Universität Paderborn
Computer networks group

Winter term 2017/2018

Mobile communications

Homework assignment 2: Decibels, antennas, Antennas

Due date: 2017-10-25

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1. **Expressing and working with ratios: decibels**

   We will often need to express large ratios. This is convenient to do with logarithmic expressions. The core concept here is to use so-called *decibel* units.

   (a) Read up on *decibels* (dB) on the Web

   (b) What is the role of a reference unit? For example, Milliwatt when taking about the ratio of power levels?

   (c) Understand the difference between dB and dBm.

   (d) Which other reference units for dB can you find?

   (e) Compute the ratio $\frac{A}{B} = \frac{1000}{10}$ in dB.

   (f) Compute the "natural" ratio that corresponds to $\frac{A}{B}_{\mathrm{dB}} = -30\,\mathrm{dB}$.

   (g) What does $A = -70\,\mathrm{dBm}$ mean? And $0\,\mathrm{dB}$?

   (h) Suppose you start with $A = 10\,\mathrm{W}$. What is the resulting power in W and in dBm if you loose 99.99% of that power?

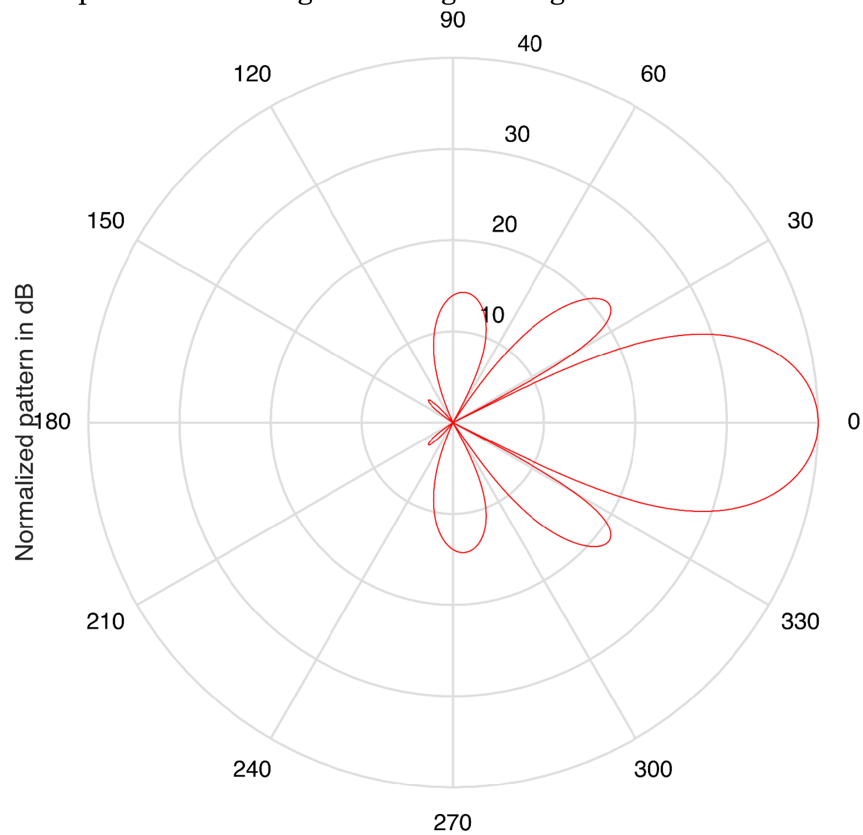
   (i) Express power loss and amplification in dB.

2. **Antennas and antenna gain**

   Regulating authorities (e.g. Bundesnetzagentur) require you to comply with the maximum **EIRP!** (**EIRP!**) of $1\,\mathrm{W}$. We start by assuming antennas with no gain at both transmitter and receiver.

   (a) Explain the concept of *antenna gain*. ***Solution:*** **Antenna gain** is the gain you archive from directing the radiation of an antenna compared to the ideal (point-) antenna.

(b) Interpret the following antenna gain diagram.



(c) Explain the concept of EIRP.

(d) What is the output power in dBm that must not be exceeded at the transmitter's radio front-end, assuming the transmit antenna has a gain of 20 dB? Base station and mobile terminal are assumed to have an ideal Line-Of-Sight (LOS) connection.

Solution: Antenna amplifies by factor 100 and the maximal signal power is 1000 mW, thus
$P_{out} = 1000\ mW/100 = 10\ mW.$

$$10mW = \frac{10}{1}mW = 10 \cdot \log_{10}\left(\frac{10}{1}\right) \text{dBm} = 10\text{dBm}$$

(e) Which received power in dBm is to be expected at the mobile terminal's radio front-end if the terminal employs an antenna with 3 dB gain, assuming that we *ignore* signal attenuation? ***Solution:***

$$\underbrace{10\text{dBm}}_{BS\ output} + \underbrace{20\text{dB}}_{Antenna\ Gain\ 1} + \underbrace{3\text{dB}}_{Antenna\ Gain\ 2} = 33\text{dBm}$$

$$33\text{dB} = 10^{\frac{33}{10}} \approx 2000\,\text{mW}$$

3. **Signal arriving over multiple paths: Multiple antennas and beamforming.**

   In Chapter 2, we looked at the interference of a signal with itself when it arrives over multiple paths. In this exercise, we look at a different source of multiple paths: Multiple antennas sending out the same signal. With cleverly chosen phase offsets when sending the signal, the interference of the multiple signals can be used to "steer" the transmitted power into various directions.

