly, the risk of poverty is the greatest for households with one adult and children (43 percent) and about three times the average (15.6 percent).\(^8\) The situation gets worse with the number of children since the chance to leave the Hartz

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\(^8\)See Statistisches Bundesamt (2012, 19).

IV system decreases respectively the needed net income increases. In the basic income approach, the unemployment trap is avoided by the linearly increasing and not nearly flat gross-wage-net-income-curve from the first earned euro on.

In Figure 2, the average combined tax and social contributions rate as a function of the gross wage for a single parent with two children is illustrated. In the case of the “Hartz IV Recipient”, the curve increases steeply, touching the blue one at a gross wage of 26524 euro. In the “HWWI Proposal”, the curve increases concavely, converging to 61 percent, cutting the blue one at a gross wage of 99798 euro. In the “Hartz IV Denier” case, the average combined tax and social contributions rate is in the range of 17 to 47 percent from a gross wage of 26524 euro on. The curve increases similarly to but flatter than the green one.

![Graph showing average combined tax and social contributions rate as a function of gross wage](image)

Figure 2: Average Combined Tax and Social Contributions Rate

One implication is that the current system is less progressive than the basic income approach. With two children, the average burden is higher up to a gross wage of 99798 euro and lower afterwards. Therefore, the basic income approach is redistributive. Another implication is that the current system is hardly progressive from a certain gross wage on. Without children, the average burden increases from 40 percent at a gross wage of 44234 euro only to 47 percent at an arbitrary high one.
In Figure 3, the marginal combined tax and social contributions rate as a function of the gross wage is illustrated. In the case of the “Hartz IV Recipient”, there is a step function, starting with zero, proceeding with 80 and 90, and closing with 100 percent. In the “HWWI Proposal”, the curve is flat at a rate of 61 percent. In the “Hartz IV Denier” case, the marginal combined tax and social contributions rate is in the range of 44 to 54 percent from a gross wage of 16779 euro on. The curve reaches its maximum at a gross wage from 60963 euro, the point from which on the income tax is linear, to 67200 euro, the point from which on all social contributions are capped.

![Graph showing marginal combined tax and social contributions rate](image.png)

Figure 3: Marginal Combined Tax and Social Contributions Rate

In the current system, there are apparently random jumps as well as ups and downs in the marginal burden. It is the highest for medium gross wages and within a relatively narrow bandwidth for the relevant gross wage range. The high transfer withdrawal rates lead to low net hourly wages within the conditional welfare system. In the basic income approach, the marginal burden is greater for Hartz IV deniers but less for Hartz IV recipients than in the current system. On the one hand, the unconditional welfare system requires higher public revenues; on the other hand, it abolishes the transfer withdrawal rates. The net hourly wage in the basic income approach accounts for 70 to 84 percent of the one in the current system for gross wages larger than 26524 euro.
3 The Neoclassical Labor Supply Model

In this section, we apply the neoclassical labor supply model to the German tax and social contributions system as well as to the basic income approach. In what follows, we apply two insights from the last section. On the one hand, it could be shown that there is a severe unemployment trap. Caught inside, there is nearly no work incentive because of the high transfer withdrawal rates. We thus assume that one either earns something outside the conditional social security system or does not earn anything at all. On the other hand, it could be shown that the marginal burden is within a relatively narrow bandwidth for the relevant gross wage range. In the basic income approach, it remains the same for all gross wages. For reasons of comparability and simplicity, we assume a constant marginal burden and thus a constant net hourly wage within each welfare system.

