Universität
Paderborn
Computer networks
group

Winter term 2017/2018

Mobile communications

Homework assignment 9: Cell capacity

Due date: 2017-12-20

Today's only assignment looks at how frequencies can be distributed over cells to combat interference. This was commonly used in GSM, abandonded in UMTS, and is taken up again in LTE, albeit in somewhat modified form. We look at the basic idea of frequency reuse and its impact on capacity in the sense of a simple, GSM-style approach. You might have to google some of the terms used below, but most of it should be fairly straightforward.

1. **Cell capacity**

   Frequency reuse allows a communication system to improve spectral efficiency throughout its deployment area. The repeating regular pattern of frequency reuse among cells is known as a *cluster*; the number of different frequency bands in use is the *cluster size*. The *cluster radius* R is the distance of the center of a cell to the furthest corner of a hexagon. In Figure 1, we represent the deployment of a cellular network composed of hexagonal cells with cluster size $K = 4$. Numbers in the hexagons indicate the frequency band used in that cell. The system operates with omnidirectional antennas.

   In this assignment, we are interested to understand two things: (1) what is the resulting signal-to-noise ratio resulting from frequency reuse and (2) what would be the impact of sectorization?

   (a) Compute the re-use distance D (distance between basestations using the same frequency bands) in relation to cell-radius R.
   Hint: it makes sense to first compute the distance r of a cell's basestation to an edge (dropping a perpendicular from center to edge) and to think about how long an edge is.
   Solution:
   Regarding the hints: $r = \frac{\sqrt{3}}{2}R$ (simple application of cosine in a triangle with a 30 degree angle). An edge is also R units long; it is part of an equilateral triangle with three times 60 degree angle.

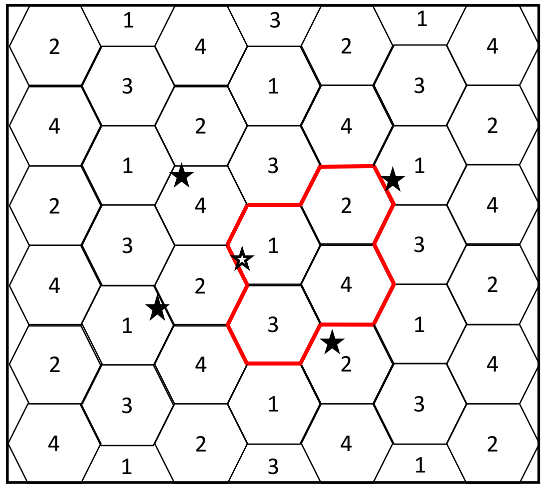

Abbildung 1: Cell layout for a cluster size of 4.

For the case of $K = 4$ as shown in Figure 1, two basestations using the same frequency hence have a distance D of $D = 4r = 2\sqrt{3}R$.

(b) Compute the worst-case co-channel interference $\frac{C}{I}$ for cell-edge located mobile stations in *uplink* and *downlink* communication; assume a frequency-division duplex, time-division multiplexing system (FDD/TDM). $\frac{C}{I}$ is the ratio of the received signal power and the sum of the power of the interfering signals. Take into account only the base stations of the first interfering ring.

Hint: start by thinking about from how many stations interference originates under an FDD/TDM assumption.

Use an attenuation coefficient of $\gamma = 3.5$ for the examples.

