A tree-form constant market share model for growth causes in international trade based on multi-level classification

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Abstract: This paper introduces a tree-form constant market share (CMS) model for analyzing growth causes in international trade based on multi-level classification. The tree-form CMS is a collection of CMS models at different levels, including the entire, branch- and leaf-models, which consists of a large amount of information and has a wide application spectrum. Basic properties of this model are investigated in detail. It is shown that the tree-form CMS model is superior to other CMS models in the literature. It is also shown that well known CMS formulations are special cases of a linear class with two parameters, which control how the interaction term is divided into the demand growth and competitive terms. Application to bilateral trade between China and Germany shows that the growth causes in different periods are clearly different. It is shown that the outputs of the tree-form CMS model can be used for further suitable statistical analysis. Furthermore, our theoretical findings are also confirmed by those data examples.

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Keywords: Tree-form CMS; international trade; growth causes; general CMS formulations

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1. Introduction

Constant market share (CMS) analysis is an accounting method for decomposing a country’s export change into the “demand growth” (or “structural change”), the “competitive” and the “interaction” components. This method, known as shift-share analysis, was first applied in the empirical studies of structural changes in industrial and regional economics (Creamer, 1943). Reviews on the development in these fields may e.g. be found in Houston (1967), Stevens and Moore (1980), and Loveridge and Selting (1998). CMS analysis became popular in applied international economics with the pioneering work of Tyszynski (1951) and it has been increasingly used and refined despite continued criticism both on its theoretical and empirical aspects (see e.g. Richardson, 1971a, Jepma 1986, Merkies and van der Meer, 1988 and Milana, 1988). In recent years, in addition to its application in international economics (see e.g. Chen et al., 2000, Simonis, 2000 and Iapadre, 2006) and regional economics (see e.g. Esteban, 2000 and Blien and Wolf, 2002), CMS or shift-share analysis is also widely applied in other fields, such as tourisms (see e.g. Sirakaya et al., 1995 and Toh et al., 2004), energy economics (see e.g. Sun, 1998, Zhang, 2003 and Mazzanti and Montini, 2009) and firm performance (see e.g. Fotopoulos, 2007 and Marini, 2010). Moreover, results of the CMS decomposition can also be used for further statistical analysis (Batista, 2008).

Despite long history and wide application of the CMS model, there are still many open questions in this context. A traditional question is with the definition of the three basic components, because they can be defined following the Laspeyres- or the Paasche-type index, or some mixture of both (Richardson, 1971a and Milana, 1988). Another question is with the further decomposition of a basic component. To this end, different ideas are proposed in the literature (see e.g. Jepma, 1986, 1989). Usually, the three components are decomposed in different ways according to the purpose of application. Furthermore, the traditional CMS model is defined for a cross-table with one observation in each entry, which cannot be applied directly, if multi-level classified data for bilateral trade is considered. For data classified at two levels, Toh et al. (2004) and Lu and Mei (2007) propose to fit a CMS model first only using data of the first-level categories and then to fit one CMS model to each first-level category using data of the second-level sub-categories. Guo et al. (2011) call this a hierarchical CMS model and discuss its theoretical background briefly, which only involves data at two levels and can be iteratively used at each level using data classified at the next level. The hierarchical CMS model is easy to understand and easy to use. But it seems that there is a lack of theoretical foundation of this proposal. In particular, for measuring the overall changes, data classified at the second level or above cannot be used, and for measuring
the change of a first-level category, data classified at the third level or above cannot be used.

To solve the above questions, we will introduce a tree-form CMS model based on multi-level classified data. Its special case for three-level classified data is a modification of the traditional CMS model in the current context. The first- and second-level branch-models and leaf-models are also defined by taking corresponding terms out of the entire tree-form CMS model, which include useful and detailed information. Theoretical properties of this model are discussed in detail. Interesting findings on the meaning and sources of the effects caused by classification at a given level, called the level-effects, are achieved, which show that under certain sufficient conditions some or all of the level-effects vanish. Particularly in the special case with only two categories, the obtained sufficient conditions such that the level-effects vanish are also necessary at the same time. Relationship between the current proposal and related ideas in the literature is also discussed. Concerning the definition of the decomposition of the CMS components at a given level, it is shown that all of the well known CMS formulations are special cases of a linear combination of the three terms following the Laspeyres-index with two parameters, which control how the interaction term is distributed into the other two. The choice of a CMS formulation hence reduces to the question how we should (or would like to) divide the interaction term. Based on this finding we propose to decompose each of the three components in the same way over all classification levels. This means that the demand growth and competitive components are treated equally. Moreover, a similar decomposition of the interaction term makes sure that the results can still be summarized according to any other specified CMS formulation. Further statistical analysis of some outputs of the tree-form CMS model shows that the proposal is very meaningful and useful in practice. In summary, this paper provides a systematic methodological discussion on some problems related to the CMS model and may open some further research topics.

The paper is organized as follows. Section 2 defines the (entire) tree-form CMS model and its branch- and leaf-models. Then the properties of effects caused by each level of classification are discussed in Section 3. In Section 4, relationship between the tree-form CMS model and existing ideas in the literature is compared and a general CMS formulation for the basic CMS decomposition is introduced. Results of application to China-Germany trade are reported in Section 5. Final remarks in Section 6 conclude the paper. Proofs of the results are put in the Appendix.

2. The tree-form CMS model

In this section we will define the tree-form CMS model for decomposing changes of total exports for multi-level classified data by modifying the definition of common CMS models in
the literature. Possible extensions of this model will also be discussed briefly.

2.1 Decomposing the export change at different classification levels

Consider the international trade between a focus country (Country A) and a single destination (Country B). In this paper we will mainly discuss the use of the CMS model based on data classified at three levels. That is Country A’s aggregate exports are first divided into $n$ first-level categories, the $i$-th first-level category consists of $n_i$ second-level sub-categories and the $(i, j)$-th second-level category is again composed of $n_{ij}$ third-level sub-categories. Country B’s total imports from the world are also classified in the same way. Denote the quantities of Country A’s exports at those classification levels in the initial and final years by $q_i^0, q_i^1, q_i^j, q_{ij}^0, q_{ij}^1, q_{ijk}^0$ and $q_{ijk}^1$, $i = 1, \ldots, n$, $j = 1, \ldots, n_i$, and $k = 1, \ldots, n_{ij}$. The corresponding quantities of Country B’s imports from the world will be denoted by $Q_i^0, Q_i^1, Q_{ij}^0, Q_{ij}^1, Q_{ij}^q, Q_{ijk}^q$, respectively. And Country A’s corresponding market shares are denoted by $s_i^0, s_i^1, s_{ij}^0, s_{ij}^1$ and $s_{ijk}^0, s_{ijk}^1$, respectively. Here we have $q_i^0 = \sum_{j} q_{ij}^0 = \sum_{j, k} q_{ijk}^0$, where $\sum_i$ and $\sum_{i, j}$ denote double and triple sums over $i$ and $j$, and over $i$, $j$ and $k$, respectively. Let $\Delta$ denote the change between the initial and final years. The change of Country A’s aggregate exports can be decomposed based on data at different levels of classification using the Laspeyres-type index (see e.g. Jepma, 1986):

$$\Delta q = s_i^0 \Delta Q_i + \Delta s_i^0 Q_i + \Delta s_i Q_i \quad \text{(overall decomposition)}$$

$$= \sum_{j} s_{ij}^0 \Delta Q_{ij} + \sum_{j} \Delta s_{ij}^0 Q_{ij} + \sum_{j} \Delta s_{ij} Q_{ij} \quad \text{(first – level decomposition)}$$

$$= \sum_{i, j} s_{ijk}^0 \Delta Q_{ijk} + \sum_{i, j} \Delta s_{ijk} Q_{ijk} + \sum_{i, j} \Delta s_{ijk} Q_{ijk} \quad \text{(second – level decomposition)}$$

$$= \sum_{i, j, k} s_{ijk}^0 \Delta Q_{ijk} + \sum_{i, j, k} \Delta s_{ijk} Q_{ijk} + \sum_{i, j, k} \Delta s_{ijk} Q_{ijk} \quad \text{(third – level decomposition)},$$

where the three terms in each row on the right-hand-side (rhs) of Model (1) are called the “demand growth component ($D_l$)”, “competitive component ($C_l$)” and “interaction component ($I_l$)”, $l = 0, 1, 2, 3$, respectively, calculated at the overall to the third-level using the aggregated, the first-, second- and third-level classified data. These terms are basic decompositions of the total export change.