In the neoclassical labor supply model, each individual maximizes his or her utility with respect to his or her budget constraint. In what follows, we limit our analysis to Cobb-Douglas utility functions with individual exponents (see equation (1)). Without a welfare system, the budget constraints are identical except for their net wages (see equation (2)). The result is an optimized leisure consumption which decreases (increases) with the individual consumption (leisure) affinity (see equation (3)):

$$u_i = U^i(c_i, l_i) = c_i^{\alpha_i} \cdot l_i^{\beta_i},$$

$$c_i = \frac{w_i}{p} \cdot (\bar{t} - l_i),$$

$$l_i^* = \frac{\beta_i}{\alpha_i + \beta_i} \cdot \bar{t},$$

where $u_i$ is the utility of individual $i$, $c_i$ is the commodity good consumption of individual $i$, $\alpha_i$ is the consumption affinity of individual $i$, $l_i$ is the leisure consumption of individual $i$, $\beta_i$ is the leisure affinity of individual $i$, $w_i$ is the net wage of individual $i$, $p$ is the price level, $\bar{t}$ is the exogenously given time, and $l_i^*$ is the optimized leisure consumption of individual $i$.

Introducing a conditional welfare system, the budget constraints either remain the same, for individuals who prefer to be employed, or are equal to the conditional social security, otherwise (see equation (4)). The optimized leisure consumption reduces to the

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10For CES utility functions, see appendix A.2.
exogenously given time if the net wage and the consumption affinity are relatively low and if the leisure affinity is relatively high (see equation (5) with appendix, equations (A.7) and (A.8)).

\[
c_i = \begin{cases} 
  \frac{w_i}{p} \cdot (\overline{t} - l_i) & \text{if } \overline{t} - l_i > 0 \\
  \frac{z}{p} & \text{else}
\end{cases}, 
\]

(4)

\[
l_i^* = \begin{cases} 
  \frac{\beta_i}{\alpha_i + \beta_i} \cdot \overline{t} & \text{if } s \leq s_i^* = w_i \cdot \frac{\alpha_i}{\beta_i} \cdot \left(\frac{\alpha_i}{\beta_i} + 1\right)^{-\frac{\alpha_i}{\beta_i + 1}} \cdot \overline{t} \\
  \overline{t} & \text{else}
\end{cases}, 
\]

(5)

where \( s \) is the conditional social security.

Introducing an unconditional welfare system, the budget constraints change. Again, they are identical except for their net wages but now, they are supplemented by the unconditional basic income (see equation (6)). The optimized leisure consumption reduces to the exogenously given time if the net wage and the consumption affinity are relatively low and if the leisure affinity is relatively high (see equation (7)).

\[
c_i = \frac{w_i}{p} \cdot (\overline{t} - l_i) + \frac{b_i}{p}, 
\]

(6)

\[
l_i^* = \begin{cases} 
  \frac{\beta_i}{\alpha_i + \beta_i} \cdot (\overline{t} + \frac{b_i}{w_i}) & \text{if } b_i \leq b_i^* = w_i \cdot \frac{\alpha_i}{\beta_i} \cdot \overline{t} \\
  \overline{t} & \text{else}
\end{cases}, 
\]

(7)

where \( b \) is the unconditional basic income.

The working condition in equation (5) is tighter than that in equation (7) if the ratio between the net wages is greater than 0.37, no matter what size the consumption and the leisure affinities have (see appendix, equation (A.14)). This means that, under reasonable assumptions, the unconditional basic income increases the participation rate in the labor market in our adapted neoclassical labor supply model.

In Figure 4, the neoclassical labor supply model without social security is drawn. The commodity good consumption \( (c_i) \) is displayed on the vertical axis, the leisure consumption \( (l_i) \) on the horizontal axis. The budget constraint starts from the maximum possible leisure consumption \( (\overline{t}) \). Its slope corresponds to the real net wage. It cuts the vertical axis at the maximum possible commodity good consumption \( (\overline{c}_i) \). We observe three individuals with a high (low) \((A)\), a medium (medium) \((B)\), and a low (high) \((C)\) consumption.
(leisure) affinity. They choose their optimal consumption-leisure-combinations on the budget constraint \((A, B, C)\), depending on their preferences. These are indicated by the shapes of their indifference curves, on which each individual’s utility level is constant.\(^{12}\)

![Figure 4: No Social Security](image)

**Figure 4: No Social Security**

In **Figure 5**, the neoclassical labor supply model without social security (gray) and with unconditional basic income (black) is drawn. After the introduction of the unconditional basic income, for all individuals, which preferred a consumption-leisure-combination left of the budget constraints’ intersection \((A)\), the substitution effect is positive while the income effect is negative and the overall effect is ambiguous. Else \((B, C)\), the substitution effect and the income effect are negative and the overall effect is positive for leisure. The implication is that in the neoclassical labor supply model the introduction of an unconditional basic income would probably lead to a decrease in the overall labor supply if there was no social security before.