Solution:

DOWNLINK

- In the uplink, a terminal interferes with only terminals from other cells (not within cell, owing to TDM; not with other BS, owing to FDD).
- Per cell, it is exactly one terminal we interfere with (TDM!).
- We limit our attention to worst-case interference from the first "ring" of cells with the same frequency band. (The result would of course be more precise if we included further away cells, but not by much.)
- The worst-case distance D BS–UE inside a cell under consideration is R. Gives weakest signal.
- Worst-case in other cell: assume a terminal closest to the BS under consideration. There are six cells to take into account. Let's suppose our

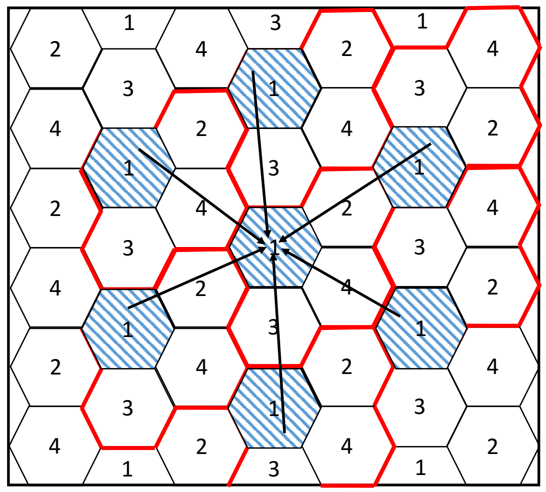

Abbildung 2: Cell layout for a cluster size of 4.

own UE is in the lower-left corner of our own cells. Then, take the interferring BSs one by one – it boils down to finding the right Pythagorean triangles.

Upper right: $d_1 = \sqrt{(1/2R)^2 + (5r)^2)} = \sqrt{1/4 + 75/4}R = \sqrt{19}R$

Lower right: $d_2 = \sqrt{(R)^2 + (4r)^2)} = \sqrt{1 + 12}R = \sqrt{13}R$

Below: $d_3 = \sqrt{(1/2R)^2 + (3r)^2)} = \sqrt{1/4 + 27/4}R = \sqrt{7}R$

Lower left: $d_4 = d_3$

Upper left: $d_5 = d_2$

Above: $d_6 = d_1$

- All terminals use same transmit power as they are all located at almost the same distance from their BS. (This is a small approximation error, but only up to a constant.)
- SIR is hence (P/R^γ cancels out):

$$\mathrm{SIR}_{\mathrm{DL}}^{\mathrm{nosect}} = \frac{P/D^\gamma}{\sum_{i=1}^{6} P/d_i^\gamma} = \frac{1}{2\sqrt{19}^{-\gamma} + 2\sqrt{13}^{-\gamma} + 2\sqrt{7}^{-\gamma}} \approx 9.96 \approx 9.98\,\mathrm{dB}$$

UPLINK

In the uplink, a UE's signal at the BS is interferred with by the BS in the neighboring cells. The UE–BS distance is R just like above. The distance from all neighboring BS to the UE's own BS is always $4r$. So the uplink SNR is slightly better than the downlink, but the difference is not dramatic:

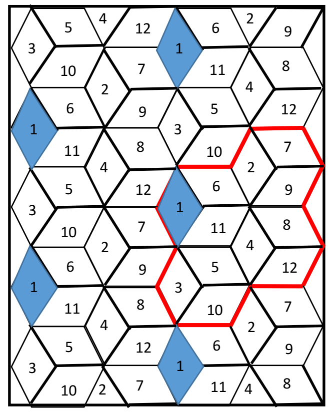

Abbildung 3: Cell layout for a cluster size of 4 with 3 sectors per cell

$$\mathrm{SIR}_{\mathrm{UL}}^{\mathrm{nosect}} = \frac{P/D^\gamma}{\sum_{i=1}^{6} P/d_i^\gamma} = \frac{1/R^\gamma}{6 \cdot 1/(4r)^\gamma} = \frac{1}{6} \cdot \frac{(4 \cdot 1/2 \cdot \sqrt{3}R)^\gamma}{R^\gamma} = \frac{1}{6}\left(\sqrt{\frac{48}{4}}\right)^\gamma \approx 12.9 \approx 11.1\,\mathrm{dB}$$

(c) In the following, assume that the network is **sectorized** so that each cell is split uniformly into three sectors.
Compute the cluster size K, the frequency reuse factor F as well as the reuse distance D.
Solution: $K = 4$, $D = \sqrt{12} \cdot R$ and $F = 4 * 3 = 12$, factor 3 due to sectorization. For 60^o sectorization there are 6 sectors and the value K needs to be multiplied by 6.

