Winter term 2017/2018

Mobile communications

Homework assignment 4: Spectral efficiency, bit errors in fading channels, DSSS

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Computer networks
group

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1. **Energy per bit vs. SNR: Spectral efficiency**

   (a) Modulation symbols are transmitted with a duration of $T_s = 4\,\mu$s. Each symbol carries 2 bits. Supposing we receive $2\,$mW, how much energy *per bit* E_b is at most available at the mobile terminal? For simplification, assume that each bit within the symbol carries the same energy. Which unit does your result have?

   (b) Now relate the energy per bit to the *average* noise power per Hertz, N_0. Discuss what the resulting E_b/N_0 ratio means for the quality of transmissions.

   (c) Explain why the bit error probability increases if higher-order modulations are used that carry many bits per symbol, e.g., Quadrature Phase Shift Keying (QPSK); all other parameters are assumed to be constant.

   (d) In which scenarios is it appealing to use higher-order modulations?

   (e) Assume you are using a modulation with M bits per symbols and a symbol duratin T_S and a symbol bandwidth of δf. Relate the E_b/N_0 ratio to the Signal-to-Noise Ratio (SNR). How can you interpret the factor between these two ratios?

2. **Bit error rates of BPSK in fading channels; diversity benefits**

   As we know from class, an AWGN channel (i.e., signal strength is fixed, noise is a Gaussian random variable) is charaterized by its *mean* SNR Γ. The bit error rate of BPSK over such a channel is approximated by the upper bound $P_{\mathrm{BER}}^{\mathrm{AWGN}} \leq \frac{1}{2}\exp(-\Gamma)$. We will work with this approximate error bound in the following.

   Now let us assume that the channel is subject to Rayleigh-fading. Recall from class: the *instantaneous* SNR in a Rayleigh-fading channel is a random variable γ, which is exponentially distributed with mean $1/\Gamma$, i.e., $\gamma \sim \exp(1/\Gamma)$.

We want to investigate the resulting bit error rate over such a channel.

(a) Give an upper bound on the bit error rate of BPSK in presence of fading, $P_{\text{BER}}^{\text{Rayleigh}}$, as a function of Γ. Which impact does fading have on the bit error rate? Compare with the given result for the AWGN channel!

Hint: Understand that a particular realization of a fading channel behaves, for a short time (e.g., a bit time), like an AWGN channel with the randomly chosen signal strength. Then, use the law of total probability to compute the BER of the fading channel as an appropriately weighted sum of BERs in such AWGN channels. Make suitable independence assumptions!

(b) We now employ L antennas at the receiver to benefit from receiver diversity (or any other form of L stochastically independent channels). We use maximum ratio combining, which allows us to model the *resulting* instantaneous SNR $\overline{\gamma}$ as a sum of random variables of each diversity branch, i.e., $\overline{\gamma} = \sum_{i=1}^{L} \gamma_i$.

What do you know about the CDF of the resulting random variable $\overline{\gamma}$? (This is easy to find in wikipedia or any stochastics textbook; look for the sum of iid exponential RVs.)

(For a bonus insight, do all the calculations below under the assumptions of selection combining.)

(c) Give an upper bound on the bit error rate of BPSK with L diversity branches in presence of fading, $P_{\text{BER}}^{\text{Rayleigh,L}}$, as a function of Γ. What is the effect of L?

(d) Is this just an effect of collecting more power via the multiple antennas? What would happen if the L channels to the L antennas were not stochastically independent, but rather perfectly identical?
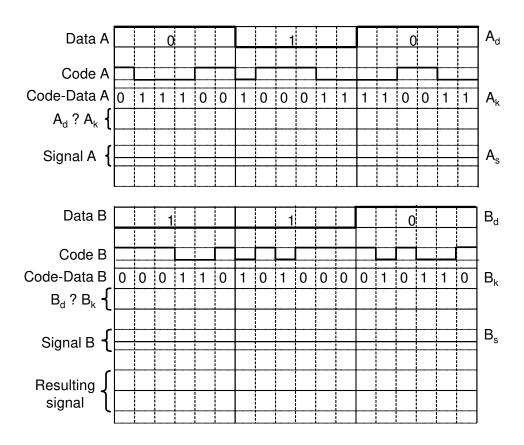
What would happen when you use L antennas with stochastically independent **AWGN** channels (as opposed to fading channels)? Does it make sense to use multi-antenna systems in AWGN scenarios?