\(^{11}\)For reasons of simplicity, we keep the nominal gross wage constant.

\(^{12}\)Algebraically, the indifference curves are derived by rearranging the utility functions on the commodity good consumptions and keeping the utility levels constant. In our example, this is \(c_i = \pi^{1/\alpha_i} l_i^{\beta_i/\alpha_i}\).
Indeed, in the neoclassical model world, the absence of a social security system would provide the greatest work incentive. Almost everyone would have to work for earning an income. However, in Germany, the basic right for a minimum subsistence level is guaranteed (Bundesverfassungsgericht 2010). This means that a social security system has to exist. In the following, we therefore consider the basic income approach as a (feasible) alternative to the prevailing conditional and not to a nonexistent social security system.

In Figure 6, the neoclassical labor supply model without social security (gray) and with conditional social security (black) is drawn. Based on the results of section 2, the new budget constraint consists of two parts. On the one hand, there is the straight line for those who are employed. It is flatter than the original budget constraint. This is the case because the social security is funded by the employees, either by direct taxes, which decrease their nominal net wage, or by indirect taxes, which increase the price level, both decreasing their real net wage. The straight line cuts the vertical axis at the new, lower maximum possible commodity good consumption \((\bar{c}_i')\). On the other hand, there is the straight line for those who are unemployed. It runs flat and cuts the vertical axis at

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13According to article 1, passage 1 of the “Grundgesetz” in conjunction with article 20, passage 1 of the “Grundgesetz” there is the claim for a humane minimum subsistence level.
the social security respectively the minimum commodity good consumption \( (c_i) \). This is a simplification since there are earning possibilities for an unemployed in a conditional welfare system. But based on the results of section 2, it is irrational to make use of these, at least from a neoclassical point of view.

It is interesting to see how the three individuals adapt their consumption-leisure-combinations to the introduction of the conditional social security. Therefore, the substitution effect \( (SE) \) and the income effect \( (IE) \) are introduced. Since the social security is funded by the employees, on the one hand, consumption (leisure) is relatively more (less) expensive than before.\(^\text{14}\) This effect leads to a substitution of consumption for leisure. On the other hand, income is lower than before.\(^\text{15}\) This effect leads to a reduction of consumption and leisure. Altogether, the overall effect \( (OE) \) is negative for consumption and ambiguous for leisure.

Therefore, in Figure 6, the consumption-leisure-combinations \( A \) and \( B \) shift downwards and keep their horizontal positions in order to indicate that they could either shift leftwards or rightwards. An exemption is the consumption-leisure-combination \( C \). The

\(^{14}\)There are two definitions of this effect, see Hicks & Allen (1934) and Slutsky (1915).

\(^{15}\)If an individual worked as much as before, it could not consume as much as before.
income of all individuals, which preferred a commodity good consumption less than the minimum commodity good consumption before the introduction of the social security, is higher than before and remains the same whether leisure consumption is increased or not. It is rational for these individuals to substitute consumption for leisure until they do no longer participate in the labor market.\footnote{Even individuals who preferred a higher commodity good consumption before could now prefer to be unemployed.}