2.2 The entire tree-form model

Now we propose to decompose each of the three CMS components further in a symmetrical way. Based on Model (1), the entire (three-level) tree-form CMS model is defined by
\[
\Delta q = s^0 \Delta Q + \Delta s^0 \Delta Q + \Delta s \Delta Q \\
+ \left( \sum_i s^0_i \Delta Q_i - s^0 \Delta Q \right) + \left( \sum_j \Delta s^0_j \Delta Q_j - \Delta s^0 \Delta Q \right) + \left( \sum_i \Delta s_i \Delta Q_i - \Delta s \Delta Q \right) \\
+ \left( \sum_{i,j} s^0_{ij} \Delta Q_{ij} - \sum_i s^0_i \Delta Q_i \right) + \left( \sum_j \Delta s^0_j \Delta Q_j - \sum_i \Delta s^0_i \Delta Q_i \right) + \left( \sum_i \Delta s_i \Delta Q_i - \sum_i \Delta s_i \Delta Q_i \right) \\
+ \left( \sum_{i,j,k} s^0_{ijk} \Delta Q_{ijk} - \sum_{i,j} s^0_{ij} \Delta Q_{ij} \right) + \left( \sum_{i,j} \Delta s^0_{ij} \Delta Q_{ij} - \sum_{i,j} \Delta s^0_{ij} \Delta Q_{ij} \right) + \left( \sum_{i,j,k} \Delta s_{ijk} \Delta Q_{ijk} - \sum_{i,j,k} \Delta s_{ijk} \Delta Q_{ijk} \right),
\]

where the three terms in the first row on the rhs of Model (2) are the overall components \((D_0, C_0, I_0)\), and those in the second, third and fourth rows are the first-, second- and third-level effects, called the demand growth effect \((E_i^C)\), competitive effect \((E_i^C)\) and interaction effect \((E_i^C)\) \((l = 1, 2, 3)\), respectively. In this paper, the concept “component” stands for a main term according to the CMS decomposition and the concept “effect” for the difference between same-named components at two levels over each other. Also note that the sum of the level-effects at each level or within any (sub-) category is always zero. Due to the data structure under consideration, the order of the indices \(i, j \) and \(k \) for each sum is fixed and the indices are not exchangeable. This fact rolls out a possible problem in a CMS formulation caused by the order of decomposition, because this is now determined by the nature of the data. Model (2) includes all level-effects due to the classification and provides us detailed sources that cause the change of exports.

2.3 The branch- and leaf-models

Furthermore, the \(i\)-th first-level branch-model is defined by taking the \(i\)-th element out of Model (2), which is based on two-levels of further categories:

\[
\Delta q_i = s^0_i \Delta Q_i + \Delta s^0_i \Delta Q_i + \Delta s_i \Delta Q_i \\
+ \left( \sum_j s^0_j \Delta Q_{ij} - s^0_i \Delta Q_i \right) + \left( \sum_j \Delta s^0_j \Delta Q_{ij} - \Delta s^0_i \Delta Q_i \right) + \left( \sum_i \Delta s_i \Delta Q_{ij} - \Delta s_i \Delta Q_i \right) \\
+ \left( \sum_{j,k} s^0_{jk} \Delta Q_{ijk} - \sum_j s^0_j \Delta Q_{ij} \right) + \left( \sum_{j,k} \Delta s^0_{jk} \Delta Q_{ijk} - \sum_j \Delta s^0_j \Delta Q_{ij} \right) + \left( \sum_{j,k} \Delta s_{ijk} \Delta Q_{ijk} - \sum_j \Delta s_{ijk} \Delta Q_{ij} \right),
\]

where the three terms in the first row on the rhs of Model (3) are the first-level components \((D_{1i}, C_{1i}, I_{1i})\), and those in the second and third rows are the second- and third-level effects, denoted by \(E_{2i}^D, E_{2i}^C, E_{2i}^L\) and \(E_{3i}^D, E_{3i}^C, E_{3i}^L\), respectively. Analogously, the \((i, j)\)-th second-level branch-model is defined by taking the \((i, j)\)-th element out of Model (2), which is based on one-level of further categories:
\[ \Delta q_{ij} = s_{ij}^0 \Delta Q_{ij} + \Delta s_{ij} Q_{ij}^0 + \Delta s_{ij} \Delta Q_{ij} \]

\[ + \left( \sum_k s_{ijk} \Delta Q_{ijk} - s_{ijk}^0 \Delta Q_{ijk} \right) + \left( \sum_k \Delta s_{ijk} Q_{ijk}^0 - \Delta s_{ijk} Q_{ijk}^0 \right) + \left( \sum_k \Delta s_{ijk} \Delta Q_{ijk} - \Delta s_{ijk} \Delta Q_{ijk} \right), \quad (4) \]

where the three terms in the first row on the rhs of Model (4) are the second-level components \((D_2i, C_2i)\) and \((I_2i)\), and those in the second row are the third-level effects, denoted by \(E_3D, E_3C\) and \(E_3I\), respectively. Finally, a so-called leaf-model can be defined for each final (leaf-)category. The \((i, j, k)\)-th leaf-model, i.e. the \((i, j, k)\)-th element of Model (2), is given by:

\[ \Delta q_{ijk} = s_{ijk}^0 \Delta Q_{ijk} + \Delta s_{ijk} Q_{ijk}^0 + \Delta s_{ijk} \Delta Q_{ijk}, \quad (5) \]

where the three terms on the rhs of Model (5) are the third-level components \((D_3ijk, C_3ijk)\) and \((I_3ijk)\). Note that all of the branches and leaf-models are parts of Model (2). Particularly, each model at a given level looks like another tree-form CMS model defined based on data from that level to the final categories. In summary, Models (2) through (5) constitute a collection of CMS models defined at different levels, which consists of a large amount of information and has a wide application spectrum.

### 2.4 Possible extensions

The tree-form CMS model above is described based on data classified at three levels. In practice, international trade data classified at four or more levels are available. The tree-form CMS model can be easily extended to analyze those more detailed data. Furthermore, the standard CMS model is proposed for modeling international trade of a focus country to more than one destinations. If exports to more than one destinations classified at multi-levels are studied, a combination of the standard and the tree-form CMS models can be defined and applied. In addition, if monthly or seasonal data are collected, then the seasonal or monthly exports can be considered as sub-categories of yearly exports. Now, it is possible to apply the idea of the tree-form CMS model for analyzing the effects caused by seasonal fluctuations. This will lead to a so-called seasonal CMS model. Here the seasons are treated as a further classification over time. Also note that the seasons are independent of the classification based on the products. Such a seasonal CMS model is hence similar to the combination of the tree-form and the standard CMS models. Finally, the tree-form CMS model can also be applied to analyze the change of market share \((\Delta s)\), relative exports growth \((q^1/q^0)\) and relative market share \((s^1/s^0)\). This can be done e.g. following Formulations (5), (13) and (14) in Milana (1988). To this end see also Fagerberg and Sollie (1987) for an analysis of the market share growth in a sample of 20 OECD countries.
3. Properties of the tree-form CMS model

Following Models (1) and (2) we can see that the $l$-th level-effects stand for a redistribution of the CMS components calculated by data classified at level $l$ instead of those at level $l - 1$. If all of the level-effects are about zero, it means that the three components calculated using data classified at level $l$ or level $l - 1$ roughly coincide with each other. Otherwise, the CMS components calculated using data classified at different levels may be quite different to each other. Now we will discuss the sources of the level-effects in detail. The following theorem provides conditions under which some or all of the level-effects $E^D_1, E^C_1$ and $E^I_1$ vanish. Results on the level-effects at other levels may be found in Section 4.2.