   To this end, implement the following simulation:

   - Place a rectangular grid of antennas in a plane. Experiment with different spacing. For example, a 18x1 or 8x2 array of antennas can provide good results.
   - Each antenna sends out the same carrier signal, but with different phase offsets.
   - Sample the points in the plane. Assume a line-of-sight connection to each antenna, calculate the resulting phase shift from each antenna and compute the resulting total signal and its strength.
   - Plot a 2D plot of the signal strength received at all sampled points in the plane.

   Solution: See beamforming.py

4. **Attenuation for two-ray ground reflection model**

   Derive an approximation formula, similar to the Friis equation, for the two-ray ground reflection model. What can you say about the path-loss exponent in the limiting case when d becomes large?

   More specifically, assume the following:

   - Transmit antenna is at height h_t
   - Receive antenna at height h_r
   - Distance between sender and receiver is d
   - Assume that d is much larger than the antenna heights
   - We use wavelength λ

   Hints:

   - The Taylor expansion for $\sqrt{1+x} = 1 + \frac{x}{2} + \ldots$ and $\mathrm{e}^x = 1 + x + \ldots$ will be useful; abort the expansion after the second term.

- Make sure to account for the phase shift of π at the ground reflection
- Look for an expression for the difference of the two path lengths (the direct line-of-sight path and the reflected path). Apply the Taylor expansion for $\sqrt{1+x}$ here.

  Hint: With a bit of clever drawing, you can do that by using Pythagoras twice.
- Then, look at the resulting signal. Understand that only the difference in the phases matters, not the absolute phases. Compute this phase difference from the difference in the path lengths and λ. Don't worry if this term turns a bit ugly.
- Now, look at the power of the received signal (it's square). You should find a term that (among others) depends on distance d in a way not foretold by Fries' equation!
- What does this result mean for large d?

Solution:

- The sketch of the setup is pretty similar to the one in the slides, Chapter 2a.
- The length of the direct path: $l = \sqrt{d^2 + (h_t - h_r)^2}$
- The length of the reflected path: $x + x' = \sqrt{d^2 + (h_t + h_r)^2}$
- Difference after some rewriting:

$$\Delta d = (x + x') - l = d\left(\sqrt{1 + \frac{(h_t + h_r)^2}{d^2}} - \sqrt{1 - \frac{(h_t + h_r)^2}{d^2}}\right)$$

  and after Taylor expansion of the roots and basic algebra:

$$\Delta d \approx 2\frac{h_t h_r}{d}$$

- The phase difference is hence:

$$\Delta\phi \approx 4\pi\frac{h_t h_r}{d\lambda}$$

- What is the received signal? We ignore the absolute path shift from transmitter to receiver; only the difference matters. Let $s(t)$ be the sent signal (details do not matter).

  Understand why the *amplitude* drops linearly with the distance!

$$r(t) = \frac{1}{l}s(t) + \frac{1}{x + x'}\mathrm{e}^{i(\Delta\phi+\pi)}s(t)$$

The additional shift by π is important! Let's look at the complex exponential term and do a Taylor expansion:
$e^{i(\Delta\phi+\pi)} = e^{i(\pi)} \cdot e^{i(\Delta\phi)} = -1 \cdot e^{i(\Delta\phi)} \approx -1 \cdot (1 + i\Delta\phi)$
Plug that in and simplify, assume $l \approx x + x' \approx d$ (true for large distances :

$$
\begin{aligned}
r(t) &\approx \frac{1}{l}s(t) + \frac{1}{x+x'}(-1 \cdot (1 + i\Delta\phi))s(t) && (1)\\
&= \left(\frac{1}{l} + \frac{1}{x+x'}(-1 \cdot (1 + i\Delta\phi))\right) s(t) && (2)\\
&\approx \left(\frac{1}{l} + \frac{1}{l}(-1 \cdot (1 + i\Delta\phi))\right) s(t) && (3)\\
&\approx \frac{-1 \cdot i\Delta\phi}{d} s(t) && (4)\\
&= \frac{-1 \cdot i4\pi h_t h_r}{d^2\lambda} s(t) && (5)
\end{aligned}
$$

- Now look at the power of $r(t)$, i.e., the square of its absolute value. Why the absolute value? Recall that we are dealing with complex numbers here; the power contained in the signal does not depend on the phase but only on the peak amplitude. That's the absolute value. Which then has to be squared, of course, to go form voltage to power.
  $P = \frac{(4\pi h_t h_r)^2}{d^4\lambda^2}$

Other derivations (e.g., without using the complex exponential channel coefficient) are of course possible and mostly a matter of taste.

The key point: in a two-ray ground reflection model, power decays with the **fourth power** of distance, not just the square.