In \textbf{Figure 7}, the neoclassical labor supply model with conditional social security (gray) and with unconditional basic income (black) is drawn. For reasons of simplicity, the basic income is equal to the social security respectively the minimum commodity consumption. The new budget constraint is even flatter than before since the funding of the basic income is more expensive than that of the social security. Therefore, it cuts the vertical axis at the new, even lower maximum possible commodity good consumption ($c_i''$). Unlike before, it never runs flat.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{conditional_social_security_unconditional_basic_income.png}
\caption{Conditional Social Security $\rightarrow$ Unconditional Basic Income}
\end{figure}

After the detachment of the conditional welfare system, for all individuals, which preferred a consumption-leisure-combination left of the budget constraints' intersection ($A$), the substitution effect is positive while the income effect is negative and the overall effect is ambiguous. Else ($B$), the substitution effect and the income effect are negative and the
overall effect is positive for leisure. The important exemptions are the individuals who did not participate in the labor market before \((C)\). On the one hand, they can increase their commodity good consumption if they decrease their leisure consumption marginally. On the other hand, they can consume as much as before if they keep their leisure consumption constant. Therefore, for them the substitution effect is negative while there is no income effect and the overall effect is either nil or negative for leisure.

In conclusion, the detachment of the conditional welfare system can lead to an increase in the labor supply of high consumption affine individuals, does lead to a decrease in the labor supply of medium consumption affine individuals and does increase the labor participation rate in our adapted neoclassical labor supply model.\(^\text{17}\)

4 Concluding Remarks

The shortcomings of the current German welfare system could be demonstrated in section 2. The present framework of Hartz IV does hardly provide any work incentives. With respect to individuals with dependent children, the situation gets even more precarious as there is just little chance to escape from poverty and leave the social benefit system. The basic income approach is a sound solution to the issue of the unemployment trap, which is an inherent part of the Hartz IV system. In the case of the basic income, the gross-wage-net-income-curve increases linearly and not nearly flat as in the Hartz IV case from the first euro of the earned income. From the foregoing it follows that work incentives are not destroyed and the unemployment trap can successfully be avoided.

Another property of the basic income approach is its rather progressive tax and social contributions system which accounts for its redistributive character. Consequently, a low income segment is better off and a high income segment is worse off as compared to the Hartz IV system, which can be hardly characterized as progressive from a certain level of gross wage on. The same can be observed if the marginal combined tax and social

\(^{17}\)Concerning the last proposition, the labor participation rate could decrease if more medium consumption affine individuals leave than low consumption affine individuals enter the labor market (compare appendix, equation (A.14)). Remember that the medium consumption affine individuals did not leave the labor market after the introduction of the conditional social security. Therefore, the substitution effect had to be very high and the income effect had to be very low if they now preferred to do so.
contributions rate is considered.

The summarized effects of all options considered within the neoclassical framework in section 3, namely the change from no social security system to an unconditional basic income, from no social security system to a conditional one, and from a conditional social security system to an unconditional basic income, are presented in Table 1.

Table 1: Effects on the Labor Supply

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<td>IE</td>
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<td>NSS → UBI</td>
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<td>NSS → CSS</td>
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<tr>
<td>CSS → UBI</td>
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In the environment of the neoclassical labor supply model, the replacement of the conditional welfare system by the unconditional basic income approach can lead to various effects, depending on the consumption affinities of the individuals. Individuals who work much may increase, those who work little will decrease, and those who do not work at all will increase their labor supply. The mathematical approach supports the graphical results in the aspect that the overall labor participation rate will increase.

Altogether, the detachment of the current conditional social security by an unconditional basic income is likely to have a great influence on the labor market. Some employees will for sure reduce their labor supply as they can consume more and work less than before. But, due to the abolishment of the unemployment trap, it is very likely that labor participation rate will increase. Therefore, an unconditional basic income provides a stronger stimulus for the low income sector than often considered.