**Theorem 1:** Under the regularity conditions $q^0_i, q^1_i, Q^0_l$ and $Q^1_l > 0$, we have

Case 1: a) If $s^0_1 = s^0_2 = ... = s^0_n$, then $E^D_1$ vanishes;

b) If $s^1_1 = s^1_2 = ... = s^1_n$, then $E^C_1$ vanishes and

c) If both conditions in a) and b) are fulfilled, then $E^D_1, E^C_1$ and $E^I_1$ all vanish.

Case 2: If $Q^1_1 : Q^0_1 = Q^1_2 : Q^0_2 = ... = Q^1_n : Q^0_n$, then all of the three first-level effects vanish.

Proof of Theorem 1 is given in the Appendix. Theorem 1 reveals a more deep relationship between different variables so that the level-effects vanish. All conditions in Cases 1 and 2 are sufficient, but may be unnecessary. In the following we will call $q^0_i$ and $q^1_i$ endogenous and $Q^0_l$ and $Q^1_l$ exogenous quantities, which reflect Country A’s export structure and its changes, and Country B’s market structure and its changes, respectively. Conditions in Case 1 show that if the market shares are the same for all commodities in the initial and final periods, respectively, all of the three level-effects caused by the classification will vanish. These conditions can be regarded as suitable mixed conditions, because market shares depend on $q^0_i$ and $q^1_i$, and $Q^0_l$ and $Q^1_l$ simultaneously, which indicate that the inhomogeneity of Country A’s market share in different commodities is one of the sources of the level-effects. The homogeneity assumption on Country A’s market share is irrelevant in practice. Conditions in Case 2 are suitable exogenous conditions only depending on the destination’s market situation, which demonstrate that if the market growth in each of the $n$ categories is (about) the same, all of the three level-effects will be (about) zero. Now the change of exports is mainly reflected by the overall components. The inhomogeneity of the market growth in different
commodities is another source of the level-effects. Data examples in Section 6 show that the market growth is sometimes roughly homogeneous.

**Corollary 1:** If the conditions of Theorem 1 hold for several categories at the same level, the corresponding level-effects will not be changed, if some of those categories are combined together.

Proof of Corollary 1 is straightforward and is hence omitted. The special case with $n = 2$ is taken to show some details. Note that now the sufficient condition in Case 2 of Theorem 1 becomes

$$Q^0_1 Q^1_2 = Q^1_1 Q^0_2.$$ Let iff stand for *if and only if*.

**Corollary 2:** Assume that the regularity conditions of Theorem 1 hold. For $n = 2$ we have

a) $E^D_1 > 0$ (=0 or <0), iff $(Q^0_1 Q^1_2 - Q^1_1 Q^0_2)(s^0_1 - s^0_2) < 0 (= 0$ or >0);

b) $E^C_1 > 0$ (=0 or <0), iff $(Q^0_1 Q^1_2 - Q^1_1 Q^0_2)(s^1_1 - s^1_2) > 0 (= 0$ or <0);

c) $E^I_1 > 0$ (=0 or <0), iff $(Q^0_1 Q^1_2 - Q^1_1 Q^0_2)(s^0_1 - s^0_2) Q^1_1 - (s^1_1 - s^1_2) Q^0_1 > 0 (= 0$ or <0) and

d) The effects $E^D_1, E^C_1$ and $E^I_1$ all vanish, iff $Q^0_1 Q^1_2 = Q^1_1 Q^0_2$ or $s^0_1 = s^0_2$ and $s^1_1 = s^1_2$.

Proof of Corollary 2 is given in the Appendix. Corollary 2 d) shows that, for $n = 2$, the conditions in Theorem 1 such that the three terms all vanish are also necessary. Corollary 2 a) to c) indicate further when the level-effects $E^D_1, E^C_1$ and $E^I_1$ will be positive or negative, respectively. Once one of the three terms vanishes, the other two are equal in absolute value and opposite in sign. Furthermore, if $(s^0_1 - s^1_2) Q^1_1 - (s^1_1 - s^1_2) Q^0_1 = 0$ but $Q^0_1 Q^1_2 - Q^1_1 Q^0_2 \neq 0$, $s^0_1 \neq s^0_2$ and $s^1_1 \neq s^1_2$, then the first-level interaction effect $E^I_1$ is completely divided into $E^D_1$ and $E^C_1$. For $n > 2$, necessary conditions such that $E^D_1, E^C_1$ and $E^I_1$ all vanish become more complex and will not be discussed.

**4. Some further results**

Now we will discuss the relationship between the tree-form CMS model and related proposals in the literature, extend the above results to multi-level tree-form CMS model and discuss the relationship between different well known CMS formulations in the literature in more detail.

**4.1 Relationship between different CMS models**

In the literature, there exists much debate about which level of classified data should be used when applying the CMS model. And most of them admit that if employing data at different levels, then the results are also clearly different. Richardson (1971b) discusses this problem in
detail, uses the classification of commodities over four options and points out “The various components ... will vary with the level of commodity aggregation. ... Altering the commodity classification led to substantial variation in the value of CMS effects”. The tree-form CMS model shows that the level-effects stand for the differences between the CMS components based on data classified at two levels. Whether these differences are clear or not, depends on the question, if the conditions such that all of the level-effects vanish are about fulfilled or not.

The following example provides a further explanation for the difference between the level-effects caused by the use of data at different levels of classification. Assume that data are classified at two levels and consider the use of two types of models. One uses only the data in the first-level categories, called Model A, the other uses only the data in all of the second-level categories, called Model B. The overall components and the first-level effects given by Model A are exactly the same as those given by the tree-form CMS model. However, Model A cannot provide the information of the second-level effects. The overall components given by Model B are also the same as those given by the tree-form CMS model. Each of the further effect of Model B is the sum of the two corresponding first- and second-level effects of the tree-form CMS model. A further decomposition of those two level-effects only using the data at the second-level is however impossible. We see that the tree-form CMS model is superior to the two types of widely used CMS models, i.e. Models A and B.

4.2 Properties of the level-effects at any level

Results of Theorem 1 and Corollary 2 also hold for the level-effects of a multi-level tree-form CMS model at each level.

**Corollary 3**: Consider data classified at $k$ levels. Under the same regularity conditions of Theorem 1, let $l = 1, ..., k$. The following holds

a) If the corresponding conditions of Cases 1 and 2 in Theorem 1 or those of Corollary 2 hold for the commodities at level $l$ of a sub-model at level $l$, then the corresponding level-effects of this sub-model vanish and

b) If the corresponding conditions of Cases 1 and 2 in Theorem 1 or those of Corollary 2 hold for all sub-models at level $l'$, then the corresponding level-effects at any level $l$ vanish, $l = l', l' - 1, ..., 1$.

Proof of Corollary 3 is straightforward and is hence omitted. The meaning of Corollary 3 a) is similar to that of Theorem 1 or Corollary 2. Conditions of Corollary 3 b) indicate that the entire level-effects at each level are caused either by the inhomogeneity of the market shares
or the market growth in all sub-categories.

4.3 Relationship between well known CMS formulations

To our knowledge, most disputes about well known CMS formulations can be attributed to how to decompose the three CMS components reasonably and validly. The following section mainly focuses on the decomposition of the overall components. We will explore the relationship between the well known decompositions proposed in the literature.

