A small example for coherence time

Holger Karl, Mobkom, WS09/10, U. of Paderborn

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1 Model assumption

Let us look at a very simple, two paths situation. A mobile receiver is distance d_1 away from the transmitter, moves away from it at speed v. The first path is direct line of sight of length d_1. The second path is reflected by an obstacle at distance d from the transmitter and at distance d_2 from the receiver (Figure 1).

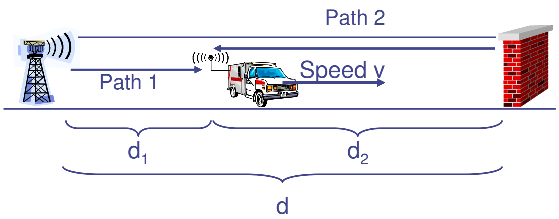

Figure 1: Scenario with a moving receiver and two paths

Since the receiver is moving, the actual distances are a function of time. Assuming the receiver is at the transmitter at time 0, it follows:

- $d_1(t) = vt$
- $d_2(t) = d + (d - vt) = 2d - vt$

We are interested in how long it takes for the received signal strength to vary substantially. Since we are only interested in small-scale effects, changes in the self-interferene of the signal are the only relevant contributing factor; we hence ignore changes in the amplitudes of the signals arriving over the two paths and treat them as equal.

Assumptions and notation:

- Speed of light $c = 300\,000\,000\,\mathrm{m/s}$
- Let f denote transmission frequency
- Let $\phi_1(t)$, $\phi_2(t)$ denote the phase at the receiver of the signal arriving over path 1, 2 at time t
- Terminal speed v is much smaller than speed of light c

2 Derivation

- Phases over the two paths:
  - Path 1: $\phi_1(t) = 2\pi\frac{vt}{\lambda} = 2\pi f t \frac{vt}{c}$
  - Path 2: $\phi_2(t) = 2\pi\frac{2d-vt}{\lambda} + \pi = 2\pi f \frac{2d-vt}{c} + \pi$

  (Note the addition π comes from the reflection on the obstacle.)

- Phase difference at time t: $\Delta\phi(t) = \phi_1(t) - \phi_2(t) = 2\pi f \frac{1}{c}(2vt - 2d)$. Recall that the phase difference is the relevant factor for small-scale fading!

- We are interested in the smallest Δt such that the phase difference of the two phases changes substantially, e.g., by $\frac{\pi}{2}$. We will call this the coherence time T_c. Formally:

  $$T_c(t) = \arg\min_{\Delta t} = \{\Delta t : \Delta\phi(t + \Delta t) - \Delta\phi(t) \geq \frac{\pi}{2}\}$$

  (It will turn out that $T_c(t)$ is actually independent of t, justifying to talk about the coherence time in this model.)

  $$\Delta\phi(t + \Delta t) - \Delta\phi(t) = 2\pi f \frac{2v}{c} \Delta t = \frac{\pi}{2}$$
  $$\Leftrightarrow \Delta t = \frac{1}{8f\frac{v}{c}} = \frac{1}{4(2f\frac{v}{c})}$$

- How does this relate to Doppler shift D_i along the two paths? Obviously, $D_1 = f - \frac{f}{c}v$ and $D_2 = f + \frac{f}{c}v$ (sign comes from the movement direction). Hence the difference between these two Doppler shifts is $D_2 - D_1 = 2\frac{f}{c}v =: D_s$, it is called the Doppler spread D_s.

  Relating this to the previous equation gives:

  $$T_c = \frac{1}{4D_s}$$

Hence: the coherence time is inversely proportional to four times the (absolute value of the) largest difference between any Doppler shifts. This holds also, in an order of magnitude sense, for more complex scenarios.

Intuition: Why to cast this as something related to Doppler shift/Doppler spread? It allows us to factor out the absolute values of movement speed vs. transmission frequency. Put another way, Doppler spread tells us how often (per unit time) the moving entity covers a half wavelength ($D = 2\frac{v}{\lambda}$). Only this ratio is relevant, not the absolute value of either v or λ.

3 Examples

The following Tables 1 and 2 give an idea of Doppler spread and coherence times at various combinations of movement speed and communication frequencies. Note that coherence time is shown in milliseconds, not seconds.

Table 4 gives a rough idea how many symbols can be transmitted inside a coherence time, for different signal bandwidths. Tread careful here; this makes serious over-simplifications. In particular, it assumes a flat fading (non-frequency-selective) channel, which is unrealistic to assume for large signal bandwidths.

Table 1: Doppler spread (in Hertz) at different speeds and frequencies

	Frequency [GHz]	0,5	1	2	4
km/h	Speed [m/s]				
1,08	0,3	10	20	40	80
2,16	0,6	20	40	80	160
4,32	1,2	40	80	160	320
8,64	2,4	80	160	320	640
17,28	4,8	160	320	640	1280
34,56	9,6	320	640	1280	2560
69,12	19,2	640	1280	2560	5120
138,24	38,4	1280	2560	5120	10240
276,48	76,8	2560	5120	10240	20480

Table 2: Coherence times (in millseconds!) at different speeds and frequencies

	Frequency [GHz]	0,5	1	2	4
km/h	Speed [m/s]				
1,08	0,3	25	12,5	6,25	3,125
2,16	0,6	12,5	6,25	3,125	1,5625
4,32	1,2	6,25	3,125	1,5625	0,78125
8,64	2,4	3,125	1,5625	0,78125	0,390625
17,28	4,8	1,5625	0,78125	0,390625	0,1953125
34,56	9,6	0,78125	0,390625	0,1953125	0,09765625
69,12	19,2	0,390625	0,1953125	0,09765625	0,048828125
138,24	38,4	0,1953125	0,09765625	0,048828125	0,024414063
276,48	76,8	0,09765625	0,048828125	0,024414063	0,012207031

Table 3: Examples for number of symbols inside a coherence time

Table 4: Examples for number of symbols inside a coherence time

		Doppler [Hz]	1	10	100	1000	10000
Signal bandwidth [kHz]	Symbol time [s]	Coherence Time [s]	0,25	0,025	0,0025	0,00025	0,000025
30	3,33333E-05		7500	750	75	7,5	0,75
200 (GSM)	0,000005		50000	5000	500	50	5
1000	0,000001		250000	25000	2500	250	25
20000	0,00000005		5000000	500000	50000	5000	500