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18Where A, B, and C stands for an individual with a high, medium, and low consumption affinity, respectively, IE means income effect, SE means substitution effect, OE means overall effect, NSS means no social security, CSS means conditional social security, UBI means unconditional basic income, and −, O, +, and −+ stands for a negative, nonexistent, positive, and ambiguous effect on the labor supply, respectively.
A Appendix

A.1 Cobb-Douglas utility functions

Inserting equation (2) in equation (1) and afterwards differentiating with respect to $l_i$ yields:

$$ u_i = \left( \frac{w_i}{p} \cdot (\bar{t} - l_i) \right)^{\alpha_i} \cdot t_i^{\beta_i}, $$

(A.1)

$$ \frac{\partial u_i}{\partial l_i} = \frac{w_i}{p} \cdot \left( \frac{w_i}{p} \cdot (\bar{t} - l_i) \right)^{\alpha_i - 1} \cdot t_i^{\beta_i - 1} \cdot (\beta_i - (\alpha_i + \beta_i)l_i)^{\frac{1}{2}}, $$

(A.2)

$$ \Rightarrow t_i^* = \frac{\beta_i}{\alpha_i + \beta_i} \cdot \bar{t}, $$

(A.3)

$$ c_i^* = \frac{w_i}{p} \cdot \frac{\alpha_i}{\alpha_i + \beta_i} \cdot \bar{t}, $$

(A.4)

$$ u_i^* = \left( \frac{w_i}{p} \right)^{\alpha_i} \cdot \frac{\alpha_i \beta_i^{\beta_i}}{(\alpha_i + \beta_i)^{\alpha_i + \beta_i}} \cdot \bar{t}^{\alpha_i + \beta_i}. $$

(A.5)

Inserting $s$ and $\bar{t}$ in equation (1) and afterwards equating the utility levels yields:

$$ u_i^* = \begin{cases} 
\left( \frac{w_i}{p} \right)^{\alpha_i} \cdot \frac{\alpha_i \beta_i^{\beta_i}}{(\alpha_i + \beta_i)^{\alpha_i + \beta_i}} \cdot \bar{t}^{\alpha_i + \beta_i}, & \text{if } s \leq s_i^* = w_i \cdot \frac{\alpha_i}{\beta_i} \cdot \left( \frac{\alpha_i}{\beta_i} + 1 \right) \cdot \left( -\frac{\beta_i}{\alpha_i + 1} \right)^{\frac{1}{2}}, \\
\left( \frac{s}{p} \right)^{\alpha_i} \cdot \bar{t}^{\beta_i}, & \text{else}
\end{cases} $$

(A.6)

Differentiating the working condition in equation (5) with respect to $\alpha_i$ and $\beta_i$ yields:

$$ \frac{\partial s_i^*}{\partial \alpha_i} = -w_i \cdot \frac{1}{\alpha_i} \cdot \left( \frac{\beta_i}{\alpha_i + \beta_i} \right)^{\alpha_i + \beta_i} \cdot \ln \left[ \frac{\beta_i}{\alpha_i + \beta_i} \right] \cdot \bar{t} > 0, $$

(A.7)

$$ \frac{\partial s_i^*}{\partial \beta_i} = w_i \cdot \frac{1}{\beta_i} \cdot \left( \frac{\alpha_i}{\alpha_i + \beta_i} \right)^{\alpha_i + \beta_i} \cdot \ln \left[ \frac{\beta_i}{\alpha_i + \beta_i} \right] \cdot \bar{t} < 0. $$

(A.8)

Inserting equation (6) in equation (1) and afterwards differentiating with respect to $l_i$ yields:

$$ u_i = \left( \frac{w_i}{p} \cdot (\bar{t} - l_i) + \frac{b}{w_i} \right)^{\alpha_i} \cdot t_i^{\beta_i}, $$

(A.9)

$$ \frac{\partial u_i}{\partial l_i} = \frac{w_i}{p} \cdot \left( \frac{w_i}{p} \cdot (\bar{t} - l_i) + \frac{b}{w_i} \right)^{\alpha_i - 1} \cdot t_i^{\beta_i - 1} \cdot \left( \beta_i \left( \bar{t} + \frac{b}{w_i} \right) - (\alpha_i + \beta_i)l_i \right)^{\frac{1}{2}}, $$