Using the Laspeyres- and Paasche-type indices, two basic CMS formulations are

\[ \Delta q = D_0 + C_0 + I_0, \quad (6) \]

with \( D_0 = s^0 \Delta Q, C_0 = \Delta sQ^0 \) and \( I_0 = \Delta s\Delta Q \) and

\[ \Delta q = D_0^* + C_0^* + I_0^*, \quad (7) \]

with \( D_0^* = s^1 \Delta Q, C_0^* = \Delta sQ^1 \) and \( I_0^* = -\Delta s\Delta Q \).

Note that \( \Delta q \) can be represented based on Models (6) or (7) as follows

\[ \Delta q = (D_0 + \alpha d_0) + (C_0 + \beta I_0) + (1 - \alpha - \beta)I_0 \] or

\[ \Delta q = (D_0^* + \alpha^* I_0^*) + (C_0^* + \beta^* I_0^*) + (1 - \alpha^* - \beta^*)I_0^*, \quad (9) \]

where \( 0 \leq \alpha \leq 1, 0 \leq \beta \leq 1 \) and \( 0 \leq \alpha^* \leq 1, 0 \leq \beta^* \leq 1 \) are two parameters. Especially, Models (8) and (9) are equivalent with \( \alpha^* = 1 - \alpha, \beta^* = 1 - \beta \). Indeed, the restrictions \( 0 \leq \alpha \leq 1, 0 \leq \beta \leq 1 \) and \( 0 \leq \alpha^* \leq 1, 0 \leq \beta^* \leq 1 \) are unnecessary, which are only used for simplicity. The relationship between most well known decompositions in the literature can be summarized as follows:

**Theorem 2:** Most of the CMS formulations in the literature are special cases of Model (8) with certain values of \( \alpha \) and \( \beta \).

Similar statement holds for Model (9). The above facts also hold for other components and level-effects of the tree-form CMS model. To clarify the meaning of Theorem 2, we will show that all of the Formulations (6) to (10) in Milana (1988) (see also Formulations (6) to (9) in Richardson, 1971a) are all special cases of Model (8) with different choices of \( \alpha \) and \( \beta \). If \( \alpha = 0 \) and \( \beta = 1 \), then Model (8) becomes Formulation (6) there, i.e. \( \Delta q = s^0 \Delta Q + Q^1 \Delta s \); if \( \alpha = 1 \) and \( \beta = 0 \), then Model (8) becomes Formulation (7) there, i.e. \( \Delta q = s^1 \Delta Q + Q^0 \Delta s \); Formulation (9) there is the same as Model (6) and Formulation (10) there is the same as Model (7), i.e. Model (8) with \( \alpha = \beta = 0 \) and Model (9) with \( \alpha^* = \beta^* = 0 \).
In particular, it can be shown that Formulation (8) there, i.e.
\[ \Delta q = \left[ \tilde{\alpha} Q^0 + (1 - \tilde{\alpha})Q^1 \right] \Delta Q + \left[ (1 - \tilde{\alpha})Q^0 + \tilde{\alpha} Q^1 \right] \Delta s, \quad 0 < \tilde{\alpha} < 1, \]
is also a special case of Model (8) with \( \alpha = 1 - \tilde{\alpha}, \beta = \tilde{\alpha} \) and \( \alpha + \beta = 1 \). Under the further restriction \( \alpha + \beta = 1 \), including Formulations (6) and (7), the interaction term will be completely divided into the other two terms. The values of \( \alpha \) and \( \beta \) can be chosen based on different purposes. For instance, \( \alpha = 0 \) and \( \beta = 1 \) or \( \alpha = 1 \) and \( \beta = 0 \) are two commonly used choices. Milana (1988) proposes to use \( \alpha = \beta = 0.5 \). This decomposition uses the Törnqvist-type index, which takes the average of the Laspeyres- and Paasche-type indices numbers. These choices are all independent of the data. We find that determining \( \alpha \) and \( \beta \) according to the ratio of the absolute values of \( D_0 \) and \( C_o \), i.e. \( \alpha = |D_0|/(|D_0|+|C_o|) \) and \( \beta = |C_o|/(|D_0|+|C_o|) \), may be a more reasonable choice.

The idea to decompose each of the three CMS components further symmetrically has a clear advantage, because it allows us to summarize the decomposition results of Model (2) according to any other choice of \( \alpha \) and \( \beta \). This is shown using data classified at two levels with \( \alpha = \beta = 0.5 \). Now we have
\[ \Delta q = 0.5 \sum_{i,j} \left( s^0_{ij} + s^1_{ij} \right) \Delta Q_{ij} + 0.5 \sum_{i,j} \left( Q^0_{ij} + Q^1_{ij} \right) \Delta s_{ij}, \tag{10} \]
where the first term on the rhs of Model (10) stands for the “structure component”, or “demand growth component”, and the second for the “competitive component”. The “interaction” term is completely divided into those two components as indicated before. If the demand growth component and the competitive component are treated in a similar way, Model (10) can be further decomposed symmetrically as follows:
\[ \Delta q = \frac{1}{2} \left( s^0 + s^1 \right) \Delta Q + \frac{1}{2} \left( Q^0 + Q^1 \right) \Delta s \]
\[ + \frac{1}{2} \left[ \sum_i \left( s^0_i + s^1_i \right) \Delta Q_i - \left( s^0 + s^1 \right) \Delta Q \right] + \frac{1}{2} \left[ \sum_i \left( Q^0_i + Q^1_i \right) \Delta s_i - \left( Q^0 + Q^1 \right) \Delta s \right] \]
\[ + 0.5 \sum_{i,j} \left( s^0_{ij} + s^1_{ij} \right) \Delta Q_{ij} - \left( s^0 + s^1 \right) \Delta Q \]
\[ + 0.5 \sum_{i,j} \left( Q^0_{ij} + Q^1_{ij} \right) \Delta s_{ij} - \left( Q^0 + Q^1 \right) \Delta s \]. \tag{11}

In Model (11) the demand growth component is divided into the “overall demand growth component”, and the “first-level and second-level demand growth effects”, corresponding to the three terms in the first column on the rhs, respectively, and the three terms in the second column on the rhs are the “overall competitive component”, and the “first-level and second-level competitive effects”. We will also call Model (11) a tree-form CMS model due to the data structure under consideration. Although Model (11) is modified from Formulation (29) in
Milana (1988), it still differs from that formulation in several ways.

If the interaction component in the third row of Model (1) is assigned to the other two components using equal weights, it will be converted into Model (10). This relationship also holds for the first three rows of Model (2) and Model (11):

**Corollary 4:** Denote the six components and effects in Model (11) by \( \tilde{D}_0, \tilde{E}^d_1 \) and \( \tilde{E}^c_2 \), respectively. Then we have the following relationship with the corresponding terms in Model (2):

\[
\tilde{D}_0 = D_0 + 0.5I_0, \quad \tilde{E}^d_1 = E^d_1 + 0.5E^l_1, \quad \tilde{E}^c_2 = E^c_2 + 0.5E^l_2
\]

5. Application to China-Germany trade

Data downloaded from the United Nations Commodity Trade Statistics Database (UN Comtrade) from 1993 to 2009 are used as examples. Both China’s exports to Germany and Germany’s exports to China will be studied. According to the Standard International Trade Classification (SITC Rev. 3, shortly SITC), the total exports are firstly divided into two first-level categories, i.e. agricultural and industrial products. Based on the 1-digit SITC, agricultural products are composed of four categories (SITC 0, 1, 2 and 4), and industrial products also consist of four categories (SITC 5-8). These are the eight second-level categories considered here. SITC 3 and 9 are excluded due to many missing values. Then based on the 2-digit SITC, each of the four agricultural categories is composed of ten, two, nine and three sub-categories and each of the four industrial categories is divided into nine, nine, nine and eight sub-categories, respectively. These are considered as the third-level categories. Hence the data under consideration are classified at three levels. In this paper the decomposition is carried out yearly, so that the end of the period in each decomposition is also the beginning of the next period. Note that the outputs of each component or effect of a CMS model within the observation period form a time series. Here it is worthy to analyze those outputs further.
5.1 Explorative analysis of the CMS outputs