(e) For a mean SNR $0\,\text{dB} \leq \Gamma \leq 30\,\text{dB}$, use a log-log graph (i.e., logarithmic scale on both the SNR and the BER axis) to plot the bit error rate for the AWGN channel and the Rayleigh channel with $1 \leq L \leq 3$ diversity branches. Discuss your results.

3. **Code division multiple access (CDMA)**

(a) The following figures show the bit strings $A_d = 010$ and $B_d = 110$ that are to be transmitted by the senders A and B, respectively. Their chipping sequences (code) are given both as signal and binary sequence A_k and B_k, respectively. The question mark "?" denotes an operator that you are free to choose.

Data A | 0 | 1 | 0 | A_d

Code A

Code-Data A: 0 1 1 1 0 0 1 0 0 0 1 1 1 1 0 0 1 1 | A_k

$A_d ? A_k$ {

Signal A { | A_s

Data B | 1 | 1 | 0 | B_d

Code B

Code-Data B: 0 0 0 1 1 0 1 0 1 0 0 0 0 1 0 1 1 0 | B_k

$B_d ? B_k$ {

Signal B { | B_s

Resulting signal {

For both senders, draw the binary sequences and signals after spreading ($A_d?A_k$ and $B_d?B_k$) as well as the resulting signal (by superposition) at the receiver.

(b) Show how you can extract the data bits from the received signal for sender A.

(c) Design your own *orthogonal* spreading code for two senders with a spreading factor of 8.

(d) Suppose that the wireless channel from these two senders to the receiver can support a data rate of 48 Mbits/s. What is the maximum achievable data rate for a particular sender when your code is used?

4. **Simulate Direct Sequence Spread Spectrum**

In this programming assignment, we develop a simple simulation of Direct-Sequence Spread Spectrum (DSSS) transmission. USe Python/numpy, Matlab, or some other tool of your choice.

Proceed as follows:

- Write a function that takes the following input:
  - An SNR level
  - A simple characterization of a multipath wireless channel as a sequence of attenuation values, one value for each delay of the channel by one chip length. (Recall the representation from slide 48 in Chapter 2a; extend that to multiple paths and different values of τ and h).
  - A chipping sequence
  - The number of chips per bit (or extract that from the chipping sequence)
  - The number of bits to simulate
- Inside this function, do the following:
  - Generate random bits to transmit, according to the number of bits. Iid bits 0 or 1.
  - Spread this sequence of bits using the chipping sequence
  - Add up multiple copies of this chip sequence, one copy per delay of the channel, multiplied with the corresponding attenuation value. Obtain the chipping sequence after transmission through a multi-path channel.
  - To this sequence of chips, add additive white Gaussian noise according to the given SNR level.
  - Despread this noisy, multi-path-channel-transmitted chipping sequence. (Hint: scalar product!)
  - Compute which bits are erroneous after despreading, calculate bit error ratio.
- Call this function for varying values of SNR.
- Plot the resulting BER-over-SNR curves. Hint: double-logarithmic plot should give nice figures. Hint: You probably need to simulate quite a lot of bits to see anything meaningful for high SNR values.

Experiment with various parameters, e.g., the channel characteristics,

5. **Simulate Direct Sequence Spread Spectrum, part 2: Rake reveicer**

In class, we briefly discussed the notion of a Rake receiver. Extend your function from the previous assignment to work as a Rake receiver. Use as additional parameters the desired number of rakes and the channel taps. Plot the same SNR curves as above, separately for varying number of rakes.

What do you observe when you use more rakes than the channel has taps?

If you are really courageous, try to estimate the channel taps from the received signal. This is not trivial, however!