(A.10)

$$ \Rightarrow (\bar{t}^*) = \frac{\beta_i}{\alpha_i + \beta_i} \cdot \left( \bar{t} + \frac{b}{w_i} \right) \leq \bar{t} \quad \text{if } b \leq b_i^* = \frac{\alpha_i}{\beta_i} \cdot \bar{t}, $$

(A.11)

$$ c_i^* = \frac{w_i}{p} \cdot \frac{\alpha_i}{\alpha_i + \beta_i} \cdot \left( \bar{t} + \frac{b}{w_i} \right), $$

(A.12)

$$ u_i^* = \begin{cases} 
\left( \frac{w_i}{p} \right)^{\alpha_i} \cdot \frac{\alpha_i \beta_i^{\beta_i}}{(\alpha_i + \beta_i)^{\alpha_i + \beta_i}} \cdot \left( \bar{t} + \frac{b}{w_i} \right)^{\alpha_i + \beta_i}, & \text{if } b \leq b_i^* = \frac{\alpha_i}{\beta_i} \cdot \bar{t}, \\
\left( \frac{b}{p} \right)^{\alpha_i} \cdot \bar{t}^{\beta_i}, & \text{else}
\end{cases} $$

(A.13)
Equating the working conditions in equation (5) and in equation (7) yields:

\[
\frac{w_i(b)}{w_i(s)} \cdot \frac{\alpha_i}{\beta_i} \cdot t \leq \frac{w_i(s)}{w_i(b)} \cdot \frac{\alpha_i}{\beta_i} \cdot \left( \frac{\alpha_i}{\beta_i} + 1 \right)^{-\left( \frac{\alpha_i}{\beta_i} + 1 \right)} \cdot t,
\]

\[
\Leftrightarrow \frac{w_i(b)}{w_i(s)} \cdot \left( \frac{\alpha_i}{\beta_i} + 1 \right)^{-\left( \frac{\alpha_i}{\beta_i} + 1 \right)} < 1,
\]

\[
\frac{\partial u_i}{\partial \frac{w_i(b)}{w_i(s)}} = \frac{\beta_i}{\alpha_i} \cdot \left( \frac{\beta_i}{\alpha_i} \cdot \ln \left( \frac{\alpha_i}{\beta_i} + 1 \right) - 1 \right) \cdot \left( \frac{w_i(b)}{w_i(s)} \right) < 0.
\]

\[
(A.14)
\]

Figure 8: Conditions for an Increase in the Labor Participation Rate (C-D)\(^{19}\)

A.2 CES utility functions

\[
u_i = U^*(c_i, l_i) = \left( \alpha_i c_i^d + \beta_i l_i^d \right)^{\frac{1}{d}}, \quad 0 \neq d \leq 1
\]

\[
(A.15)
\]

Inserting equation (2) in equation (A.15) and afterwards differentiating with respect to \(l_i\) yields:

\[
u_i = \left( \alpha_i \left( \frac{w_i}{p} \cdot (t - l_i) \right)^d + \beta_i l_i^d \right)^{\frac{1}{d}},
\]

\[
(A.16)
\]

\[
\frac{\partial u_i}{\partial l_i} = \cdot \left( \alpha_i \left( \frac{w_i}{p} \cdot (t - l_i) \right)^d + \beta_i l_i^d \right)^{\frac{1}{d} - 1} \cdot \left( \beta_i l_i^{d-1} - \frac{w_i}{p} \cdot \alpha_i \left( \frac{w_i}{p} \cdot (t - l_i) \right)^{d-1} \right) = 0.
\]

\[
(A.17)
\]