We will give some explorative analysis of selected results to show this. The demand growth, competitive and interaction components and their corresponding level-effects of the three-level tree-form CMS model in China’s total exports to Germany and Germany’s total exports to China from 1994 to 2009 are displayed in Figures 1 and 2, respectively. Firstly, the three components are discussed. For China’s exports to Germany (see the first row in Figure 1), both the demand growth and interaction components exhibited a great drop, i.e. a structural break, in 2009 due to the financial crisis, which however did not happen in the competitive component. That is the competitiveness of China’s exports to Germany was not affected by the financial crisis. Furthermore, there is a clear positive jump in the interaction component between 2002 and 2003, which may indicate the impact of China’s accession to WTO on the growth causes of its exports to Germany. To investigate this in detail, a simple linear and a constant regression model with a rolling dummy variable either in the intercept or in the slope is applied to the three series (without the value in 2009). See also Guo et al. (2011) for a related proposal. Then the AIC is used to detect a possible structural break. It is found that all of the three series exhibit a highly significant positive structural break between 2002 and 2003. And the structural break in the demand growth and competitive components is a rate-shift (a jump in the growth rate), while the interaction component exhibits a level-shift (a jump in the intercept). Jump in the interaction component is jointly caused by those in the demand growth and competitive components and is hence much clear. We can also see that the development of the demand growth component between 1994 and 2008 is relatively regular, the variation in the interaction component between 2003 and 2008 is clearly larger than before and that in the competitive component is very large. Moreover, the competitive components seem to correlate to each other in a negative way. This means that if China’s competitiveness strongly increases in one year, its increment in the next year tends to be smaller. Results of the Durbin-Watson test and the correlogram show that this negative correlation is however insignificant at the 5%-level. For Germany’s exports to China (see the first row in Figure 2), the common feature is that the developments of the three components are relatively stable between 1994 and 2002, which however exhibit large fluctuations between 2003 and 2009. The reasons could be the official launch of the Euro, the EU’s CAP (Common Agricultural Policy) reform in 1999 and further reforms thereafter. Similarly to the components in Figure 1, the demand growth and interaction components also showed a clear reduction in 2009 mainly caused by the financial crisis. But the competitive component still had a gain instead of loss, which suggests that Germany’s export competitiveness improved and its market share in China
increased during the financial crisis. Then we look at the level-effects from the first- to the third-level. For both countries, there is no obvious rule in their developments.

Figure 1: The three-level tree-form CMS model for China’s total exports to Germany.

Now consider the behavior of the first-level branch-models. Since industrial products accounts for about 97% of the total exports for both countries, the results of the first-level branch-models for this category are very similar to the first, third and fourth rows in Figures 1 and 2, respectively. These two branch-models are hence omitted. From the two first-level branch-models for agricultural products only that of China’s exports to Germany is chosen and displayed in Figure 3. Here the three components (the first row) are mainly considered. We see that the demand growth component kept positive from 2001 to 2008, which indicates Germany’s import demand was expanding during that period. But in 2009, it decreased significantly and became negative because of the financial crisis. For the competitive
Figure 2: The three-level tree-form CMS model for Germany’s total exports to China.

component, it shows that the development was very irregular without clear structural breaks. In contrast, the competitive component of China’s exports to Germany in industrial products had a clear structural break between 2002 and 2003 and kept a relatively high level from then on (compare with Figure 1 (b)). This suggests that China’s accession to WTO exhibited a more clearly positive impact on its exports of industrial products than agricultural products. Similarly to Figure 1 (b), the competitive components also appear to correlate to each other in a negative way while the correlation is insignificant at the 10%-level. The interaction component had a sudden reduction during 2008 to 2009, which also reflects the impact of the financial crisis.

Finally, some chosen second-level branch-models and leaf-models are displayed in Figure 4. For China’s exports to Germany, machinery and transport equipment (SITC 7) and one of its biggest sub-categories—electrical machinery etc. (SITC 77) are shown in the first and third
Figure 3: The first-level branch-model for China’s exports to Germany in agricultural products.

The first-level branch-model for China’s exports to Germany in agricultural products. For Germany’s exports to China, food and live animals (SITC 0) and its most representative sub-category—vegetables and fruit (SITC 05) are given in the second and fourth rows of Figure 4, respectively. For the second-level branch-models, only the three components are provided and the third-level effects are omitted to save space. Similarly to Figure 1 (b), the competitive component of machinery and transport equipment (see Figure 4 (b)) also exhibits a clear positive jump between 2002 and 2003. And we see that the variation of this component between 2003 and 2008 is very large. In contrast to the corresponding result in Figure 1 (b), the negative short-term impact of the financial crisis on the competitiveness of China’s exports in this category is clearer with a great drop in 2009.

The demand growth component of food and live animals (see Figure 4 (d)) was positive in 2009, which indicates that China’s import demand in this category still increased despite the financial crisis. Furthermore, vegetable and fruit is the only sub-category that its demand growth component (see Figure 4 (j)) had a clear increment in 2009. Last but not least, the interaction component is difficult to explain and often with large fluctuations. If the competitive component is positive and its development is stable, then the shape of the interaction component looks like the demand growth component (see Figure 4 (i)). Similarly, if the demand growth component is positive and its development is stable, then the shape of
the interaction component looks like the competitive component. This is e.g. true for the category of general industrial machinery and equipment etc, which is omitted to save space.

![Graphs of different components](image)

Figure 4: Some chosen second-level branch-models and leaf-models for both countries.

### 5.2 Numerical evidence of the theoretical results

In this sub-section, we will use some examples to confirm the theoretical results in Sections 3 and 4 in detail. The period from 2004 to 2008 is considered, because the trade development between China and Germany within this period is more stable without structural breaks (see Guo et al., 2011). At first, results of the three-level tree-form CMS model are listed in Table 1, which corresponds to the terms of Model (2). From Table 1, we can see that each sum of the effects at the first-, second- and third-level is zero. For instance, in Germany’s exports to China from 2004 to 2005, the three third-level effects are -1898, 1511 and 387 million US dollars, respectively, which sum up to zero. Results of Corollary 1 can be now confirmed
numerically. Firstly, the three first-level effects of China’s exports to Germany from 2005 to 2006 are all about zero. According to Corollary 1 d), this can only happen if \( Q^0_1 : Q^0_2 = Q^0_2 : Q^0_1 \) or \( s^0_1 = s^0_2 \) and \( s^1_1 = s^1_2 \). After checking the details, we find \( s^0_1 \neq s^0_2 \) and \( s^1_1 \neq s^1_2 \), but \( Q^0_1 : Q^0_2 = 91.6 : 78.4 \approx 1.168 : 1 \) and \( Q^0_1 : Q^0_2 = 674.4 : 577.2 \approx 1.168 : 1 \) and conditions of Corollary 1 d) are roughly fulfilled. By Germany’s exports to China from 2007 to 2008 the first-level demand growth effect \( E^D \) is negative and the first-level competitive effect \( E^C \) is positive. The reasons for this are \( Q^0_1 Q^1_2 - Q^1_2 Q^0_1 \left(s^0_1 - s^0_2\right) > 0 \) and \( Q^0_1 Q^1_2 - Q^1_2 Q^0_1 \left(s^1_1 - s^1_2\right) > 0 \), according to Corollary 1 a) and b). Results in Table 1 also show that the absolute values of the second- or third-level effects may also be larger than that of the corresponding first-level effect.