(d) Compute the co-channel interference for cell-edge located mobile terminals in *uplink* and *downlink* direction again. Think which cells you need to include in the interference calculation.
Solution: An example of sectorization is given in Figure 3:
DOWNLINK Let us start with the donlink again. The main point to observe here is from which other BSs a UE would receive interference. Because of the 120 degree angle, there are only two instead of six BS that produce interference! The relative distances are the same as in the example above. For the sector orientation as shown in Figure 3, the cells on the right interfere. Worst case is a UE on the left side of the cell. Hence, BS–UE distance is again R. The two interference distances are:

Upper right cell: $d_1 = \sqrt{(1/2R)^2 + (5r)^2} = \sqrt{19}R$

Lower right cell: $d_2 = d_1$; scenario is symmetric around a horizontal line passing through the base station of interest

Hence, we get:

$$\mathrm{SIR}_{\mathrm{DL}}^{\mathrm{sect}} = \frac{P/D^\gamma}{\sum_{i=1}^{2} P/d_i^\gamma} = \frac{1}{2\sqrt{19}^{-\gamma}} \approx 86,45 \approx 19.3\,\mathrm{dB}$$

UPLINK

Here, the main point is that an antenna is directional for both receive and sending! Similarly, we have a UE–BS distance of again $D = R$ and an inter-BS distance of $d_i = 4r$.

$$\mathrm{SIR}_{\mathrm{UL}}^{\mathrm{sect}} = \frac{P/D^\gamma}{2 \cdot P/d_i^\gamma} = \frac{1/R^\gamma}{2 \cdot 1/(4r)^\gamma} = \frac{1}{2} \cdot \frac{(4 \cdot 1/2 \cdot \sqrt{3}R)^\gamma}{R^\gamma} = \frac{1}{2}\left(\sqrt{\frac{48}{4}}\right)^\gamma \approx 38,68 \approx 15.87\,\mathrm{dB}$$

So the uplink improvement is smaller than the downlink improvement.

(e) What does that mean for the capacity improvement by secotrization? Assume a Shannon-capacity-achieving setup!

Note

Solution:

Scenario	Uplink	Downlink
SIR: No sectorization	12.9	9.98
SIR: Sectorization	38.68	86.45
Normalized Shannon capacity: No sectorization	7.1	10.1
Normalized Shannon *user* capacity: Sectorization	3.98	4.36
Normalized Shannon *cell* capacity: Sectorization	11.95	13.09

Tabelle 1: Approximately achieved SIR and spectral efficiency

Under these assumptions, sectoriaztion is detrimental for the capacity of an individual user. But it improves cell capacity provide the cell is full enough (have to divide the unsectorized spectral efficiency by three to obtain user spectral efficiency; multiply by three again to get back the cell capacity of a loaded cell).

Keep in mind that this applies to loaded cells. To understand how this works out for lightly loaded cells, assume a spatial Poisson process with rate λ arrivals per cell area, divide it in three non-overlapping processes with rate

$\lambda/3$ for each of the cells, and look at the probability of at least one terminal being there. (Assuming all terminals are always backlogged.)

Then, the probability that sectorization outperforms non-sectorized systems is the probability that for a given λ, each of the sectors has at least one terminal. (Approximation for small λ.)

$$P = (1 - P(0\,\text{terminals}))^3 = \left(1 - \frac{(\lambda/3)^0}{0!}\mathrm{e}^{-\lambda/3}\right)^3 = \left(1 - \mathrm{e}^{-\lambda/3}\right)^3 \approx \left(1 - (1 - \frac{\lambda}{3})\right)^3 = \frac{\lambda^3}{9}$$

So this probability grows as $O(\lambda^3)$, so quite quickly.