\(^{19}\)The labor participation rate is larger in the unconditional basic income case than in the conditional social security case assuming that \(w_i(b)/w_i(s) = (1 - 0.61)/(1 - 0.4431)\).
\[ l_t^* = \frac{\alpha_i^{d-1} \left( \frac{w_i}{p} \right)^{d-1}}{\beta_i^{d-1} + \alpha_i^{d-1} \left( \frac{w_i}{p} \right)^{d-1}} \cdot \bar{t}, \tag{A.18} \]

\[ c_i^* = \frac{w_i}{p} \cdot \frac{\beta_i^{1-1}}{\beta_i^{1-1} + \alpha_i^{1-1} \left( \frac{w_i}{p} \right)^{1-1}} \cdot \bar{t}, \tag{A.19} \]

\[ u_i^* = \left( \frac{1}{\beta_i} \left( \frac{\alpha_i^{1-1} \left( \frac{w_i}{p} \right)^{1-1}}{\beta_i^{1-1} + \alpha_i^{1-1} \left( \frac{w_i}{p} \right)^{1-1}} \right)^{d-1} \right)^{\frac{1}{d}} \cdot \bar{t}. \tag{A.20} \]

Inserting \( s \) and \( \bar{t} \) in equation (A.15) and afterwards equating the utility levels yields:

\[ u_i^* = \begin{cases} 
(\alpha_i c_i^{d} + \beta_i l_i^{d})^{\frac{1}{d}} & \text{if } s \leq s_i^* = p \cdot \left( \frac{\beta_i}{\alpha_i} \right)^{\frac{1}{d}} \cdot \left( \frac{\alpha_i^{1-1} \left( \frac{w_i}{p} \right)^{d-1}}{\beta_i^{1-1} + \alpha_i^{1-1} \left( \frac{w_i}{p} \right)^{d-1}} \right)^{d-1} - 1 \right)^{\frac{1}{d}} \cdot \bar{t}, \\
(\alpha_i \left( \frac{s}{p} \right)^{d} + \beta_i \bar{t}^{d})^{\frac{1}{d}} & \text{else} 
\end{cases} \tag{A.21} \]

Differentiating the working condition in equation (A.20) with respect to \( \alpha_i \) and \( \beta_i \) yields:

\[ \frac{\partial s^*}{\partial \alpha_i} = -\frac{1}{d} \cdot \frac{1}{\alpha_i} \left( 1 - \frac{\beta_i^{1-1}}{\beta_i^{1-1} + \alpha_i^{1-1} \left( \frac{w_i}{p} \right)^{1-1}} \cdot \left( \frac{\alpha_i^{1-1} \left( \frac{w_i}{p} \right)^{d-1}}{\beta_i^{1-1} + \alpha_i^{1-1} \left( \frac{w_i}{p} \right)^{d-1}} \right)^{d-1} \right) \cdot s_i^* \leq 0, \tag{A.22} \]

\[ \frac{\partial s^*}{\partial \beta_i} = \frac{1}{d} \cdot \frac{1}{\beta_i} \left( 1 - \frac{\beta_i^{1-1}}{\beta_i^{1-1} + \alpha_i^{1-1} \left( \frac{w_i}{p} \right)^{1-1}} \cdot \left( \frac{\alpha_i^{1-1} \left( \frac{w_i}{p} \right)^{d-1}}{\beta_i^{1-1} + \alpha_i^{1-1} \left( \frac{w_i}{p} \right)^{d-1}} \right)^{d-1} \right) \cdot s_i^* \leq 0. \tag{A.23} \]

Inserting equation (6) in equation (A.15) and afterwards differentiating with respect to \( l_i \) yields:

\[ u_i = \left( \alpha_i \left( \frac{w_i}{p} \cdot (\bar{t} - l_i) + \frac{b}{p} \right)^{d} + \beta_i l_i^{d} \right)^{\frac{1}{d}}, \tag{A.24} \]