Table 1: Results of the three-level tree-form CMS model from 2004 to 2008 ($US$ million)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( D )</td>
<td>( C )</td>
<td>( I )</td>
<td>( D )</td>
</tr>
<tr>
<td>Level</td>
<td></td>
<td>China’s exports to Germany</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Overall</td>
<td>2998</td>
<td>5105</td>
<td>649</td>
<td>5446</td>
</tr>
<tr>
<td>( E_1 )</td>
<td>-22.1</td>
<td>23.7</td>
<td>-1.6</td>
<td>-1.2</td>
</tr>
<tr>
<td>( E_2 )</td>
<td>-231</td>
<td>270</td>
<td>-38.3</td>
<td>-336</td>
</tr>
<tr>
<td>( E_3 )</td>
<td>150</td>
<td>-42.3</td>
<td>-107</td>
<td>-913</td>
</tr>
<tr>
<td>Level</td>
<td></td>
<td>Germany’s exports to China</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Overall</td>
<td>3988</td>
<td>-3415</td>
<td>-549</td>
<td>4448</td>
</tr>
<tr>
<td>( E_1 )</td>
<td>-156</td>
<td>113</td>
<td>43.6</td>
<td>25.3</td>
</tr>
<tr>
<td>( E_2 )</td>
<td>-35.9</td>
<td>45.9</td>
<td>-9.9</td>
<td>386</td>
</tr>
<tr>
<td>( E_3 )</td>
<td>-1898</td>
<td>1511</td>
<td>387</td>
<td>350</td>
</tr>
</tbody>
</table>

Now we will analyze the results of the first-level branch-models and will only focus on agricultural products. Results of the first-level branch-model for agricultural products are given in Table 2, which correspond to Model (3) with \( i = 1 \). Based on Models (2) and (3), the results for industrial products can be obtained by simple calculation of corresponding terms in Tables 1 and 2 and are hence omitted. Table 2 confirms again that \( E^D_2 + E^C_2 + E^I_2 = 0 \) and \( E^D_3 + E^C_3 + E^I_3 = 0 \). It is found that all of the three second-level effects of Germany’s exports to China in agricultural products from 2007 to 2008 are relatively very small, because China’s
market growth in the four sub-categories of agricultural products during that period was roughly homogeneous with $Q_i^0 : Q_0^0 : Q_3^0 : Q_4^0 \approx 1:0.12:10.3:0.65$ and $Q_i^1 : Q_1^1 : Q_3^1 : Q_4^1 \approx 1:0.14:11.9:0.76$. That is the condition of Case 2 in Theorem 1 is approximately fulfilled.

Table 2: Results of the first-level branch-model for agricultural products ($US$ million)

<table>
<thead>
<tr>
<th>Year</th>
<th>$D_{11}$</th>
<th>$C_{11}$</th>
<th>$I_{11}$</th>
<th>$E_{21}^D$</th>
<th>$E_{21}^C$</th>
<th>$E_{21}^I$</th>
<th>$E_{31}^D$</th>
<th>$E_{31}^C$</th>
<th>$E_{31}^I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>04-05</td>
<td>96.4</td>
<td>174</td>
<td>23.8</td>
<td>0.94</td>
<td>-2.4</td>
<td>1.5</td>
<td>-0.79</td>
<td>7.8</td>
<td>-7.0</td>
</tr>
<tr>
<td>05-06</td>
<td>169</td>
<td>16.1</td>
<td>2.7</td>
<td>-0.77</td>
<td>13.4</td>
<td>-12.6</td>
<td>-63.5</td>
<td>60.2</td>
<td>3.2</td>
</tr>
<tr>
<td>06-07</td>
<td>174</td>
<td>78.6</td>
<td>11.5</td>
<td>-0.52</td>
<td>2.7</td>
<td>-2.2</td>
<td>-46.9</td>
<td>47.1</td>
<td>-0.18</td>
</tr>
<tr>
<td>07-08</td>
<td>201</td>
<td>72.3</td>
<td>10.0</td>
<td>9.3</td>
<td>-6.7</td>
<td>-2.7</td>
<td>-33.5</td>
<td>33.1</td>
<td>0.45</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Year</th>
<th>China’s exports to Germany</th>
<th>Germany’s exports to China</th>
</tr>
</thead>
<tbody>
<tr>
<td>04-05</td>
<td>118</td>
<td>9.0</td>
</tr>
<tr>
<td>05-06</td>
<td>119</td>
<td>286</td>
</tr>
<tr>
<td>06-07</td>
<td>469</td>
<td>-185</td>
</tr>
<tr>
<td>07-08</td>
<td>541</td>
<td>-342</td>
</tr>
</tbody>
</table>

Furthermore, results for some chosen second-level branch-models are shown in Table 3, which correspond to Model (4) with certain $i$ and $j$. Both agricultural and industrial products are composed of four second-level categories, so the number of corresponding second-level branch-models is eight. It indicates that $E_{2j}^D + E_{2j}^C + E_{2j}^I = 0$ for $i = 1, 2$ and $j = 1, 2, 3, 4$ in all cases. According to the 1-digit SITC, we choose food and live animals (0), and machinery and transport equipment (7) as examples to show this.

Finally, results for some chosen leaf-models are listed in Table 4, which correspond to Model (5) with given $i, j$ and $k$. For China’s exports to Germany, two representative 2-digit SITC commodities, i.e. fish etc. (03) and office machines etc. (75) are chosen. For Germany’s exports to China, another two typical 2-digit SITC commodities, i.e. coffee, tea, cocoa etc. (07) and road vehicles (78) are selected. The results are very different among those third-level categories. Each leaf-model based on each third-level category could provide us some detailed information that causes the change of exports, in particular the growth causes for that category.
Table 3: Results of the chosen second-level branch-models ($US million)

<table>
<thead>
<tr>
<th>Name</th>
<th>China’s exports to Germany</th>
<th>Machinary and Transport Equipment</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$D_{211}$</td>
<td>$C_{211}$</td>
</tr>
<tr>
<td>Year</td>
<td>China’s exports to Germany</td>
<td>Machinary and Transport Equipment</td>
</tr>
<tr>
<td>Year</td>
<td></td>
<td></td>
</tr>
<tr>
<td>04-05</td>
<td>68.3</td>
<td>50.3</td>
</tr>
<tr>
<td>05-06</td>
<td>64.1</td>
<td>81.5</td>
</tr>
<tr>
<td>06-07</td>
<td>103</td>
<td>103</td>
</tr>
<tr>
<td>07-08</td>
<td>161</td>
<td>0.84</td>
</tr>
</tbody>
</table>

Table 4: Results of the chosen leaf-models ($US million)

<table>
<thead>
<tr>
<th>China’s exports to Germany</th>
<th>Germany’s exports to China</th>
</tr>
</thead>
<tbody>
<tr>
<td>Name</td>
<td>04-05</td>
</tr>
<tr>
<td>Fish etc.</td>
<td>$D_{3113}$</td>
</tr>
<tr>
<td></td>
<td>$C_{3113}$</td>
</tr>
<tr>
<td></td>
<td>$I_{3113}$</td>
</tr>
<tr>
<td>Office machines</td>
<td>$D_{3235}$</td>
</tr>
<tr>
<td></td>
<td>$C_{3235}$</td>
</tr>
<tr>
<td></td>
<td>$I_{3235}$</td>
</tr>
</tbody>
</table>

Results of the CMS model using different types of index are given in Table 5, which takes 2005 to 2006 and 2007 to 2008 at the overall-level as examples. The following facts can be extended to the first-, second- and multi-level correspondingly. In Table 5 the abbreviations “CN” for China and “DE” for Germany are used. Based on Theorem 2 and Corollary 4, the relationship between different types of the CMS model is that they can be transformed into each other under some conditions. It is clear that if the Paasche-type index is used, then
\[ s^1 \Delta Q + \Delta s^0 \Delta Q^1 - \Delta s \Delta Q = s^0 \Delta Q + \Delta s \Delta Q^0 + \Delta s \Delta Q; \]

if the Törnpvist-type index is used, then \( \tilde{D}_0 = D_0 + 0.5I_0 \) and \( \tilde{C}_0 = C_0 + 0.5I_0 \); and if the ratio of the absolute value of \( D_0 \) and \( C_0 \) is used as weights, then \( \alpha + \beta = 1 \) and the interaction component also vanishes. Table 5 shows that e.g. \( 5782+2330-336 \approx 5614+2162 = 5691+2085 = 5446+1994+336 \) (Table 1) for China’s exports to Germany from 2005 to 2006. In addition, \( \alpha \) and \( \beta \) can also be defined differently at different levels.