\[ \frac{\partial u}{\partial l_i} = \left( \alpha_i \left( \frac{w_i}{p} \cdot (\bar{t} - l_i) + \frac{b}{p} \right)^{d} + \beta_i l_i^{d} \right)^{\frac{1}{d}-1} \cdot \left( \beta_i l_i^{d-1} - \frac{w_i}{p} \cdot \alpha_i \left( \frac{w_i}{p} \cdot (\bar{t} - l_i) + \frac{b}{p} \right)^{d-1} \right) \cdot 0, \tag{A.25} \]
\[ l_i^* = \frac{\alpha_i^1 \left( \frac{w_i}{p} \right)^{\frac{d-1}{2}}}{\beta_i^1 + \frac{\alpha_i^1}{\beta_i^1} \left( \frac{w_i}{p} \right)^{\frac{d-1}{2}}} \cdot \left( \bar{t} + \frac{b}{w_i} \right) \leq \bar{t} \quad \text{if} \quad b \leq b_i^* = \frac{1}{\beta_i} \cdot \left( \frac{\alpha_i}{\beta_i} \right)^{\frac{1}{d}} \cdot \bar{t}, \]

(A.26)

\[ c_i^* = \frac{w_i}{p} \frac{\beta_i^{\frac{1}{d}}}{\beta_i^{\frac{1}{d}} + \frac{\alpha_i^1}{\beta_i^1} \left( \frac{w_i}{p} \right)^{\frac{d-1}{2}}} \cdot \left( \bar{t} + \frac{b}{w_i} \right), \]

(A.27)

\[ u_i^* = \begin{cases} 
\left( \beta_i^1 \left( \frac{\alpha_i^{\frac{1}{d}} \left( \frac{w_i}{p} \right)^{\frac{d-1}{2}}}{\beta_i^{\frac{1}{d}} + \frac{\alpha_i^1}{\beta_i^1} \left( \frac{w_i}{p} \right)^{\frac{d-1}{2}}} \right) \right)^{\frac{d-1}{d}} \cdot \left( \bar{t} + \frac{b}{w_i} \right) & \text{if} \quad b \leq b_i^* = \frac{1}{\beta_i} \cdot \left( \frac{\alpha_i}{\beta_i} \right)^{\frac{1}{d}} \cdot \bar{t} \\
\left( \frac{\alpha_i}{\beta_i} \right)^{\frac{d}{d}} + \beta_i^1 d & \text{else} \end{cases} \]

(A.28)

Equating the working conditions in equation (A.21) and in equation (A.28) yields:

\[
\frac{w_i(b)}{w_i(s)} \cdot \left( \frac{\alpha_i}{\beta_i} \right)^{\frac{1}{d}} \cdot \bar{t} \geq \frac{p}{\alpha_i} \cdot \left( \frac{\alpha_i^{\frac{1}{d}} \left( \frac{w_i(s)}{p} \right)^{\frac{d-1}{2}}}{\beta_i^{\frac{1}{d}} + \frac{\alpha_i^1}{\beta_i^1} \left( \frac{w_i(s)}{p} \right)^{\frac{d-1}{2}}} \right)^{\frac{d-1}{d}} - 1 \cdot \bar{t},
\]

(A.29)

\[
\frac{w_i(b)}{w_i(s)} < 1 \quad \Rightarrow \quad \left( \frac{\alpha_i}{\beta_i} \right)^{\frac{1}{d}} \left( \frac{w_i(s)}{p} \right)^{\frac{d-1}{d}} \cdot \left( \frac{(\alpha_i^{\frac{1}{d}} \left( \frac{w_i(s)}{p} \right)^{\frac{d-1}{2}}}{\beta_i^{\frac{1}{d}} + \frac{\alpha_i^1}{\beta_i^1} \left( \frac{w_i(s)}{p} \right)^{\frac{d-1}{2}}} \right)^{\frac{d-1}{d}} + 1 \right)^{-1},
\]

Figure 9: Conditions for an Increase in the Labor Participation Rate (CES)

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20 The labor participation rate is larger in the unconditional basic income case than in the conditional social security case if \( d \leq 0.75 \Leftrightarrow \sigma \leq 4 \) assuming that \( w_i(b)/w_i(s) = (1 - 0.61)/(1 - 0.4431) \).
References