Table 5: Results of the CMS model using different types of index ($US million)

<table>
<thead>
<tr>
<th>Type</th>
<th>Component</th>
<th>2005-2006</th>
<th>2007-2008</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>CN to DE</td>
<td>DE to CN</td>
</tr>
<tr>
<td>Paasche</td>
<td>( D )</td>
<td>5782</td>
<td>4986</td>
</tr>
<tr>
<td></td>
<td>( C )</td>
<td>2330</td>
<td>3540</td>
</tr>
<tr>
<td></td>
<td>( I )</td>
<td>-336</td>
<td>-538</td>
</tr>
<tr>
<td>Törnpvist</td>
<td>( D )</td>
<td>5614</td>
<td>4717</td>
</tr>
<tr>
<td></td>
<td>( C )</td>
<td>2162</td>
<td>3271</td>
</tr>
<tr>
<td>Relative Weight</td>
<td>( D )</td>
<td>5691(( \alpha = 0.73 ))</td>
<td>4771(( \alpha = 0.6 ))</td>
</tr>
<tr>
<td></td>
<td>( C )</td>
<td>2085(( \beta = 0.27 ))</td>
<td>3217(( \beta = 0.4 ))</td>
</tr>
</tbody>
</table>

6. Final remarks

This paper provides a deep insight into the CMS analysis by introducing a tree-form CMS model. It is shown that the tree-form CMS model can be extended to multi-level classified data and in other ways. It is also indicated that, when using the CMS model, not only the final but also the intermediate results contain useful information for decision making. Furthermore, it is shown that the well known CMS formulations indeed form a linear class with two parameters, which control how the interaction term is divided into the demand growth and the competitive terms. A symmetric further decomposition of the three components is suggested. This allows us to report the results according to different CMS formulations after the decomposition. Sources of effects caused by each level of classification are discussed in detail. Advantages of the new CMS model to related ones in the literature are clarified. Application to trade between China and Germany shows that the proposals are very useful and flexible. Analysis of the outputs of the three-level tree-form CMS model shows that China’s accession to WTO has had a positive long-term impact on all of the three CMS components in China’s.
exports to Germany. It is found that the growth causes before and after some remarkable economic events are clearly different. Furthermore, our theoretical findings are also confirmed by data examples. We see that despite a huge number of theoretical studies on the CMS model and its wide application, there still seems to be a big play room for further development of theory and practice of the constant market share analysis. Hence, the current paper may open a new research direction in this area.

**Acknowledgements**

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References


Appendix: Proofs of the results

Proof of Theorem 1

Case 1: a) Since \( s_1^0 = s_2^0 = \ldots = s_n^0 \), we have \( s^0 = s_1^0 \). This results in

\[
E_1^D = \sum s_i^0 \Delta Q_i - s^0 \Delta Q = s_1^0 \Delta Q_1 + s_2^0 \Delta Q_2 + \ldots + s_n^0 \Delta Q_n - s^0 \Delta Q
\]

\[
= s_1^0 \Delta Q_1 + s_1^0 \Delta Q_2 + \ldots + s_1^0 \Delta Q_n - s_1^0 \Delta Q
\]

\[
= s_1^0 (\Delta Q_1 + \Delta Q_2 + \ldots + \Delta Q_n) - s_1^0 \Delta Q
\]

\[
= s_1^0 \Delta Q - s_1^0 \Delta Q = 0.
\]

b) Since \( s_1^1 = s_2^1 = \ldots = s_n^1 \), we have \( s^1 = s_1^1 \). This results in

\[
E_1^C = \sum s_i^1 \Delta Q_i^0 - \Delta s Q^0 = \left( s_1^1 - s_1^0 \right) Q_1^0 + \left( s_2^1 - s_2^0 \right) Q_2^0 + \ldots + \left( s_n^1 - s_n^0 \right) Q_n^0 - \left( s_1^1 - s_1^0 \right) Q^0
\]

\[
= s_1^1 Q_1^0 - s_1^0 Q_1^0 + s_1^1 Q_2^0 - s_1^0 Q_2^0 + \ldots + s_1^1 Q_n^0 - s_1^0 Q_n^0 - s_1^0 Q^0 + s_1^0 Q^0
\]

\[
= s_1^1 Q_1^0 + s_1^1 Q_2^0 + \ldots + s_1^1 Q_n^0 - s_1^1 Q^0 - q_1^0 - q_2^0 - \ldots - q_n^0 + q^0
\]

\[
= s_1^1 (Q_1^0 + Q_2^0 + \ldots + Q_n^0) - s_1^1 Q^0 - q^0 + q^0
\]

\[
= s_1^1 Q^0 - s_1^1 Q^0 = 0.
\]

c) Under the conditions of a) and b), we have \( E_1^D = 0 \) and \( E_1^C = 0 \).

Hence \( E_1^t = 0 \), because \( E_1^D + E_1^C + E_1^t = 0 \).

Case 2: Since \( Q_1^0 : Q_2^0 : \ldots : Q_n^0 = Q_1^1 : Q_2^1 : \ldots : Q_n^1 = 1 : C_2 : \ldots : C_n \), we have

\[
\sum s_i^0 \Delta Q_i = s_1^0 \Delta Q_1 + s_2^0 \Delta Q_2 + \ldots + s_n^0 \Delta Q_n = s_1^0 (Q_1^1 - Q_1^0) + s_2^0 (Q_2^1 - Q_2^0) + \ldots + s_n^0 (Q_n^1 - Q_n^0)
\]

\[
= s_1^0 \left( Q_1^1 - Q_1^0 \right) + s_2^0 \left( C_2 Q_1^1 - C_2 Q_1^0 \right) + \ldots + s_n^0 \left( C_n Q_n^1 - C_n Q_n^0 \right)
\]

\[
= s_1^0 \left( Q_1^1 - Q_1^0 \right) + C_2 s_2^0 \left( Q_1^1 - Q_1^0 \right) + \ldots + C_n s_n^0 \left( Q_1^1 - Q_1^0 \right)
\]

\[
= \left( Q_1^1 - Q_1^0 \right) \left( \frac{q_1^0}{Q_1^0} + C_2 \frac{q_2^0}{Q_2^0} + \ldots + C_n \frac{q_n^0}{Q_n^0} \right)
\]

\[
= \left( Q_1^1 - Q_1^0 \right) \left( \frac{q_1^0}{Q_1^0} + C_2 \frac{q_2^0}{C_2 Q_1^1} + \ldots + C_n \frac{q_n^0}{C_n Q_1^1} \right)
\]

\[
= \left( Q_1^1 - Q_1^0 \right) \left( q_1^0 + q_2^0 + \ldots + q_n^0 \right)
\]

\[
= \left( Q_1^1 - Q_1^0 \right) \frac{q_1^0 + q_2^0 + \ldots + q_n^0}{Q_1^0}
\]

\[
= \frac{Q_1^1 - Q_1^0}{Q_1^0} \left( 1 + C_2 + \ldots + C_n \right)
\]
And $s^0 \Delta Q = s^0(\Delta Q^1 - Q^0) = s^0\left(\Delta Q^1 + \Delta Q^2 + ... + \Delta Q^n\right) - \Delta Q^0 = s^0\left((\Delta Q^1 - \Delta Q^0) + (\Delta Q^2 - \Delta Q^0) + ... + (\Delta Q^n - \Delta Q^0)\right) = s^0\left((\Delta Q^1 - \Delta Q_0) + C_2(\Delta Q^1 - \Delta Q_0) + ... + C_n(\Delta Q^1 - \Delta Q_0)\right) = s^0(\Delta Q^1 - \Delta Q_0)(1 + C_2 + ... + C_n)

Hence $E_i^D = \sum_i s_i^0 \Delta Q_i - s^0 \Delta Q = 0$.

Furthermore, $\sum_i \Delta s_i Q_i^0 = \Delta s_1 Q_1^0 + \Delta s_2 Q_2^0 + ... + \Delta s_n Q_n^0$

$= \Delta s_1 Q_1^0 + \Delta s_2 C_2 Q_1^0 + ... + \Delta s_n C_n Q_1^0$

$= Q_1^0 \left(\Delta s_1 + C_2 \Delta s_2 + ... + C_n \Delta s_n\right)$

$= Q_1^0 \left(\frac{q_1^0 - q_0^0}{Q_1^0} + C_2 \left(\frac{q_2^0 - q_2^0}{Q_2^0} - \frac{q_2^0}{C_2 Q_1^0}\right) + ... + C_n \left(\frac{q_n^0}{C_n Q_1^0} - \frac{q_0^0}{C_n Q_1^0}\right)\right)$

$= Q_i^0 \left(\frac{q_i^0}{Q_i^0} + C_2 \left(\frac{q_2^0 - q_2^0}{Q_2^0} - \frac{q_2^0}{C_2 Q_i^0}\right) + ... + C_n \left(\frac{q_n^0}{C_n Q_i^0} - \frac{q_0^0}{C_n Q_i^0}\right)\right)$

$= \sum_i \Delta s_i Q_i^0 = \sum_i \Delta s_i \left(\frac{q_i^0}{Q_i^0} + \frac{q_i^0}{Q_i^0} + ... + \frac{q_i^0}{Q_i^0}\right)$

$= Q_1^0 \left(\frac{q_i^1}{Q_i^0} + C_2 Q_1^0 + ... + C_n Q_1^0\right)$

$= Q_i^0 \left(Q_1^0 \left(\frac{q_i^1}{Q_i^0} + C_2 Q_1^0 + ... + C_n Q_1^0\right) - \frac{q_i^0}{Q_i^0} + \frac{q_i^0}{Q_i^0} + ... + \frac{q_i^0}{Q_i^0}\right)$

$= Q_i^0 \left(1 + C_2 + ... + C_n\right) \left(\frac{q_i^1}{Q_i^0} - \frac{q_i^0}{Q_i^0}\right)$

and $\Delta s Q^0 = \left(\frac{q_i^1}{Q_i^0} - \frac{q_i^0}{Q_i^0}\right)Q_i^0 + \frac{q_i^0}{Q_i^0} + ... + \frac{q_i^0}{Q_i^0} = \left(\frac{q_i^1}{Q_i^0} - \frac{q_i^0}{Q_i^0}\right)Q_i^0 + \frac{q_i^0}{Q_i^0} + ... + \frac{q_i^0}{Q_i^0}$

$= Q_i^0 \left(1 + C_2 + ... + C_n\right) \left(\frac{q_i^1}{Q_i^0} - \frac{q_i^0}{Q_i^0}\right)$

Hence $E_i^C = \sum_i \Delta s_i Q_i^0 - \Delta s Q^0 = 0$.

Finally we have $E_i' = 0$, because $E_i^D + E_i^C + E_i' = 0$. 

\[\square\]
Proof of Corollary 1

a) When \( n = 2 \), we have

\[
E^D_i = s_i^0 \Delta Q_1 + s_2^0 \Delta Q_2 = q_i^0 - q_i^0 \left( Q_i^0 - Q_o^0 \right) + \frac{q_i^0}{Q_i^0} \left( Q_1^o - Q_2^o \right) - \frac{q_2^0}{Q_2^o} \left( Q_1^o + Q_2^o \right) - q_i^0 + q_2^0 \left( Q_i^0 + Q_2^o - Q_o^0 \right)
\]

\[
= \frac{Q_1^0}{Q_2^o} q_i^0 - \frac{Q_i^0 + Q_2^0}{Q_2^o} q_i^0 - \frac{Q_i^0}{Q_2^o} q_2^0 + \frac{Q_2^0}{Q_2^o} q_2^0
\]

\[
= \left( Q_i^0 - Q_i^0 \right) s_2^0 - s_i^0
\]

Hence \( E^D_i \) is positive (zero or negative), iff \( \left( Q_i^0 Q_2^o - Q_i^0 Q_o^0 \right) \left( s_i^0 - s_2^0 \right) < 0 \) (= 0 or > 0).

b) Furthermore, we have

\[
E_i^C = \Delta s_i Q_i^0 + \Delta s_2 Q_2^o - \Delta s Q = \left( \frac{q_i^0}{Q_i^0} - q_i^0 \right) Q_i^0 + \left( \frac{q_2^0}{Q_2^o} - q_2^0 \right) Q_2^o - \left( \frac{q_i^0 + q_2^0}{Q_i^0 + Q_2^o} - q_i^0 \right) Q_i^0 + Q_2^o
\]

\[
= \frac{Q_1^0}{Q_2^o} q_i^0 - \frac{Q_i^0 + Q_2^0}{Q_2^o} q_i^0 - \frac{Q_i^0}{Q_2^o} q_2^0 + \frac{Q_2^0}{Q_2^o} q_2^0
\]

\[
= \left( Q_i^0 Q_2^o - Q_i^0 Q_2^o \right) \left( s_i^0 - s_2^0 \right)
\]

Hence \( E_i^C \) is positive (zero or negative), iff \( \left( Q_i^0 Q_2^o - Q_i^0 Q_2^o \right) \left( s_i^0 - s_2^0 \right) > 0 \) (= 0 or < 0).

c) For \( E_i^D \), we have

\[
E_i^D = \Delta s_i \Delta Q_1 + \Delta s_2 \Delta Q_2 - \Delta s \Delta Q
\]

\[
= \left( \frac{q_i^0}{Q_i^0} - q_i^0 \right) \left( Q_i^0 - Q_o^0 \right) + \left( \frac{q_2^0}{Q_2^o} - q_2^0 \right) \left( Q_2^o - Q_1^o \right) - \left( \frac{q_i^0 + q_2^0}{Q_i^0 + Q_2^o} - q_i^0 \right) Q_i^0 + Q_2^o
\]

\[
= \frac{Q_1^0}{Q_2^o} q_i^0 - \frac{Q_i^0 + Q_2^0}{Q_2^o} q_i^0 - \frac{Q_i^0}{Q_2^o} q_2^0 + \frac{Q_2^0}{Q_2^o} q_2^0
\]

\[
= \left( Q_i^0 Q_2^o - Q_i^0 Q_2^o \right) \left( s_i^0 - s_2^0 \right)
\]

\[
E_i^D \text{ is positive (zero or negative)}, \text{ iff } \left( Q_i^0 Q_2^o - Q_i^0 Q_2^o \right) \left( s_i^0 - s_2^0 \right) Q^0 > 0 \text{ (= 0 or < 0).}
\]

d) It is clear that the terms \( E_i^0, E_i^C \) and \( E_i^D \) all vanish, if \( Q_i^0 Q_1^o = Q_i^0 Q_2^o \) or \( s_i^0 = s_1^0 \) and \( s_i^0 = s_2^0 \).

On the other hand, if \( Q_i^0 Q_2^o \not= Q_i^0 Q_2^o \) and \( s_i^0 \neq s_2^0 \) or \( s_i^0 \neq s_2^0 \), at least one of \( E_i^0 \) and \( E_i^C \) is non-zero. This finishes the proof.